Indirect Taxes in Multisectoral Macroeconomic Models

A Contribution from the Inforum group

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(Some) Matrices behind a Macroeconomic Multisectoral Model

- Supply and Use matrices
- Domestic and imported flows
- Symmetric Input-output matrices
- Margin matrix
- Excise Tax Matrix
- *ad valorem* tax flows (EU VAT Matrix)
- Bridge Matrices (Consumption, Investments)

Tax Flows in IO tables

Let us consider intermediate flows in terms of :

$\boldsymbol{q}$ the real flow

$\boldsymbol{p}$ the corresponding price

$\boldsymbol{s}$ the amount of excise tax

$\boldsymbol{t}$ the rate of the *ad valorem* tax

INTERMEDIATE FLOWS AT BASIC PRICES

Let us consider an intermediate consumption element X_{ij} at basic prices

$$X_{ij} = q_{ij} * p_i$$

where:

q_{ij} is the input i in sector j in‘real’ term

p_i is the basic price of input i

INTERMEDIATE FLOWS AT BASIC PRICES PLUS EXCISE TAX

Now we have

$$X_{ij} = q_{ij} * p_i + s_{ij}$$

where:

s_{ij} is the amount of excise tax on q_{ij}

The *ad valorem* Tax and Excise tax in the intermediate consumption flows

Now, the intermediate consumption flow is

$$X_{ij} = q_{ij}*p_i*(1+t_{ij})+ s_{ij}$$

or

$$X_{ij}=(q_{ij}*p_i + s_{ij})*(1 + t_{ij})$$

where:

$\boldsymbol{t_{ij}}$ is the *ad valorem* tax rate related to $\boldsymbol{q_{ij}*p_i}$

Peeling off the IO table

Reshaping the IO table thorough the removal of excise and ad valorem taxes

Excise tax flows in IO matrix

CI_1	CI_2	CI_3	FD	Q
$q_{11}p_1 + s_{11}$	$q_{12}p_1 + s_{12}$	$q_{13}p_1 + s_{13}$	$c_1p_1 + s^1$	$q_1p_1 + s_{11} + s_{12} + s_{13} + s^1$
$q_{21}p_2 + s_{21}$	$q_{22}p_{21} + s_{22}$	$q_{23}p_2 + s_{23}$	$c_2p_2 + s^2$	$q_2p_2 + s_{21} + s_{22} + s_{23} + s^2$
$q_{31}p_3 + s_{31}$	$q_{32}p_3 + s_{32}$	$q_{33}p_3 + s_{33}$	$c_3p_3 + s^3$	$q_3p_3 + s_{31} + s_{32} + s_{33} + s^3$
VA_1	VA_2	VA_3		
q_1p_1	q_2p_2	q_3p_3		
II_1	II_2	II_3		
qq_1	qq_2	qq_3		

II_i **excise tax row sum of sector** ***i***

qq_i **column sum of sector** i

$$II_i = s_{i1} + s_{i2} + s_{i3} + s^i$$

$$qq_i = q_i p_i + II_i$$

IO table after the removal of the excise flows

CI_1	CI_2	CI_3	FD	Q
$q_{11}p_1$	$q_{12}p_1$	$q_{13}p_1$	c_1p_1	q_1p_1
$q_{21}p_2$	$q_{22}p_{21}$	$q_{23}p_2$	c_2p_2	q_2p_2
$q_{31}p_3$	$q_{32}p_3$	$q_{33}p_3$	c_3p_3	q_3p_3
VA_1	VA_2	VA_3		
IA_1	IA_2	IA_3	IA^c	
q_1p_1	q_2p_2	q_3p_3		

The removal of excise tax flows makes room for the excise tax row in the Value Added zone

Where

$$IA_i = s_{1i} + s_{2i} + s_{13}$$

and

$$IA^c = s^1 + s^2 + s^3$$

the excise tax on final demand components **not necessarily recorded** in IO tables

Ad valorem tax flows in IO matrix

CI_1	CI_2	CI_3	FD	Q
$q_{11}p_1$	$q_{12}p_1$	$q_{13}p_1$	$c_1p_1(1+t_1)$	$q_1p_1 + VATRS_1$
$q_{21}p_2$	$q_{22}p_{21}$	$q_{23}p_2(1+t_2)$	$c_2p_2(1+t_2)$	$q_2p_2 + VATRS_2$
$q_{31}p_3$	$q_{32}p_3(1+t_3)$	$q_{33}p_3(1+t_3)$	$c_3p_3(1+t_3)$	$q_3p_3 + VATRS_3$
VA_1	VA_2	VA_3		
q_1p_1	q_2p_2	q_3p_3		
$VATRS_1$	$VATRS_2$	$VATRS_3$		
TOT_1	TOT_2	TOT_3		

Where VATRS stands for VAT Row Sum

$$VATRS_1 = c_1 p_1 t_1$$

$$VATRS_2 = c_2 p_2 t_2 + q_{23} p_2 t_2$$

$$VATRS_3 = c_3 p_3 t_3 + q_{32} p_3 t_3 + q_{33} p_3 t_3$$

$$VAT_1 = 0$$

$$VAT_2 = q_{32} p_3 t_3$$

$$VAT_3 = q_{23} p_2 t_2 + q_{33} p_3 t_3$$

$$VAT^c = c_1 p_1 t_1 + c_2 p_2 t_2 + c_3 p_3 t_3$$

IO table when VAT flows are removed

CI_1	CI_2	CI_3	FD	Q
$q_{11}p_1$	$q_{12}p_1$	$q_{13}p_1$	c_1p_1	q_1p_1
$q_{21}p_2$	$q_{22}p_{21}$	$q_{23}p_2$	c_2p_2	q_2p_2
$q_{31}p_3$	$q_{32}p_3$	$q_{33}p_3$	c_3p_3	q_3p_3
VA_1	VA_2	VA_3		
VAT_1	VAT_2	VAT_3	VAT^c	
q_1p_1	q_2p_2	q_3p_3		

Basic prices from the Leontief's price equation

$$\begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}_{basic} = \begin{bmatrix} 1-a_{11} & -a_{21} & -a_{31} \\ -a_{12} & 1-a_{22} & -a_{32} \\ -a_{13} & -a_{23} & 1-a_{33} \end{bmatrix}^{-1} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

Price equation with excise tax (rates)

$$a_{11}p_1 + a_{21}p_2 + a_{31}p_3 + a_{11}\alpha_1 + a_{21}\alpha_2 + a_{31}\alpha_3 + v_1 = p_1$$

$$a_{12}p_1 + a_{22}p_2 + a_{32}p_3 + a_{12}\alpha_1 + a_{22}\alpha_2 + a_{32}\alpha_3 + v_2 = p_2$$

$$a_{13}p_1 + a_{23}p_2 + a_{33}p_3 + a_{13}\alpha_1 + a_{23}\alpha_2 + a_{33}\alpha_3 + v_3 = p_3$$

Price equation with excise tax (rates)

In the language of matrix algebra:

$$\begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}_{excise} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}_{basic} + \begin{bmatrix} 1-a_{11} & -a_{21} & -a_{31} \\ -a_{12} & 1-a_{22} & -a_{32} \\ -a_{13} & -a_{23} & 1-a_{33} \end{bmatrix}^{-1} \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix}$$

Here the excise tax produces a clear tax schifting

Price equation with ad valorem taxes (rates) such as European VAT

$$a_{11}p_1 + a_{21}p_2 + a_{31}p_3 + v_1 = p_1$$

$$a_{12}p_1 + a_{22}p_2 + a_{32}p_3(1+t_3) + v_2 = p_2$$

$$a_{13}p_1 + a_{23}p_2(1+t_2) + a_{33}p_3(1+t_3) + v_3 = p_3$$

Firms may not be entitled to full deduction

- Firms exempted from VAT have no right to deduct the VAT paid on their taxed purchases
- Some intermediate transactions may be allowed only for partial deduction
- “Forfeit” (or “ standardized”) systems
- The Agricultural special system
- Government Agencies

Price equation with ad valorem taxes (rates) such as European VAT

- In the language of matrix algebra:

$$\begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}_{vat} = \begin{bmatrix} 1-a_{11} & -a_{21} & -a_{31} \\ -a_{12} & 1-a_{22} & -a_{32}(1+t_3) \\ -a_{13} & -a_{23}(1+t_2) & 1-a_{33}(1+t_3) \end{bmatrix}^{-1} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

- The ad valorem tax shifting is not additive to the basic price solution

Long run Price formation with *ad valorem* and excise tax in sector j

$$p_j = \sum_{i=1}^{n} a_{ij} p_i (1 + t_{ij}) + \sum_{i=1}^{n} a_{ij} \alpha_{ij} + v_j$$

Indirect tax flows in the Intermediate Consumption Table of the Italy IO Matrix

Some statistics

- Excise taxes are recorded in 73% flows
- Ad valorem taxes are recorded in 64% flows

Indirect Taxes between Intermediate Consumption and Final Demand

Intermediate Consumption is the tax base of:

- 14% of the *ad valorem* taxes
- 45% of the excise taxes

VAT in the Intermediate Consumption

- This ad valorem tax applied by the European Union Member States has been designed to be an indirect tax on Personal Consumption Expenditure.
- So called VAT ‘impurities’ extend VAT tax base to intermediate consumption flows

- About VAT ‘impurities’, see Bardazzi,Grassini,Longobardi (1991) “Value-Added Taxes and Other Indirect Taxes in EEC Country Model. The Italian Case “, Economic Systems Research, n. 1, pp. 37-47

Main VAT bases in Intermediate Consumption

82% of the VAT tax yield comes from 5 sectors out of 59 (Eurostat standard Input-output EU country matrices)

- Financial intermediation services
- Insurance and pension funding services
- Services auxiliary to financial intermediation
- Public administration and defence services
- Health and social work services

Main VAT bases in Intermediate Consumption

Being mostly industry specific, the treatment of an *ad valorem* tax such as the European VAT deserves a a special attention in modelling price equations

After all

The construction of time series of excise and *ad valorem* tax matrices and the design of indirect tax scenarios is a challenging task up to the macroeconomic multisectoral model builder.

Thank you for your attention

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