

# The IdLift Model: A Brief Description

## GENERAL STRUCTURE OF THE I-O TABLE

The input-output table for the *IdLift* model is a 97 sector commodity by commodity table. It is derived from the 1987 Benchmark I-O study using the purification algorithm described by Almon (1999), with certain redefinitions of sectors. The first 87 sectors comprise the private economy, 88 and 89 are government enterprises, and 93 is government industry. The other sectors from 90 to 97 are special industries. The sectoring classification is listed at the back of this document.

## FINAL DEMAND EXPENDITURE COMPONENTS

### *Categories to be Bridged*

- Personal Consumption Expenditures (92 categories), passed through consumption bridge (97x92)
- Equipment Investment (55 categories), passed through capital flow matrix (97x55)
- Private Construction (25-order, 1<sup>st</sup> 19, plus category 25 are private), passed through construction bridge (97x25)
- Federal Defense Consumption and Investment (25 categories), passed through defense bridge (97x25)
- Federal Nondefense Consumption and Investment (8 categories), passed through nondefense bridge (97x8)
- State & Local Education Consumption and Investment (7 categories), passed through education bridge (97x7)
- State & Local Health Consumption and Investment (7 categories), passed through health bridge (97x7)
- State & Local Other Consumption and Investment (7 categories), passed through other bridge (97x7)
- Government Construction (5 categories), passed through government structures bridge (25x5), then passed through construction bridge.

### *Final Demands by I-O Commodity*

- Personal Consumption Expenditures
- Producers' Durable Equipment
- Private Structures by Commodity
- Government Structures by Commodity
- Inventory Change
- Federal Defense
- Federal Nondefense
- State & Local Education
- State & Local Health
- State & Local Other
- Exports
- Imports

### *Value Added Components (51 industries)*

- Labor Compensation
  - Wages and Salaries
  - Supplements
    - Employer Contributions for Social Insurance
    - Benefits
      - Health Benefits
      - Pension Benefits
      - Other Benefits
- Return to Capital
  - Corporate Profits
  - Proprietors' Income
  - Corporate Capital Consumption Allowances
  - Noncorporate Capital Consumption Allowances
  - Corporate Inventory Valuation Adjustment
  - Noncorporate Inventory Valuation Adjustment
  - Net Interest Payments
  - Business Transfer Payments
  - Rental Income of Persons
  - Government Subsidies Less Surplus
- Indirect Business Taxes
  - Excise and Sales Tax
  - Value Added Tax
  - Energy Tax
  - Other Indirect Tax

### *Prices*

- Producers' Prices (97 commodities)
- Consumer Prices (92 categories)
- Equipment Investment Prices (55 industries)
- Construction Prices (25 categories), used for both private and government construction
- Foreign Price of Imports (97 commodities)
- Foreign Price of Competing Exports (97 commodities)
- Federal Defense Deflators (25 categories)
- Federal Nondefense Deflators (8 categories)
- State & Local Education Deflators (7 categories)
- State & Local Health Deflators (7 categories)
- State & Local Other Deflators (7 categories)

## OTHER MATRICES USED IN THE MODEL

- *Consumption Bridge Matrix* - This bridge links personal consumption expenditures by 92 consumer categories to personal consumption at the commodity level, stripping off trade and transportation margins. It is also used in the reverse direction to form consumer prices from producers' prices.
- *Investment Bridge Matrix* – The investment bridge matrix, or “capital flow table” links investment by 55 purchasing industries to producers' durable equipment at the commodity level, allocating expenditures by type of investment good, and stripping off trade and transportation margins. This matrix is also used in the reverse direction to form equipment investment prices at the 55 industry level.
- *Construction Bridge Matrix* – Links construction by 25 categories of public and private to the 97 commodity level. It distributes construction expenditures by type of material, as well as allocating margin expenditures. Value added for construction is in the new construction row (7). This bridge serves double duty, bridging private construction and public construction in two different places in the model. It is also used to form private and government construction prices.
- *Defense Bridge Matrix* – Links defense consumption and investment expenditures by 25 categories (similar to those found in the national accounts) to expenditures by input-output commodity. It is used in the reverse direction to obtain prices by defense expenditure category.
- *Nondefense Bridge Matrix* – Links federal nondefense consumption and investment expenditures by 8 categories to the commodity level.
- *State & Local Education Bridge Matrix* – Links 7 categories of education consumption and investment to the commodity level.
- *State & Local Health Bridge Matrix* – Links 7 categories of education consumption and investment to the commodity level.
- *State & Local Other Bridge Matrix* – Links 7 categories of education consumption and investment to the commodity level.
- *Product to Industry Bridge* – Relates commodity value added (97 commodities) to industry value added (51 industries), and vice-versa.

## THE REAL SIDE

### Personal Consumption

The personal consumption equations have been estimated using PADS (the Perhaps Adequate Demand System) for 92 categories of consumption expenditure, divided into 9 groups. A cross-sectional equation is used to get the impacts of demographic variables and to estimate the Engel curve for each good. The result of this equation is then used as the “income” term in the time-series equations,

The cross-section equations are of the following form:

$$C_i^* = \left( a + \sum_{k=1}^K b_k Y_k + \sum_{l=1}^L d_l D_l \right) \left( \sum_{g=1}^G w_g n_g \right)$$

where:

$C_i^*$  = household consumption expenditures on good  $i$

$Y_k$  = the amount of per-capita income (expenditures) within income category  $k$

$D_j$  = dummy variable used to show membership in the  $j$ th demographic group

$n_g$  = number of household members in age category  $g$

$w_g$  = adult equivalency weights

$K$  = the number of income groups

$L$  = the number of demographic categories

$G$  = the number of age groups

The demographic categories  $D$  include region of residence, family size, working status of spouse, college education, and age of household head, all estimated using dummy variables. The left-hand side result of this equation is known to us by the name of “C-star”. The two terms in the first factor of the equation are the “piecewise linear Engel curve” and the demographic term. The second factor of this equation is the age-weighted population.

The PADS equations take the form:

$$x_i(t) = \left( a_i(t) + b_i \left( \frac{C_c^*}{P} \right) + c_i \Delta \left( \frac{C_c^*}{P} \right) \right) \left( \frac{p_i}{P} \right)^{I_i} \prod_{k=1}^n \left( \frac{p_i}{p_k} \right)^{-I_i s_k} \left( \frac{p_i}{p_G} \right)^{-m_g} \left( \frac{p_i}{p_g} \right)^{n_g}$$

where:

$C_c^*$  = cross-section expenditures for corresponding cross-section category  $c$

$P$  = overall consumption price index

$p_i$  = the price of good  $i$

$p_G$  = the average price index of group  $G$

$p_g$  = the average price of subgroup  $g$

$I_i$  = individual good price response parameter

$m_g$  = group price response parameter

$n_g$  = subgroup price response parameter

## Exports

The Inforum bilateral trade model (BTM) forecasts bilateral trade flows for 120 commodities, by 14 trading partners and two regions covering the rest of the world. The forecasting equations are based on annual OECD and UN data on international trade by commodity and country of origin. For 13 of the trading partners, Inforum models are available, which forecast imports and domestic prices endogenously.

As an alternative to taking exports exogenously from BTM, we have also estimated export equations similar to those used in the previous Inforum international system. The form of these export equations is:

$$E_{it} = \left( b_0 + b_1 \sum_{k=1}^{15} \frac{w_k m_{kit}}{m_{ki0}} \right) \left( \frac{p_{it}^d}{p_{it}^f} \right)^h$$

where:

$E_{it}$  = U.S. exports of commodity  $i$  for year  $t$

$w_k$  = the fraction of U.S. imports which went to trading partner  $k$  in the base year (1987)

$m_{kit}$  = imports to country  $k$  for commodity  $i$  in year  $t$

$m_{ki0}$  = imports to country  $k$  for commodity  $i$  for the base year

$p_{it}$  = a moving average of the domestic price

$f_{it}$  = a moving average of a weighted exchange rate adjusted price of competing

exporters.

In contrast to the older LIFT equations of this form, these equations were all estimated with data from the BTM database, and the forecasts from BTM are still used to obtain the foreign import demands and foreign prices. However, this form allows the IdLift equations to adjust independently from BTM. These equations, like all others in the model, can be turned on or off.

## Equipment Investment

IdLift forecasts purchases of equipment investment for 55 industries comprising the U.S. economy. Sales of investment goods at the 97 commodity level are then determined by passing equipment investment by buyer through the investment bridge matrix. The investment equations are estimated in a two-stage, three equation framework. Factor demands for equipment capital, labor and energy are estimated simultaneously. In the first stage, optimal capital-output, labor-output and energy-output ratios are estimated. In the second stage, the parameters from the first stage are treated as fixed, and equations for net investment, labor and energy are estimated. In this stage, investment is based upon a distributed lag on past changes in output, whereas labor and energy demand are based upon a distributed lag of levels of output.

The first stage equation for the optimal capital-output ratio is obtained by using Shephard's Lemma with a generalized Leontief cost function with equipment, labor and energy to obtain:

$$\left( \frac{K}{Q} \right)_t^* = e^{-a_{k1}t + a_{k2}t^2} \left[ \sum_{j=K,L,E} b_{kj} \left( \frac{p_j}{p_K} \right)^{\frac{1}{2}} \right]$$

where:

$K$  = capital stock

$Q$  = output

$\left( \frac{K}{Q} \right)_t^*$  = the optimal capital-output ratio

$p_j$  = price of factor  $j$ , where  $j = K, L, E$

$t_1$  = time trend

$t_2$  = 2<sup>nd</sup> time trend, starting in 1970

This equation is used in a three equation system to fit the historical capital-output, labor-output and energy-output ratios.

The equation for net investment is derived from the first difference of the optimal capital stock equation and can be expressed by:

$$N_t = e^{-a_k t_1 + a_k z t_2} \left[ \sum_{j=K,L,E} b_{Kj} \left( \frac{p_j}{p_K} \right)^{\frac{1}{2}} \right] \sum_{j=0}^3 w_j^K \Delta Q_{t-j}$$

where:

$N_t$  = net investment

$\Delta Q$  = the change in output

The price of capital  $p_K$  is the commonly used neoclassical measure:

$$p_K = \frac{p_{eq} (r + dep)(1 - tz - C)}{1 - T}$$

where:

$p_{eq}$  = the equipment price deflator for this purchasing industry

$r$  = the real AAA bond rate

$dep$  = the average depreciation rate for this industry

$T$  = the effective corporate tax rate

$z$  = the present value of depreciation of one dollar worth of investment

$c$  = the investment tax credit

Replacement investment is determined by multiplying the optimal capital output ratio by the losses to capacity (as the level of optimal output given the current capital stock) occurring in the current year. Since the optimal capital-output ratio is a function of relative prices, price change affects both the demand for net investment and replacement investment.

### Private Construction by Type

The equations for private purchases of plant and other structures are for 19 categories of construction available from the National Income and Product Accounts. These purchases are generally aggregated into two major divisions. Residential construction consists of single- and multi-family homes, and additions and alterations. Non-residential construction is comprised of a motley of different types: hotels, industrial buildings, office buildings, schools, farm buildings, oil wells, railroads, telephone and communications, electric and gas utilities, and petroleum

pipelines. The residential equations are estimated in per-capita form, and based on disposable income per capita, the mortgage interest rate, and the percent of households of home-buying age. The non-residential constructions are each unique, but often based on the output of the related industry or group of industries, the relative price of the related industry (especially in the case of oil and drilling rigs), interest rates, and a variety of demographic variables. Some of the equations also use a measure of capital stock of structures of that type, to model replacement investment needs.

## Government Consumption and Investment

Only consumption purchases are included in the presentation of the government budget. However, investment is accumulated into a book value stock, and the depreciation of this stock is the capital consumption of government. This capital consumption is part of current consumption expenditures.

IdLift has an accounting for the government capital stock, and capital consumption equations that relate capital consumption to the calculated depreciation from this stock. This capital consumption is part of the current government budget, and also shows up in the non-corporate capital consumption vector in the income side of the model.

Aside from capital consumption, the other categories of government consumption and investment are exogenous. There are five categories of government spending: federal defense, federal nondefense, state & local education, state & local health and state & local other. For each category, there is a bridge that translates purchases by type to purchases by input-output commodity. For example, the bridge for federal defense spending has 97 rows and 25 columns.

## Output Calculation

To calculate output by industry, the sources of demand by commodity arising from personal consumption, equipment investment, structures investment, government and exports are first added together to form a vector of final demand. Then the Seidel iterative technique is used to jointly solve for domestic output, imports and inventory change by industry.

The Seidel algorithm in IdLift solves by iteratively calculating

$$q_i^{k+1} = \frac{\sum_{j < i} a_{ij} q_j^{k+1} + \sum_{j > i} a_{ij} q_j^k + v_i - m_i + d_i}{1 - a_{ii}}$$

where:

$q_i^k$  = the solution for output of commodity  $i$  in iteration  $k$

$a_{ij}$  = the direct input-output coefficient from commodity  $i$  to commodity  $j$

$f_i$  = the sum of final demand for commodity  $i$  excluding imports, inventory change, and discrepancy

$v_i$  = inventory change for commodity  $i$

$m_i$  = imports for commodity  $i$

$d_i$  = final demand discrepancy for commodity  $i$

A triangulation ordering typically optimizes the loop over commodities, so that the solution is as recursive as possible. Within the loop for each commodity, the equations for inventory change and imports are also calculated.

One complication is the final demand discrepancy, which is the approach we have taken to handle the inconsistencies between input-output tables, final demand data and output data. The discrepancy  $\mathbf{d}$  is formed in the last year of output data as  $\mathbf{d} = \mathbf{q} - \mathbf{A}\mathbf{q} - \mathbf{f} - \mathbf{v} + \mathbf{m}$ , where  $\mathbf{d}$  is the final demand discrepancy,  $\mathbf{q}$  is output,  $\mathbf{A}$  is the direct requirements matrix,  $\mathbf{f}$  is other final demand, and  $\mathbf{v}$  is inventory change and  $\mathbf{m}$  is imports. In the forecast periods, the discrepancy is kept constant, and added back in during the Seidel solution.

The import equations are based on domestic demand, calculated in the regressions as  $\mathbf{dd} = \mathbf{q} + \mathbf{m} - \mathbf{x}$ , where  $\mathbf{dd}$  is domestic demand, and  $\mathbf{x}$  is exports. However, when solving for imports,  $\mathbf{m}$  and  $\mathbf{q}$  are still unknown, so we use instead  $\mathbf{dd} = \mathbf{A}\mathbf{q} + \mathbf{f} + \mathbf{v} + \mathbf{d}$ , where  $\mathbf{q}$  is the best current guess of output for the current iteration.

### Inventory Change

The form of the inventory equations is:

$$v_i = \mathbf{b}_0 + \mathbf{b}_1 USE_i + \mathbf{b}_2 \Delta USE_i$$

where:

$$USE_i = q_i(1 - a_{ii}) + m_i - v_i$$

### Labor Productivity, Average Hours Worked and Employment

The labor productivity equations we use can be written as:

$$\ln\left(\frac{q}{h}\right) = \mathbf{b}_0 + \mathbf{b}_1 t_1 + \mathbf{b}_2 t_2 + \mathbf{b}_3 qup + \mathbf{b}_4 qdown$$

where:

$q$  = output

$h$  = hours worked

$t_1$  = a simple linear time trend

$t_2$  = a second time trend, starting in 1972

$qup = dq$ , when positive, 0 otherwise

$qdown = -dq$ , when  $dq$  negative, 0 otherwise

$dq = \ln q_t - \ln qpeak_{t-1}$

$qpeak_t = q_t$ , if  $q_t > qpeak_{t-1}(1 - spill)$ , otherwise  $= qpeak_{t-1}(1 - spill)$

$spill$  = depreciation rate of capacity, both in the sense of capital and “hoarded” labor



The second time trend picks up a change in the rate of labor productivity growth that began sometime in the 1969 to 1973 period. The *qup* and *qdown* terms model the increase in labor productivity that is observed in periods of increasing output, and vice-versa. This phenomenon of procyclical labor productivity is sometimes associated with “labor hoarding”, where firms retain trained workers in periods of slack output. When output increases again, they put the hoarded labor back to work before making new hires. The *qpeak* variable attempts to measure capacity output, both in the sense of capital and “hoarded” labor.

The equations for hours worked relate annual hours worked per employee to a time trend and cyclical changes in output, much like the labor productivity equations. Therefore, they are also essentially time trends.

Hours worked by industry can be obtained by dividing output by productivity. Then employment by industry is simply total hours divided by average hours worked per employee. This yields hours and employment for all industries comprising the private economy. Public sector employment, domestic employment and rest of world employment are specified exogenously.

## THE PRICE-INCOME SIDE

### Hourly Labor Compensation

In the price-income side of the model, the wage equations are really the backbone, for labor compensation comprises the largest share of income, and the most significant contributor to the core inflation rate is wage inflation. It is perhaps appropriate that it is here that we introduce the growth of money into the model, as the long-run determinant of average inflation.

The wage equations in IdLift are estimated in a stacked system, and the left-hand side variable is the percent change in the hourly labor compensation in each industry. An alternative formulation, used in LIFT, which we plan to test against the current version, is to estimate a *relative wage* equation. In this specification, an equation is estimated for the aggregate wage, and then the sectoral wage equations are estimated as ratios of the sectoral wage over the aggregate wage.

The wage equations are estimated as:

$$dw_i = \mathbf{b}_1 pj + \mathbf{b}_{2i} dlp_i + \mathbf{b}_{3i} supply$$

where:

$dw_i = \ln(wag_{it}) - \ln(wag_{i,t-1})$ , and  $wag_i$  is the labor compensation rate for industry  $i$

$pj$  = a five-year weighted average of the percent change in the growth of M2/real GNP

$supply = ([pc(p_{ag}) - pc(p_{gnp})] + [pc(p_{oil}) - pc(p_{gnp})]) / 2$ , the average difference of the rate of price change in agriculture and crude petroleum with respect to GNP inflation

$dlp_i = \ln(q_{it} / h_{it}) - \ln(q_{i,t-1} / h_{i,t-1})$ , the percent change in industry labor productivity

The stacked system estimation imposes the same coefficient on the  $pj$  variable across all industries, letting the parameters of the other variables differ by industry. Although the main

motive of introducing the monetary aggregate into this equation is to provide a mechanism whereby money affects prices, there is also a rationale supported by anecdotal evidence. This evidence suggests that when the money supply increases more rapidly, it stimulates aggregate demand. This creates pressure in the labor markets, which puts upward pressure on wages. An alternate story is the rational expectations version, that workers bid up wages in expectation of the higher inflation which they know will be generated by the money supply growth.

## Profits

The profits equations are another very important part of the income side of the model. Corporate profits are a large and volatile component of value added by industry. The equations are based on profits as a mark-up over labor compensation, with a response to variables representing both demand and cost changes affecting the markup rate. The data available from the Gross Product Originating (GPO) is corporate profits before tax, with no capital consumption allowance adjustment, and no inventory valuation adjustment. For the equations, these two variables are added to profits before tax to get a measure that is more of an economic measure of costs than a purely accounting measure. Formally:

$$\Pi_i = CPR_i + IVA_i + \frac{CCC_i}{CCCSUM} CCADJ$$

where  $\Pi_i$  = economic profits

$CPR_i$  = corporate profits before tax

$IVA_i$  = inventory valuation adjustment

$CCC_i$  = corporate capital consumption allowance

$CCCSUM$  = aggregate corporate capital consumption allowance

$CCADJ$  = aggregate capital consumption allowance adjustment

The ratio in the last term is used because the capital consumption adjustment is not available by industry. The dependent variable in the equations is the ratio of economic profits to labor compensation for each industry.

Corporate profits depend on both changes in demand and changes in unit costs. The demand effect is measured by both industry-specific and aggregate measures of demand. The industry-specific variable is the change in output. The aggregate measure is a measure of the tightness in the labor market, the ratio of the natural rate of unemployment to the actual rate of unemployment squared. The input cost variable is the unit intermediate cost per unit of output. The general form of the profits equations is:

$$\frac{\Pi_i}{LAB_i} = f(PCOUT_{it}, PCOUT_{i,t-1}, GNP GAP, NAUNSQ, PCUC_{it}, PCUC_{i,t-1}, PCOIL, PCAG)$$

where

$PCOUT$  = the percent change in industry output

$GNP GAP$  = the GNP gap (  $\frac{GNP}{POTGNP} * 100$  )

$NAUNSQ = \left[ \frac{NAIRU}{UN} \right]^2$ , where *NAIRU* is the natural rate of unemployment, and *UN* is the actual rate

*PCUC* = percent change in unit intermediate cost

*PCOIL* = percent change in oil prices

*PCAG* = percent change in agricultural prices

Note that not all variables will appear in every equation. In particular, we will choose either the *GNPGAP* or the *NAUNSQ* variable, as they are both measures of tightness in the aggregate economy.

### Proprietors Income

Proprietor income by industry, or noncorporate profits, the returns to entrepreneurs by industry. For the few industries in which proprietors income is large, it appears to be linked to the average level of labor compensation. Many of the equations relate proprietors income to the average level of labor compensation over the past three years. Some additional variables used in the other equations are: percent change in GDP, industry change in output, the industry capital-output ratio, and the square of the ratio of the natural rate of unemployment to the actual rate.

### Capital Consumption Allowances

Capital consumption allowances, or depreciation, comprise a fairly large share of total value added. The *IdLift* model contains separate sets of equations for corporate and noncorporate capital consumption allowances. Although the capital consumption allowances calculated by the BEA differ from accounting measures of depreciation, they still appear to be closely related to a capital stock that would be formed by cumulating nominal values of investment. The equations for capital consumption are fairly simple, and for most industries, fit very well. The explanatory variables are nominal stocks of equipment and structures capital.

### Producers' Prices

One way of calculating prices is to forecast value added per unit of output, or value added directly, and then calculate prices through the second fundamental input-output identity:

$$\mathbf{p}' = \mathbf{p}'\mathbf{A} + \mathbf{v}$$

An alternative approach is to forecast wages, then forecast prices as markups over wages plus intermediate costs, and then determine the sum of the other categories of value added as a residual. Forecasts of profits, proprietors' income and capital consumption allowances are then scaled into consistency with the price forecast. Both versions of the price income model are present in *IdLift*, and can be switched in or out. The price equations are markup equations, which are estimated as:

$$mrkup_{it} = \mathbf{b}_0 + \mathbf{b}_1 gnpgap_t + \mathbf{b}_2 pcgap_t + \mathbf{b}_3 pcgap_{t-1} + \mathbf{b}_4 pcq_t + \mathbf{b}_5 pcq_{t-1} + \mathbf{b}_6 p_{ag} + \mathbf{b}_7 p_{oil}$$

where:

$$mrkup_{it} = \frac{p_{it}}{(lab_{it}/q_{it}) + \sum_i a_{ij} pw_i}, \text{ the ratio of price over unit labor costs plus unit}$$

intermediate cost

$gnpgap_i$  = as described above

$pcgap_i$  = percent change in GNP gap

$pcq_{it}$  = percent change in output of commodity  $i$

$p_{ag}$  = agriculture price

$p_{oil}$  = crude oil price

$pw_i$  = average of domestic and import prices (weighted by import share)

Therefore, both an aggregate variable (GNP gap) and an industry variable (change in output) are used in explaining price, as well as the raw materials supply price variables, which are significant in explaining the increase in inflation stemming from major supply shocks.

### The Product-Industry Bridge

The input-output table and the vectors of final demand expenditures are compiled for the model at the 97 commodity level. However, the categories of value added are compiled for 51 industries. When value added is used to calculate prices using the equation  $p' = p'A + v$ , it must first be converted to the commodity level, which is basis of the A matrix and the price vector. If the price equations are used, or if prices are fixed exogenously, then industry value added must be modified so as to be consistent with these commodity prices. For these purposes and others, we make use of the product to industry bridge.

The product-industry bridge is derived from the make matrix, which in the case of Idlift has 51 rows (industries) and 97 columns (commodities). The bridge is stored and updated in flows, which enables converting in both directions.

In the base year, which we designate as year 0, the following relationship holds:

$$\sum_{i \in \text{industry}} g_i^0 \frac{V_{ij}^0}{\sum_i V_{ij}^0} = vaa_j^0, \text{ for all } j \text{ commodities}$$

where:

$g^0$  = the total vector sum of 13 categories of value added by industry, including a discrepancy column, so calculated to that the above relationship holds

$V^0$  = the product-industry bridge matrix for the base year

$vaa$  = the commodity level vector of value added allocated. In the base year, this is the vector of value added in the input-output table, which satisfies the fundamental identities

An important concept to grasp is that of real value added weighted output, or *revawo*. Denote the ratio  $V_{ij}^0/q_j^0$  the *value-added fraction*. This is the share of total output of commodity  $j$  in the base year, accounted for by output produced in industry  $i$ . To transfer output through the bridge, we multiply each output by its value added fraction, and distribute the result to the various industries in their proportion to that commodity in the base year. The result is *revawo*, and may be written:

$$revawo_i^t = \sum_j \frac{V_{ij}^0}{q_j^0} q_j^t$$

In the years beyond the base year, the  $\mathbf{V}$  matrix is updated as follows:

$$V_{ij}^t = V_{ij}^0 \frac{q_j^t}{q_j^0} \frac{g_i^t}{revawo_i^t}$$

In this equation  $g_i^t$  is the sum of industry value added plus the value added discrepancy column. This equation adjusts the bridge matrix so as to prorate each industry's value added back to products in accordance with that product's contributions to the industry *revawo*. To obtain value added by product we then just sum down the column of the updated bridge.

If prices are forecast directly, as we are currently doing, we first calculate commodity value added from prices and intermediate cost and price discrepancy:

$$vaa_j = p_j q_j - \sum_i p w_i a_{ij} q_j - pdisc_j$$

where:

$p w_i$  = the weighted foreign and domestic price

$pdisc_j$  = the price discrepancy, calculated as  $\mathbf{p}' - \mathbf{p}'\mathbf{A} - \mathbf{v}$  in the last year of price data

Next the *vaa* vector is passed through the bridge in the reverse direction to obtain value added by industry. Finally, several components of value added are scaled so that total value added by industry is equal to the desired total. These components are corporate profits, proprietor's income, and corporate and non-corporate consumption allowances.

## THE ACCOUNTANT

Even if forecasting the prices directly we still need to develop consistent estimates of the other components of income besides labor compensation. Rental income, interest income, proprietor's income and that part of profits paid out as dividends all contribute to personal income. Corporate profits taxes and indirect business taxes contribute to the revenue of governments. Capital consumption allowances and retained earnings are part of business savings, which is an important component of national savings. It is the job of the Accountant to aggregate the components of value added on the price income side and obtain the aggregate variables needed to state the relationships between GNP, national income, personal income and disposable income. Along the way, the important components of the household and government

balance sheets are obtained. The Accountant also forms aggregates of the expenditure vectors on the real side of the model, and forms implicit deflators from current and constant price aggregates.

The operation of the Accountant can be viewed in several stages. In the first stage, several aggregates of income are created from the price-income side, and summed to form nominal GNP. Factor imports are added and factor exports subtracted to obtain GDP. Supplements to labor compensation such as employer contributions are first aggregated across industries and then distributed to different funds based on exogenous ratios. Components of other labor income are also calculated in this stage. The second stage computes capital consumption adjustments, and forms proprietor's income, rental income and profits with and without capital consumption adjustments and inventory valuation adjustments. In the third stage, national income is formed by summing labor compensation, proprietor's income, rental income, corporate profits and interest income. Corporate profits, net interest and contributions for social insurance are subtracted, and transfer payments, personal interest income, personal dividend income, and business transfer payments are added back in to obtain personal income. This stage is quite lengthy due to a detailed set of identities and regression equations calculating the different components of transfer payments and interest payments. In the fourth stage, the components of federal and state and local receipts and expenditures are calculated. In the fifth and final stage, personal taxes and non-tax payments are removed to obtain disposable income. At this point, the loop in the model has been closed, and it returns to the real side, with the new guess at disposable income.

## SEVERAL IMPORTANT MACRO EQUATIONS

### The Personal Savings Rate

This equation has gone through many revisions since the first version of the model. Common variables used were the unemployment rate, usually with a coefficient of close to  $-1$ , and the share of motor vehicles in personal consumption, also with a coefficient of  $-1$ . The argument for using motor vehicles share was that consumers may view purchases of consumer durables generally as a substitute for savings. The reason for using the unemployment rate is to introduce an automatic stabilizer in the model.

The current equation is rather simple,

$$savrat = \mathbf{b}_0 + \mathbf{b}_1(gnpgap-100) + \mathbf{b}_2rtb + \mathbf{b}_3dummy$$

where:

$rtb$  = the treasury bill rate

$dummy$  = a dummy variable starting in 1986

The parameter  $\mathbf{b}_1$  is constrained to be about  $.75$ . The dummy variable takes the value 1 after 1986, when a significant decrease in the U.S. personal savings rate was observed. Despite its importance in the structure of the model, the savings rate equation is in trouble. The U.S. has recently seen a personal savings rate become *negative*. The economy is very strong now, so this would usually be a cause for the savings rate to be higher. However, there has always been

something problem of direction of causality in our specification of the savings equation. In the current U.S. situation, something is making consumer spending very strong, and this is both stimulative to the economy, as well as resulting in negative measured savings. It could be the wealth effect from the strong stock market, or the price effect of cheap imports, but any of our traditional forms of savings rate equation cannot explain the current phenomena. As soon as the savings rate equation rate in the model comes into action, the savings rate jumps up about 7 points, and therefore, this is a variable which needs to be modified in the first few years of the forecast.

## Interest Rate Equations

Interest rates are important to the expenditure side of the model, in that they affect construction and equipment investment, the most cyclical components of expenditure. Previous versions of the model had interest rates in the personal consumption equations as well, and we intend to revisit this specification. Interest rates are also important in the income side of the model, where they affect net interest income, and in various other variables, such as interest paid by federal and state and local governments, and consumer payments of interest to business.

The two most important interest rates are the short- and long-term Treasury bill rates. The short-term rate, or 3-month Treasury bill rate, has the following equation:

$$rtb = \mathbf{b}_0 + \mathbf{b}_1 \text{expinf} + \mathbf{b}_2 \text{grmbase} + \mathbf{b}_3 (\text{gnpgap}[1] - 100) + \mathbf{b}_4 \text{funds}$$

Where:

*expinf* = "Expected" inflation, formed as a three-year average of inflation

*grmbase* = Percent change in the real monetary base

*gnpgap[1]* - 100 = Percentage of actual GNP over potential GNP

*funds* = The share of equipment investment, structures investment plus the federal deficit over GNP

The expected inflation rate illustrates the Fisher effect, where expected increases in inflation tend to get translated into higher nominal interest rates. The base money growth variable is especially effective in predicting interest rates over the last several years. Since 1990, the monetary base has been growing fairly quickly, whereas M2 has been growing slowly. The GNP gap variable is used to pick up cyclical demand effects on interest rates. The last variable, called *funds*, is intended to capture financial market demand pressure on interest rates. Higher values of this variable indicate a higher demand for finance. The equation for the long-term Treasury bill rate includes the short-term rate, expected inflation, and the *funds* variable. The other interest rate variables are the commercial paper rate, the mortgage rate, and the AAA bond rate. The first two are regressed directly on the long-term rate, and the bond rate is regressed on the long-term rate and the share of profits in GNP.