Effective Rates of Sectoral Productivity Change

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ABSTRACT - The *effective rates* of change in sectoral productivity depend not only on the direct change of productivity originating in the sector in question, but also on productivity gains/losses in the production of at least part of its inputs. Both the direct rate of sectoral productivity growth as well as all the effective rates derived in the static input-output framework are equal to the relative decrease in the production price of the output of the sector. For the traditional direct measure all input prices have to be treated as exogenous constants. In the case of the effective measures, prices of the inputs regarded as produced are treated as variables depending on production technology, while the rest of the input prices are still exogenous constants.

The *fully effective rate* of sectoral productivity growth is derived from the price equations of the closed dynamic input-output model after that the absolute level of the relative prices has been fixed. It is equal to the rate of decrease in the sectoral production price when the prices of all inputs, human capital and human time included, depend on production technology. It can be interpreted as the rate of change in the quantity of growth potential of the economy used up per unit of output in the sector. The overall rate obtained as a weighted sum of the fully effective sectoral rates is equal to the relative change in the growth potential of the economy.

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1. Introduction

The impact of technical change on the efficiency of producing the final output of a sector is not limited to the direct productivity change originating in it. The effective rate of productivity change introduced by Hulten (1978) takes into account the fact that this impact depends also on productivity gains/losses obtained in the production of intermediate inputs for the sector. The term "effective" was used by Hulten to point out the analogy with effective tax incidence. In fact it was obvious already from the aggregation rule derived by Domar (1961) that the contribution of a sector to the overall productivity growth depends besides the direct productivity growth within it also on the changes in the efficiency in the production of its intermediate inputs. Hulten (1978) proved that a weighted average of his effective rates produced the same aggregate rate of TFP growth as the Domar (1961) aggregation of the direct rates.

The idea of treating intermediate inputs as produced inputs in sectoral productivity analysis has been developed further in the input-output framework by Peterson (1979), Wolff (1985) and Cas and Rymes (1991). The logical next step, taken by all of them, is to regard also physical capital as produced. When restricted to replacement of fixed capital (Wolff, 1985) this can be done in a rather straightforward manner, provided that depreciation is in fixed proportion to output. Including also that part of capital input, which materialises in capital stocks being tied up in the production process, is more problematic. Peterson (1979) suggests this to be done by treating gross additions to capital stock as produced input. Cas and Rymes (1991) have a same type of approach.

In this paper it is demonstrated that the closed dynamic input-output model provides an appropriate framework for treating even capital stocks tied up in the production process as produced inputs in productivity analysis. It makes it also possible, and even necessary, to include human capital and human time as well (see e.g. Aulin-Ahmavaara, 1997). This on the other hand is also an obvious conclusion from the tendency of regarding more and more inputs as produced in sectoral productivity analysis. Therefore it is only logical for this paper to end up introducing a sectoral productivity measure based on the closed dynamic input-output model.

The paper starts by showing that the traditional direct rate of sectoral TFP growth is, in static input-output framework, equivalent to the relative decrease in the price of output of the sector when all the input prices are treated as exogenous constants. It can be derived directly from the price equation of the input-output model. It goes on by showing that all the different effective rates sectoral productivity growth can also be derived in the same way directly from this same price equation. The only difference is that in case of the effective rates the prices of all the inputs regarded as produced are treated as variables depending on production technology, while the rest of the input prices are still treated as exogenous constants.
Finally, in section 5 an effective rate of sectoral productivity growth is derived from the price equation of the closed dynamic input-output model after that the absolute level of the relative prices has been fixed. In this case the prices of all inputs, human

capital and human time included, are treated as variables. Therefore this rate is here called the fully effective rate of sectoral productivity change. The overall rate obtained as a weighted average of these fully effective sectoral rates is shown to be equal to the relative change in the balanced rate of growth possible for an economy utilising the production technology represented by the two technological matrices.

2. Direct measure: All input prices exogenous

The derivation of the traditional measure of sectoral productivity change can be based on the following accounting identity

(1) $$p_i X_i = \sum_j p_j X_{ji} + wL_i + rK_i, \quad j, i = 1 \cdots n,$$

where X_i is the gross output of sector i;

X_{ji} is the intermediate input from sector j to sector i;

L_i is the labour input to sector i;

K_i is the capital input to it,

p_i is the unit price of its output

w is the uniform price of labour

r is the uniform price of capital input

Competitive equilibrium being assumed prices of labour and capital can be assumed to be homogenous across sectors. Variation in the prices with the type of labour or capital inputs can always be allowed by including different categories of these inputs.

The rate of direct sectoral productivity growth is traditionally defined as the difference of the growth rates of the output and the inputs:

(2)

$$d\log t_j = d\log X_j - (p_j X_j)^{-1}\left[\sum_i (p_i X_{ij}\, d\log X_{ij}) + wL_j\, d\log L_j + r K_j d\log K_j\right].$$

Since all the derivatives in this paper are with respect to time the notation has been simplified by writing dz instead of dz/dt.

Adding together the sectoral accounting identities in (1) yields:

(3) $$\sum_i p_i X_i = \sum_i \sum_j p_j X_{ji} + w\sum_i L_i + r\sum_i K_i.$$

On the other hand the balance of supply and demand in the product and factor market requires that

(4) $$X_j = Y_j + \sum_i X_{ji}\ ; \quad \sum_j L_j = L \quad \text{and} \sum_j K_j = K,$$

where Y_j is the final output of sector j

L is the total supply of labour and

K is the total supply of capital input.

Substituting (4) to (3) results, since the two sums representing the total value of intermediate inputs cancel out, in the following accounting identity for a national economy:

(5) $\sum_i p_i Y_i = wL + rK.$

The rate of overall TFP growth at the national level can now be defined analogously to the sectoral rates as the difference between the growth rates of outputs and the inputs:

(6) $d\log T = \left(\sum_i p_i Y_i\right)^{-1}\left[\sum_i (p_i Y_i d\log Y_i) - wLd\log L - rKd\log K\right].$

Or since

(7) $dz = z(d\log z)$

equivalently utilising matrix notation

(8) $d\log T = (\mathbf{pY})^{-1}(\mathbf{p}d\mathbf{Y} - wdL - rdK),$

where $\mathbf{p}$ is the row vector of prices p_i and

$\mathbf{Y}$ is the column vector of final output by sector.

The Domar (1961) aggregation gives the overall rate as a weighted sum of the sectoral rates:

(9) $d\log T = (\mathbf{pY})^{-1}(d\log \mathbf{t})\hat{\mathbf{p}}\,\mathbf{X},$

where $\mathbf{t}$ is the row vector of sectoral rates of productivity change;

$\hat{\mathbf{p}}$ is the diagonal matrix with the elements of vector $\mathbf{p}$ on the diagonal and

$\mathbf{X}$ is the column vector of sectoral outputs.

For the proof of equation (9) the balance equations are differentiated to get:

(10) $dX_j = dY_j + \sum_i dX_{ji}$; $\sum_j dL_j = dL$ and $\sum_j dK_j = dK_j$.

Substituting (2) in (9) and making use of (7) and (10) results in

(11) $d\log T = (\mathbf{pY})^{-1}\left[\sum_j p_j dY_j + \sum_j\sum_i p_j dX_{ji} - \sum_j\sum_i p_i dX_{ij} - wdL - rdK\right]$

where the second and third terms in brackets are both equal to the total value of the change in the consumption of intermediate products.

Furthermore by introducing the input coefficients

(12) $a_{ij} = (X_j)^{-1}X_{ij}$, $l_j = (X_j)^{-1}L_j$ and $k_j = (X_j)^{-1}L_j$

the balance equations in (4) can be rewritten as follows:

(13) $\mathbf{X} = \mathbf{Y} + \mathbf{AX}$, $\mathbf{L} = \mathbf{lX}$ and $\mathbf{K} = \mathbf{kX}$,

where $\mathbf{A}$ is the $n \times n$ matrix of coefficients a_{ij} ;

$\mathbf{l}$ is the row vector of coefficients l_i and

$\mathbf{k}$ is the row vector of coefficients k_i .

Substituting the logarithmic derivatives

$d\log X_{ij} = d\log a_{ij} + d\log X_j$,

$d\log L_j = d\log l_j + d\log X_j$ and $d\log K_j = d\log k_j + d\log X_j$.

into (2) yields by (1)

(14) $d\log t_j = -(p_j X_j)^{-1}\left[\sum_i p_i X_{ij}\, d\log a_{ij} + wL_j\, d\log l_j + rK_j d\log k_j\right].$

From this form it is easy to see that the index of sectoral factor productivity change actually is a continuous analogue of the Leontief (1953) index of structural change. The latter, in its original discrete form, is equal to the weighted average of the relative changes in the input coefficients (both intermediate and primary) with the average shares of the inputs in total input (or output) as weights.

The formula in (14) can be simplified, by utilising (7) and (10), to

(15) $d\log\mathbf{t} = -(\mathbf{p}d\mathbf{A} + wd\mathbf{l} + rd\mathbf{k})\hat{\mathbf{p}}^{-1}$.

On the other hand dividing (1) through by X_i produces together with (13) the following price equations

(16) $\mathbf{p} = \mathbf{pA} + w\mathbf{l} + r\mathbf{k}$.

On the assumption that all the input prices in (16) are exogenous constants differentiating it (with respect to time) and multiplying both sides by $-\hat{\mathbf{p}}^{-1}$ gives the equation (15) directly. Thus we have:

Result 1. The traditional direct measure of sectoral TFP growth in (2) is equivalent to the relative decrease in the production price (unit production cost) of the output of the sector in equation (16), when all the input prices are treated as exogenous constants

2. Effective Rates (1): Prices of intermediate inputs determined by production technology

Part of the output of a sector can be used as an intermediate input within the sector itself. This adds to the impact of technological change on the productivity growth in the sector. When the change in production price of the output of the sector used as input by the sector itself is taken into account, the relative change in the unit production cost becomes:

$$d\log p'_j = (p_j)^{-1}\left[\sum_i p_i d a_{ij} + wdl_j + rdk_j + \mathrm{a}_{\mathrm{jj}} dp'_j\right].$$

This is related to the direct traditional measure of TFP change as follows:

(17) $$d\log p'_j = (1-a_{jj})^{-1}(p_j)^{-1}\left[\sum_i p_i d a_{ij} + wdl_j + rdk_j\right] = -(1-a_{jj})^{-1} d\log t_j .$$

The right hand side of the second equation in (17) is the negative of the measure of total factor productivity growth in which the productivity gains/losses within the sector *j* itself have been taken into account, suggested by Cas and Rymes (1991, p.36). The derivation of the equation (17) shows that it is equal to the relative decrease in the unit production cost. In this case the price of the input originating from the sector itself is allowed to change along with the production technology, while all the other input prices are treated as constants.

But all the intermediate inputs are actually produced by the production system and their prices are determined simultaneously by the system of equations in (16). Thus the logical next step is to let all the product prices to change while keeping the factor prices constant. For this purpose the product prices are expressed as a function of the input coefficients:

(18) $\mathbf{p} = w\mu + r\gamma$,

where

(19) $\mu = \mathbf{l}(\mathbf{I}-\mathbf{A})^{-1}$ and $\gamma = \mathbf{k}(\mathbf{I}-\mathbf{A})^{-1}$.

The elements μ_j and γ_j of the two row vectors μ and γ in (19) represent the total requirements of labour and capital per unit of final output in sector *j*.

Differentiating (18) and postmultiplying by $\hat{\mathbf{p}}^{-1}$ yields

(20) $d\log\mathbf{p}^* = (wd\mu \; + rd\gamma \;)\hat{\mathbf{p}}^{-1} = -d\log\mathbf{t}^*$.

Here $\mathbf{t}^*$ is the row vector with elements identical with the rate of "inverse" technical change as defined by Wolff (1985). Substituting (18) in (20) reveals that they are also identical with the TFP growth rates for the vertically integrated sectors defined by Peterson (1979). On the other hand differentiating the original price equation (16) keeping only the prices of capital and labour constant gives

$d\log\mathbf{p}^* = (\mathbf{p}d\mathbf{A} + wd\mathbf{l} + rd\mathbf{k} + d\mathbf{p}^*\mathbf{A})\hat{\mathbf{p}}^{-1}$

and furthermore

(21) $d\log\mathbf{p}^* = (\mathbf{p}d\mathbf{A} + \mathbf{w}d\mathbf{l} + rd\mathbf{k})(\mathbf{I}-\mathbf{A})^{-1}\hat{\mathbf{p}}^{-1} = -d\log\mathbf{t}^*$

The equivalence with (20) can be established by substituting

$$dl = d\mu(\mathbf{I}-\mathbf{A}) - \mu d\mathbf{A} \quad \text{and} \quad dk = d\gamma(\mathbf{I}-\mathbf{A}) - \gamma d\mathbf{A},$$

calculated from (19), into (21) and then simplifying and making use of (18).

The relationship between the traditional direct measure and the effective rate of sectoral productivity change can easily be established. Post multiplying both sides of equation (15) by $\hat{\mathbf{p}}$ and substituting the result in (21) gives

(22) $d\log\mathbf{t}^* = d\log\mathbf{t}\,\hat{\mathbf{p}}(\mathbf{I}-\mathbf{A})^{-1}\hat{\mathbf{p}}^{-1}$,

which is in agreement with Wolff (1985) and shows that the effective rate defined in (21) is also equivalent with the "new measure" of Cas and Rymes (1991, p. 40) in case only intermediate inputs are treated as produced inputs. Thus we have:

Result 2. All the effective rates of sectoral productivity growth in which intermediate inputs are treated as produced inputs are equal to the relative decrease in the unit production cost or production price when the prices of intermediate inputs are allowed to be determined by production technology. The formula for this effective rate of sectoral TFP growth could be derived directly by differentiating the original price equation (16) by treating the prices of intermediate inputs as variables depending on production technology and the prices of primary inputs as exogenous constants.

Note. At the level of a national economy intermediate inputs cancel each other out. Therefore treating them as produced inputs does not have any effect on the measure of overall TFP growth in (6). Solving equation (22) for $d\log\mathbf{t}$ and substituting the result into (9) gives by (13) the following aggregation rule:

(23) $d\log T = (\mathbf{pY})^{-1} d\log\mathbf{t}^*\hat{\mathbf{p}}\mathbf{Y}$.

This is identical with the aggregation rule given by Hulten (1978) for his effective rates. It is also identical with the aggregation rule given by Peterson (1979) for this TFP growth rates for the vertically integrated sectors, as well as the one given by

Wolff (1985) for the rates of "inverse" technical change and Cas and Rymes (1991) for their "new measure" (in case only intermediate inputs are treated as produced inputs). This is natural since all these measures are, as was shown in this section, equivalent to the negative of the rate of change in sectoral production price when prices of intermediate inputs are determined by production technology while prices of primary inputs are exogenous constants.

3. Effective rates (2): Capital as produced input

The definition of capital input has so far not been discussed in this paper. It will be discussed here only to the extent that is necessary for the exposition of the ideas presented in this paper. Obviously capital input has to be, in one way or other, related to the capital stock involved in the production process. Likewise it is evident that the physical capital input consists besides fixed capital also of inventories of raw materials as well as those of semi finished and even finished goods indispensable for running the production process. The level of the inventories is part of the description of the production technology. In fact as Domar [1961, p. 722] has pointed out "the distinction between raw materials and short-lived capital is arbitrary, and it can become troublesome if the length of the time period is changed (from a year to a decade and vice versa, for instance)". As to the intangible assets the ones that are consumed in the production process and therefore have to be reproduced sooner or later should be included. Those assets that are used to improve production technology, i.e. to change the momentary value of productivity which is being measured, should obviously be excluded.

Some authors draw an analogy between capital input and labour input. Capital input is seen as services of capital in the same way as labour input involves the services of work force (Jorgenson et al., 1991, p. 109). There is however a significant difference between these two types of input. The production unit that uses labour input does not own the (human) capital needed to produce that input. It buys part of the services of the unit of human capital needed to produce labour. It is of course possible to buy services of physical capital as well. But then they are treated as an output of a separate activity, called operating leasing, in the SNA. In this paper labour is regarded as an output of the household sector. Accordingly the human capital needed to produce labour belongs to the households. In this sense labour input has a closer resemblance to operating leasing than to the capital input proper. Therefore human capital that is tied up in the workforce will not, in this paper, be treated as part of the capital input until labour is treated as a product, which takes place in the next section.

Capital input can be divided into two different parts: 1) depreciation of fixed capital and 2) the presence of capital at the production process, i.e. keeping the capital stocks (fixed capital and inventories) tied up in the production process. Utilisation rate of capital stock is here treated as a property of the production technology. The failure of a sector to adapt its capital stocks to the variation of final demand, or vice versa, is indicative of inefficiency. This means that the possible effect of the utilisation rate on the decay of fixed capital has to be taken into account in measuring depreciation. On the other hand all the capital stocks that are present in the production process and are not at the same time used to produce something else are part of the capital stock tied up in the production process.

If that part of capital stock that has depreciated during a period is replaced during the same period, it is rather natural to treat depreciation or replacement requirements as a produced input. This can be done, following Wolff (1985), by adding to the production system a new sector. The output of this sector is "capital". It uses only products of the rest of the sectors as inputs. It doesn't need any capital or labour inputs itself. The balance equations of product market in the augmented system are

$$(24)\qquad \begin{bmatrix}\mathbf{Y}-\mathbf{hgX}\\ \mathbf{0}\end{bmatrix}=\left(\mathbf{I}-\begin{bmatrix}\mathbf{A} & \mathbf{h}\\ \mathbf{g} & \mathbf{0}\end{bmatrix}\right)\begin{bmatrix}\mathbf{X}\\ \mathbf{gX}\end{bmatrix}$$

where $\mathbf{g}$ is the row vector showing the amount of capital stock replaced per unit of sectoral output and
$\mathbf{h}$ is the column vector showing the inter-industry inputs required to produce one unit of capital stock.

The first equation in (24) can be rewritten as follows:

$$(25)\qquad \tilde{\mathbf{Y}}=\mathbf{Y}-\mathbf{h}(\mathbf{gX})=(\mathbf{I}-\mathbf{A}-\mathbf{hg})\mathbf{X},$$

where $\tilde{\mathbf{Y}}$ is that part of the original final output vector that is not needed to replace depreciation of fixed capital.

The price equations of this augmented system are:

$$(26)\qquad [\tilde{\mathbf{p}}\quad \tilde{\mathbf{r}}]=[\tilde{\mathbf{p}}\quad \tilde{r}]\begin{bmatrix}\mathbf{A} & \mathbf{h}\\ \mathbf{g} & \mathbf{0}\end{bmatrix}+w[\mathbf{l}\quad \mathbf{0}]$$

or equivalently

$$(27)\qquad \tilde{\mathbf{p}}=\tilde{\mathbf{p}}\mathbf{A}+\tilde{r}\mathbf{g}+w\mathbf{l}\quad \text{and}\quad \tilde{r}=\tilde{\mathbf{p}}\mathbf{h}$$

or, after substitution of the second equation to the first one:

$$(28)\qquad \tilde{\mathbf{p}}=\tilde{\mathbf{p}}(\mathbf{A}+\mathbf{hg})+w\mathbf{l}.$$

In (27) the price of capital $\tilde{r}$ is expressed as a weighted sum of product prices with the intermediate input coefficients to the sector producing capital as weights. Since capital is now treated as produced input these intermediate input coefficients are part of the description of production technology in (28) and the price of capital does not explicitly appear in it.

Differentiating (28) on the assumption that all the input prices are constant yields

$$(29)\qquad d\log\tilde{\mathbf{t}}=-[\tilde{\mathbf{p}}d\mathbf{A}+\tilde{\mathbf{p}}d\mathbf{hg}+wd\mathbf{l}]\,\hat{\tilde{\mathbf{p}}}^{-1}$$

which is analogous to the direct sectoral measures of productivity growth in (15). Total direct and indirect requirements of the only primary input, labour per unit of final output, now are equal to

$$\tilde{\mu}=\mathbf{l}(\mathbf{I}-\mathbf{A}-\mathbf{hg})^{-1}.$$

The effective sectoral rates of TFP growth analogously to (20) and (21) are equal to

$$(30)\qquad d\log\tilde{\mathbf{t}}^{*}=-d\log\tilde{\mathbf{p}}^{*}=-wd\tilde{\mu}\hat{\tilde{\mathbf{p}}}^{-1}=(\tilde{\mathbf{p}}d\mathbf{A}+\tilde{\mathbf{p}}d(\mathbf{hg})+wd\mathbf{l})(\mathbf{I}-\mathbf{A}-\mathbf{hg})^{-1}\hat{\tilde{\mathbf{p}}}^{-1}.$$

Thus we have:

Result 3. The effective sectoral rate of productivity growth is equal to the rate of decrease in the production price even when, besides ordinary intermediate inputs, also replacement requirements of fixed capital are treated as produced inputs. In this case the prices of both ordinary intermediate and replacement inputs are allowed to change with production technology and only the price of the sole primary input labour is kept constant.

Note 1. In the measures defined by equations (29) and (30) the replacement of fixed capital is treated similarly to intermediate inputs. The only difference is that fixed capital of all the production sectors has exactly the same input structure given by the vector **h**. This shortcoming can be easily avoided by replacing the matrix **hg** by a new matrix of replacement coefficients **R.** Replacement requirements of fixed capital are however in fixed proportion to output, and accordingly can be treated as a feature of production technology only, if either geometric decay of capital is assumed or if the economy is in a state of balanced growth. In the latter case the decay of capital stock can assume any pattern (see Aulin-Ahmavaara, 1991).

Note 2. The second part of capital input, i.e. the presence of capital stock at the production process, is not taken into account in the measures defined by the equations (29) and (30). It is possible to give these equations a different interpretation. Following Peterson (1979) the vector **g** can be interpreted as the gross increase in capital stock per unit of output in each sector. But in this case coefficients **g** of course depend besides the production technology also on the growth of output in each of the sectors.

Note 3. A similar approach to include the second part of capital input has been presented by Cas and Rymes (1991), who want to treat the services of (net) capital stock as produced inputs. Their price equations (p. 23) take, after having been simplified by merging capital consumption allowances with ordinary intermediate inputs, the following form

(31) $\mathbf{p} = \mathbf{pA} + \mathbf{pE} + w\mathbf{l}$,

where **E** is the matrix of inter industry coefficients of capital services per unit of output. The direct rates of sectoral productivity growth derived from this price equation, analogously to (15) now are

$$d\log \mathbf{t} = -(\mathbf{p}d\mathbf{A} + \mathbf{p}d\mathbf{E} + wd\mathbf{l})\hat{\mathbf{p}}^{-1}$$

The effective rates when only the prices of primary inputs are held constant are analogously to equations (21) and (30) equal to

$$d\log \mathbf{t}^* = (\mathbf{p}d\mathbf{A} + \mathbf{p}d\mathbf{E} + wd\mathbf{l})(\mathbf{I} - \mathbf{A} - \mathbf{E})^{-1}\hat{\mathbf{p}}^{-1},$$

where the relationship between the direct measure and effective one is the same as the one given by Cas and Rymes (1991, p. 43). On the hand (Cas and Rymes, 1991, p.23)

(32) $\mathbf{E} = r\mathbf{B}$

where r is the uniform rate of return on capital and **B** the matrix of inter industry stock coefficients. This indicates that in the productivity measures above the rate of return on capital is treated as a technological property of the production system. The problem is how justify this.

4. Fully effective rates: All prices determined by production technology

If however also labour (or more generally human time) is produced within the system, then the input-output model can be closed. The production of labour can simply be included by adding one row and column for the household sector to matrices **A** and **B**. But besides the services of human capital, i.e. labour, households also produce human capital itself. The production of a unit of human capital, capable of producing labour, can take up to 25 years. Therefore a more accurate description of the production technologies of different types of labour and human capital (see e.g. Aulin-Ahmavaara, 1991) would improve the precision of productivity measurement.

The output proportions $\overline{\mathbf{X}}$ and the balanced rate of growth λ of a closed system are, if the economy is capable of balanced growth, uniquely determined by the technological matrices **A** and **B** as follows:

(33) $(\mathbf{I}-\mathbf{A})\overline{\mathbf{X}} = \lambda \mathbf{B}\overline{\mathbf{X}}$

The equilibrium price proportions $\overline{\mathbf{p}}$, as well as the normal rate of profit λ, are determined by the technological matrices in the dual system:

(34) $\overline{\mathbf{p}} = \overline{\mathbf{p}}\mathbf{A} + \lambda \overline{\mathbf{p}}\mathbf{B}$.

In a steady state equilibrium the normal rate of profit, determined by price equations, and the balanced rate of growth, determined by the output equations, are equal. Therefore they are both here denoted by λ.

The formula for direct sectoral rates, with all the input prices held constant, can now, analogously to (15), be written as follows:

(35) $d\log\overline{\mathbf{t}} = -(\overline{\mathbf{p}}d\mathbf{A} + \lambda \overline{\mathbf{p}}d\mathbf{B})\hat{\overline{\mathbf{p}}}^{-1}$

This time however labour and human capital, are treated in the same way as the rest of the intermediate and capital inputs. Accordingly these measures of productivity cover also the sectors producing them. The matrix **A** includes also the coefficients of replacement for fixed capital. Capital input consists only of the stocks of different types of fixed capital and inventories, with human capital and labour included. (Labour and other services are tied up in semifinished products.) Prices of capital inputs consist of two components, viz. the production prices $\overline{\mathbf{p}}$ and the normal rate of profit λ.

The application of the aggregation rule in (9) to equations (35) gives the following overall measure of TFP growth:

(36) $d\log\overline{T} = -\overline{\mathbf{p}}(d\mathbf{A} + \lambda d\mathbf{B})\overline{\mathbf{X}}(\overline{\mathbf{p}}\overline{\mathbf{Y}})^{-1}$.

On the other hand multiplying both sides of (33) by $\overline{\mathbf{p}}$ and solving for λ yields:

(37) $\lambda = \overline{\mathbf{p}}(\mathbf{I}-\mathbf{A})\overline{\mathbf{X}}(\overline{\mathbf{p}}\mathbf{B}\overline{\mathbf{X}})^{-1}$.

Furthermore differentiating the equations in (33), multiplying both sides by $\overline{\mathbf{p}}$ and making use of (34) yields as a linear approximation:

(38) $d\lambda = -\overline{\mathbf{p}}(d\mathbf{A} + \lambda d\mathbf{B})\overline{\mathbf{X}}(\mathbf{p}\mathbf{B}\overline{\mathbf{X}})^{-1}$.

Finally, division by (37) yields in view of (36):

(39) $d\lambda / \lambda = d\log\overline{T}$,

Thus the relative change in the balanced rate of growth is approximately equal to the rate of overall TFP growth calculated on the basis of the balanced growth solution. Since the balanced rate of growth is inversely related to the average turnover time (see Brody, 1970) the overall productivity growth is also equal to the rate of decrease in the average turnover time, i.e. to the success of the economy in economising time.

Since the vector $\overline{\mathbf{p}}$ only gives the price proportions, it is necessary to make the prices absolute, in order to be able to calculate their relative changes. This can be done by setting the following condition:

(40) $\sum p_i = 1$,

which together with the rewritten price equations

(41) $\overline{\mathbf{p}} = \overline{\mathbf{p}}\lambda\mathbf{B}(\mathbf{I}-\mathbf{A})^{-1}$

define a system of $n+1$ equations with $n+1$ unknowns. Substituting (41) into (40) gives

(42) $\lambda = \left(\sum_i \left[\overline{\mathbf{p}}\mathbf{B}(\mathbf{I}-\mathbf{A})^{-1}\right]_i \right)^{-1}$,

and substituting (42) into (41) results in

(43) $\overline{\mathbf{p}} = \overline{\mathbf{p}}\mathbf{B}(\mathbf{I}-\mathbf{A})^{-1}\left(\sum \left[\overline{\mathbf{p}}\mathbf{B}(\mathbf{I}-\mathbf{A})^{-1}\right]_i\right)^{-1}$.

Hence

(44) $d\log\overline{\mathbf{p}} = d\log\left[\overline{\mathbf{p}}\mathbf{B}(\mathbf{I}-\mathbf{A})^{-1}\right] - d\log\sum\left[\overline{\mathbf{p}}\mathbf{B}(\mathbf{I}-\mathbf{A})^{-1}\right]_i$

and , by (42),

(45) $d\log\overline{\mathbf{p}} = d\log\left[\overline{\mathbf{p}}\mathbf{B}(\mathbf{I}-\mathbf{A})^{-1}\right] + d\log\lambda$.

Equations (44), and (45), divide the relative change in the production prices into two parts, keeping their sum equal to one. The first part is the change in the production prices caused by technological change without imposing the condition that the sum of the changed prices should be equal to one again i.e. it is the relative change in the absolute prices. The second part imposes this condition.

The interpretation of the first part as a measure of technological change becomes obvious by considering the following method of solving the price equations in (41) empirically. Because $\mathbf{B}(\mathbf{I}-\mathbf{A})^{-1}$, denoted here by **D,** is a positive, Frobenius, matrix we have (see Tsukui and Murakami, 1979 and Aulin-Ahmavaara, 1987) for any semipositive row vector $\mathbf{s}(t)$:

(46) $\lim_{t\to\infty}(\mathbf{s}\mathbf{D}^t)\left(\sum_j\left[\mathbf{s}\mathbf{D}^t\right]_j\right)^{-1} = \overline{\mathbf{p}}\left(\sum_j \overline{p}_j\right)^{-1} = \overline{\mathbf{p}}$,

Let us assume that after T iterations the convergence is close enough for us to write:

(47) $\overline{\mathbf{p}} \cong \mathbf{s}\mathbf{D}^T\left(\sum_j\left[\mathbf{s}\mathbf{D}^T\right]_j\right)^{-1}$

and furthermore

(48) $d\log\overline{\mathbf{p}} \cong d\log(\mathbf{s}\mathbf{D}^T) - d\log\sum_j\left[\mathbf{s}\mathbf{D}^T\right]_j$.

In view of equations (43) and (47) the vector $\overline{\mathbf{p}}\mathbf{B}(\mathbf{I}-\mathbf{A})^{-1}$ has to be approximately equivalent to a multiple of the vector $\mathbf{s}\mathbf{D}^T$. Accordingly the partition of the price changes in (48) is identical with the one in (44). As is obvious from (48) both of components of the relative change of the price vector depend only on the changes in the elements of the matrix **D.** The vector **s** consists of arbitrary constants.

Equation (45) can be rewritten, by utilising equation (41), as

(49) $$d\overline{\mathbf{p}}(\hat{\overline{\mathbf{p}}})^{-1} = \lambda d\left[\overline{\mathbf{p}}\mathbf{B}(\mathbf{I}-\mathbf{A})^{-1}\right](\hat{\overline{\mathbf{p}}})^{-1} + d\lambda(\lambda)^{-1}.$$

The first term on the right hand side is designated as the vector of *fully effective rates of sectoral productivity change* for which the formula accordingly is:

(50) $$d\log\overline{\mathbf{t}}^{*} = -\lambda d\left[\overline{\mathbf{p}}\mathbf{B}(\mathbf{I}-\mathbf{A})^{-1}\right](\hat{\overline{\mathbf{p}}})^{-1} = -d\log\overline{\mathbf{p}}^{*}.$$

The i'th element of the vector $d\log\overline{\mathbf{t}}^{*}$ in equation (50) quite obviously represents productivity change. It is equal to the negative of the relative change in the production price of the sector concerned, as are all the rest of the measures of sectoral productivity change discussed in this paper. Besides, it is determined by the changes in the elements of the two matrices used to represent production technology in the input-output context. It can be called fully effective since it covers all the changes in production technology that has however indirect impact on the unit cost of the sector concerned. This is obvious from its equivalence with the first term of the right hand side of the equation (44). Thus we have obtained:

Result 4. The fully effective rate of sectoral productivity growth is equal to relative decrease in the production price of the sector, when all the prices that have an effect on it are allowed to change. Even the effect of changes in the rate of growth or normal rate of profit λ on the production price is taken into account. Only the change in the rate of profit that is needed to make the sum of production prices equal to one is excluded.

Note. This fully effective rate of sectoral productivity change can be, in view of (41) and (50), interpreted as the rate of change in the quantity of growth potential λ used per unit of output in the sector. The average turnover time however is inversely proportional to growth rate (Brody, 1970). Thus the fully effective rate of sectoral productivity change can also interpreted to indicate the relative change in the turnover time used by the sector per unit of output.

The second term of the sum in (49) was already, see equation (41), shown to be equal to the rate of overall productivity change in the closed growth system. Furthermore postmultiplying (49) through by $\hat{\overline{\mathbf{p}}}$ and summing over the sectors results in view of (40) in:

(51) $$d\log\lambda = -\sum \lambda d\left[\overline{\mathbf{p}}\mathbf{B}(\mathbf{I}-\mathbf{A})^{-1}\right]_i.$$

Accordingly a decrease in the sum of absolute production cost allows the rate of growth or the rate of profit λ to increase until the sum of the production cost is equal to one again. It is of course also possible to write:

(52) $$d\log\lambda = -\sum \frac{\lambda d\left[\overline{\mathbf{p}}\mathbf{B}(\mathbf{I}-\mathbf{A})^{-1}\right]_i}{\overline{p}_i}\frac{\overline{p}_i}{\sum \overline{p}_i}.$$

Hence in view of equation (39) we have:

Result 5. The equation

(53) $d \log \overline{T} = d \log \lambda = d \log \overline{\mathbf{t}}^* \hat{\overline{\mathbf{p}}}(\overline{\mathbf{p}}\mathbf{e})^{-1}$

gives the overall productivity growth in the case of the closed dynamic input-output model as a weighted average of the fully effective sectoral rates.

5. Summary of the results

It has been shown in this paper that the sectoral measures of productivity change derived in the static input-output framework can be expressed as changes in the production price i.e. the unit cost of the output of the sector. This applies both to the direct traditional measure as well as to the effective measures in which also the productivity gains/and losses in the sectors producing inputs are taken into account. In the direct traditional measure the change in the unit cost of the output is calculated on the assumption that the prices of all of inputs, both intermediate and primary, are exogenous constants, i.e. are not affected by changes in production technology. The first step in the direction of effective measures is to take into account the changes in the prices of intermediate inputs caused by technological change itself. The next and still a relatively straightforward step is to include also replacement of fixed capital, in which case also the prices of capital goods needed for replacement are determined by production technology.

Changing over to the closed dynamic input-output model provides a natural way of treating also the other part of capital input, i.e. the presence of capital stock (both fixed capital and inventories) at the production process, as a produced input. It also makes it possible, and in fact necessary to treat even human capital and labour as products. The two technological matrices of the dynamic input output model now determine all the prices, up to an arbitrary coefficient. The fully effective rate of sectoral productivity change is defined as the relative change in the production price, which have been made absolute by imposing the condition that the sum of the prices has to be equal to one. The balanced rate of growth, which in the case of the closed dynamic input-output model is equal to the overall rate of productivity growth, is a weighted average of the fully effective sectoral rates.

What do changes in the fully effective rates indicate? The balanced rate of growth associated with the technological matrices gives the growth rate that the economy could have if it would apply the same technology continuously. But it can of course use this potential growth to any purpose and normally technology would change from one year to the next. The fully effective rate of sectoral productivity growth tells the relative decrease of the growth potential consumed by the sector, per unit of output, as a consequence of the changes in production technology.

On the other hand since the balanced rate of growth is inversely related to the average turnover time, the fully effective rate of sectoral productivity change can also be interpreted to represent the relative decrease in the turnover time consumed by the sector per unit of output. It shows the change in the success of the sector in economising time, time being in this case the only factor of production. The actual population, its natural talents and other natural resources of the country set the limits within which the economy is able to choose its production technology.

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