THE CORE OF THE MULTISECTORAL INFORUM MODEL[1]

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1. Introduction

In this paper, a formal representation of the skeleton of a standard Inforum multisectoral model is given. The aim of the paper is twofold. Firstly, a compact algebraic representation of the "common" part of the Inforum model system - the "core" - is presented in order to reveal the theoretical background of this kind of input-output model; this description will show how these models differ from other models using input-output data. Secondly, new users or potential builders of an Inforum model can find here a quick overview of the nature of these models. Each model, of course, has its own particular features; they can be found in articles and books in which Inforum partner models are described (see, for example, Almon *et al*, 1974; Almon, 1997; Antille *et al*., 1996; Arango *et al*., 1987; Buckler McCarthy, 1991; Grassini, 1983, 1987; I.T.I., 1996; Orlowski *et al*., 1991; Richter, 1991; Meyer *et al*., 1995;Nyhus, 1997; Werling, 1992; Yu, 1997).

The name Inforum originally stood for "INterindustry FORecasting at the University of Maryland" and is a registered trade mark. It is now used by groups in many countries that work with the Maryland group. A name more descriptive of the nature of the models might be Interindustry Macroeconomic (IM) models — "Interindustry" to stress the presence of an input-output structure and many industries in the models and "Macroeconomic" to stress that all of the normal variables of macroeconomics (GDP, inflation, interest rates, employment, and unemployment) are covered. Like macroeconometric models, they use regression analysis of time-series. They do not, however, begin from a macro projection and allocate it to industries. Rather, the macro totals are obtained by summing the industry details: total employment is calculated by summing up the employment computed for each sector, and so on. In this article, I shall generally use IM when speaking of the nature of the models and use Inforum to describe the group of model builders.

One important "common" aspect of the Inforum models escapes the algebraic description and should be mentioned at the outset. That is the fact that model builders in different countries all work with a common software. Everyone who has worked on an Inforum-type model knows the importance of this common software; it is not a simple technical tool undeserving of any

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intellectual attention. Rather, it embodies an extensive understanding of what is needed for multisectoral modeling. It makes it possible to build a wide variety of models with relative ease, to penetrate into the working of the model, and to use it flexibly. It also facilitates international cooperation in the construction of the models and makes their linking possible.

The original Inforum software, called Slimforp, was in Fortran (Sichra, 1980). While Slimforp continues in use in a few models, most have now converted to Interdyme (Inforum, 1997), which began to be functional about 1993 and reached its second release and major improvement in 1995, at which point it had all the features originally planned. Interdyme is written in C++ and allows the user to employ a convenient matrix notation wherever appropriate. It also has extensive provision for reading in data in convenient formats and using regression results directly in the form produced by the G regression program. A versions of this software for a 32-bit compiler and Windows95 has recently become available (Almon,1998).

No matter how powerful or seductive the software, however, it is necessary to know clearly and theoretically how the model is to work. Studying the code and equations of existing models is certainly one way of discovering their structure, and the author has spent many an hour doing so. This paper sets out in analytic form what I believe to have learned through this study so that the way may be shorter for newcomers.

2. Historical remark

The evolution of the Inforum models illustrates a trend noted with approval years ago by Richard Stone.

> The development of the I/O model seems to be leading in directions in which its I/O core is becoming less and less discernible. This is as it should be, because it shows the possibility of improving the very simple relationships which were used initially. (Stone, 1984)

This statement is still valid; the I/O core is fading away as time goes by. The basic accounting identities are still there, but model builders surround them with "structural econometric equations"[2] in the process of improving the seminal Leontief equation.
The development of the I/O model has followed many ways; some of them have produced interesting quantitative enrichments of the original model based upon empirical data; others have led to the abstractions of mathematical economics.

As in other scientific fields, lack of communication has all too often wasted intellectual energies in discovering what was already known. It is still not difficult to find those who believe that the I/O framework excludes interdependence between prices and quantities or who believe that an I/O model must be driven by an aggregate macroeconomic model or who suppose that there is a fundamental problem of the stability of dynamic I/O models. In fact, all these problems arise from

[2] West (1995) names this kind of “sophisticated models”: IO+econometrics or IOE. He compares IOE models with the standard IO and CGE models and gives an interesting listing of the model characteristics.

misconceptions. The Inforum partners, which now work in over a dozen countries, produce dynamic forecasts of multisectoral models, including both industry outputs and prices with significant effects of the prices on the quantities and vice-versa. The models have no trace of an aggregate driver model, and no problems of the instability beyond that which actually appears in business fluctuations.

3.The I/O table and the notation

Let us consider the following four basic components of an I/O table: IC the Intermediate Consumption flows, FD the final demand components, VA the value added (compensation of primary factors and others), TO the total output. These components fill a simple input-output table as in the following figure.

IC	FD	TO
VA		
TO		

The table summarizes the two fundamental accounting identities, that is to say

Intermediate Consumption flows + Final Demand = Total Output

Total Output = Intermediate Consumption flows + Value Added

These two identities come from traditional (Italian!) double-entry bookkeeping. By equating the two expressions for total output and canceling Intermediate Consumption from both sides, we get that Final Demand is equal to Value Added. If the table represents an economy disaggregated into n industries, the final demand into k components and the value added into l components, then IC is a matrix nxn, FD a matrix nxk, VA a matrix lxn and TO a vector with n elements; using appropriate sum vectors h, the two accounting identities can be rewritten as

$$ICh + FDh = TO$$

$$IC'h + VA'h = TO$$

By means of simple algebraic manipulation, the two accounting identities are transformed into two sets of equations which are respectively the basis of the real side and the price side of a multisectoral model. The notation we shall use is shown in the Table of Notation.

For the moment, we will assume that all the flows along a single row of the table were conducted at the same price. With this assumption, the two identities described above can

Table of Notation	
Vectors and matrices of quantities	
q	vector of sectoral total outputs
$\hat{q}$	diagonal matrix with elements of q on the main diagonal
m	vector of imports
f	vector of final demand with net exports
F_i	vector of the i-th final demand component in national account classification
B_i	bridge matrix related to the i-th final demand
Q	matrix of intermediate consumption flows ($Q=Q^d+Q^m$)
z_R	vector of exogenous variables in the real side
z_P	vector of exogenous variables in the nominal side
Vectors of prices	
p	vector of sectoral production prices
$\hat{p}$	diagonal matrix with elements of p on the main diagonal
p^m	vector of import prices
Vectors and matrices of nominal flows	
IC	matrix of Intermediate consumption flows ($IC=\hat{p}Q$)
FD	matrix of Final Demand flows ($FD=FD^d+FD^m$)
VA	matrix of Value Added components
TO	vector of total outputs
fd	vector of final demand ($fd=FDh=\hat{p}f$)
va	vector of value added ($va=VA'h$)
$\hat{va}$	diagonal matrix with elements of va on the main diagonal
v	vector of value added per unit of output ($v=\hat{q}^{-1}va$)

be written as follows

$$[\ \hat{p}Q \quad \hat{p}f\ ]h = \hat{p}q \qquad h'\begin{bmatrix}\hat{p}Q\\ va\end{bmatrix} = (\hat{p}q)'$$

Premultiplying the first set of identities by $\hat{p}^{-1}$, postmultiplying the second set of identities by $\hat{q}^{-1}$, using $h=\hat{q}^{-1}q$, noting that $\hat{p}q=\hat{q}p$ and defining the traditional technical coefficients

$$A = Q\hat{q}^{-1} = [\ a_{ij} = q_{ij}\ /\ q_j\ ] \qquad for \quad i,j = 1,2,.....,n$$

we get

$$\hat{p}^{-1}[\ \hat{p}Q\hat{q}^{-1}q \quad \hat{p}f\ ]h = \hat{p}^{-1}(\hat{p}q) \qquad h'\begin{bmatrix}\hat{p}Q\hat{q}^{-1}\hat{q}\\ va\end{bmatrix}\hat{q}^{-1} = (\hat{q}p)'\hat{q}^{-1} = p'\hat{q}\hat{q}^{-1} = p'$$

from which we obtain

$$Aq + f = q \qquad p'A + v' = p'.$$

These two systems are the basic equations of an I/O model; more precisely, the two sets are respectively the basis of the real side and of the price side.

4.The accounting identities and the model

The construction of a model should begin with establishing the accounting system.
This accounting system is, in fact, already a model but with many exogenous variables; adding econometrically estimated equations just reduces the number of exogenous variables. Without the behavioral equations, the model would be all framework with little content; without the identities, the content could be self-contradictory.

The accounting identities of the I/O real side are

$$Qh + f = q$$

These n equations can be used to explain n (endogenous) variables; but the system involves n^2+2n variables (the n^2 elements of Q and n each of f and q) so a way to reduce the number of unknowns must be found. The usual device is to introduce the "input-output technical coefficients" a_{ij} defined by $q_{ij}=a_{ij}q_j$ where a_{ij} may be a function of time, prices, interest rates, levels of output and so on. (One extreme case is to assume them constant; it is still not rare to find people who suppose that this one possibility is the only one ever used in input-output work and therefore wrongly assert: "input-output assumes fixed coefficients".)

Now, we see that the introduction of "input-output technical coefficients" reduces the number of variables to 2n. In order to solve them, we must in some way determine n of them. There are, in general, three alternatives: (a) q is left endogenous and f is given, (b) f is endogenous and q is given, (c) n_1 components of q and n_2 components of f are endogenous ($n_1+n_2=n$) and the others are left exogenous. Although the above three alternatives are all interesting for tackling specific real problems, in the economic literature (a) is practically the only one considered. Likewise, in the price side there are 2n variables, p and v. One can determine v and deduce p, or determine p and deduce v, or determine some elements of p and others of v. If in both cases we choose the first alternative, then, from the I/O table used to build a simple I/O model, it is possible to obtain

$$q = g(f) \qquad \text{and} \qquad p = f(v).$$

These are the solutions used to investigate the mathematical properties of the "static" Leontief model; the solutions of modern input-output models involve yet other variables and equations.

5. The real side of the model

The structure of the real side of the model can be conveniently presented starting from the Leontief equation. An IM model cannot be confined into the narrow set of variables contained into vectors q and f. In fact, final demand components (private consumption expenditure, government expenditure, investments, imports, exports, inventory changes and so on) must be evaluated at a specified level of aggregation. In general, we can state that the final demand vector is equal to the sum of k final demand components

$$f = f_1 + f_2 + f_3 \ldots\ldots\ldots f_k.$$

Some components of the final demand, let us say r<k, are explained by means of econometric equations because the model builder does not want to consider all of them as exogenous. Private consumption expenditures, investments, imports and exports are usually explained by means of econometric equations; then, these final demand components are no longer exogenous; consequently, new exogenous variables appear in the framework of the I/O model; these new variables belong to the set of the explanatory variables of the endogenized final demand components.

Now, total output and the final demand components represent the set of endogenous variables of the (real side) I/O model. The total output, q, is defined by the Leontief equation; the r final demand vectors require an econometric estimate of their components. This is accomplished as in every econometric model by using time-series and cross-section statistical data. Much of the time-series data usually comes from national accounts. But I/O and national account classifications in many cases do not match. For example, the National Accounts give a structure of household expenditures with categories quite different from the sectors of the I/O table. The differences in classification are due to real statistical problems with interesting economic contents. For consumption expenditures, national account time series generally reflect categories in which consumers are able to think and answer survey questions rather than the sectors in which the industrialist thinks. The modeling of consumer behavior should also be done in these categories, but it is then necessary to convert a vector of consumption in these consumer categories into a vector of consumption in industrial categories. Similarly, if investment decisions are to be modeled, data on investment by user of the investment goods is necessary. The resulting vector of investment by purchaser must be converted into a vector of investment by product purchased. In both cases, the link between variables available in different classifications is done by means of *bridge* matrices which, in general, should be made available by every statistical bureau producing both national accounts and I/O tables. (In fact, these matrices are not always available from the statistical office; the work of constructing them falls on the model builder.)

Every bridge matrix has rows corresponding to I/O sectors and columns to the specific national accounts classification. Thus, all bridge matrices have the same number of rows, though their number of columns may be different. We shall assume that such a bridge matrix is available for each final demand vector; this matrix is such that the correspondence between final demand vector f_i and the national account vector F_i is

$$f_i = B_i F_i$$

When there is a perfect correspondence between the two classifications (as it often is for imports and exports) the bridge matrix will be equal to the identity matrix. Using F's instead instead of f's, the Leontief equation becomes

$$Aq + B_1F_1 + B_2F_2 + \ldots\ldots\ldots + B_rF_r + f_{r+1} + \ldots + f_k = q$$

As previously stated, the *r* final demand vectors F_i are explained by econometrically estimated equations; we have, therefore, for vector F_i as many equations as there are elements of this vector. Among the determinants of the F_i may be found elements of the q vector and lagged values of q (as in the dependence of investment on increases in output) or variables such as personal income which derive from q, as we shall see below. F_i may also depend on variables belonging to the price side of the I/O model (mainly prices) and other variables, such as interest rates which are not explicitly shown in the input-output flow table. These other variables may be true exogenous variables or they may be determined in supplementary equations included in the model but not depicted in the I/O accounting scheme. When q has been computed the real side goes on to compute labor productivity and, from it and output, employment which will be - as we shall see - an important link between real and nominal sides. In short, we are very far from the simple determination of q given f .

So far, we have written equations without specifying the years to which they belong. Of course, for different years we have different final demand vectors, f_t, and input-output coefficient matrices, A_t. Thus, including the time subscript, we would write the real side as

$$q_t = A_t q_t + f_t .$$

6. The price side of the model

The structure of the price side of the model can be introduced by using the Leontief price equation $p=A'p+v$. Prices are measured as price indexes; the base year coincides with base year of the I/O table. That is to say, all prices are equal to 1.0 in the base year.

In other years, prices vary according to changes in the vector of value added per unit of output, v, and the A matrix. By using the t subscript to denote the time index, the price equation becomes

$$p_t = A_t' p_t + v_t .$$

The model must describe an open economy, so we must take into account two sources of goods: domestic and foreign industries. The total output vector is the amount of resources provided by domestic industries; the import vector is the amount of resources provided by foreign industries. Then, we have that $q_{ij}=q^d_{ij}+q^m_{ij}$ from which we get the ratios $h_{ji}=q^d_{ij}/q_j$ and $t_{ji}=q^m_{ij}/q_j$, with $a_{ij}=h_{ji}+t_{ji}$; defining $H=[h_{ij}]$ and $T=[t_{ij}]$ we have $A'=H(h_{ij})+T(t_{ij})$. While the elements of matrix A can be interpreted as technical coefficients, their division between the H and T matrices is based primarily on commercial rather than technological considerations. They are used here to compute the cost of intermediate consumption when domestic and import prices differ. The price equation becomes

$$p_t = H_t p_t + T_t p_t^m + v_t$$

For a national model, vector p^m is exogenous; in a naive I/O model, the vector v_t is assumed exogenous as well. In an IM model, the value added per unit of output is considered as the sum of l value-added components such as Wages and salaries, Contributions for social insurance, Capital consumption, Profits, Interest payments, and Indirect taxes. These value-added components are partially exogenous and partially endogenous; subsidies are mainly considered as exogenous, while wages are usually (econometrically) explained. One can even find exogenous as well endogenous variables within the same component; for example, if wages in manufacturing sectors are mainly endogenous, wages in Government sectors may be treated as exogenous. The result of modeling these components is an *nxl* matrix, *V*, of different types of value added per unit of output. Summing the columns gives the *v* vector.

7. Real side and nominal side cross over

In our presentation, we have artificially separated the real and nominal side for exposition. In actual modeling, of course, there are many cross-overs. In modeling value added components, we may use explanatory variables such as output or investment, from the real side. On the real side, we may use prices in determination of the input-output coefficients or final demands. Some explanatory variables, such as interest rates, may fall entirely outside the I/O table; they may be exogenous or may be endogenized in the overall model by equations in what is generally called the “macro” part of the model. Lagged values from one side may enter the equations for determination of values in either side. For instance, formal or informal price indexation makes lagged values of prices a good explanatory variable for wages; price formation under fixed or flexible mark-up implies that profits depend on prices; cost of labor depends on labor productivity, that is to say on total outputs and employment, and so on.

8. Do we assume fixed I/O matrices?

We have mentioned repeatedly the concern of the IM model builder with time series of historical data. Usually these are annual series and include at least series for output, *q*, imports, *m*, exports, and the various F_i vectors from the national accounts. Input-output tables, however, are often not available annually. Can we assume that the A matrices derived from them are constant? In general, certainly not. In order to construct a consistent data base for our work, we should try to produce a series of balanced tables using all the data that is readily available to us, including information on individual flows. Some statistical offices, such as the Dutch and French, routinely do this work and release national accounts that are consistent with generally plausible input-output tables. Others do not bother with this check on the consistency of their national accounts data. In the best of cases, the model builder finds that reasonable changes in the input-output coefficient matrices can reconcile data on output, exports, and imports with the national accounts. In other cases, it becomes clear that the national accounts are inconsistent with other official data sources, thus posing a difficult choice for the model builder. To the best of our knowledge, *in no case have we found data consistent with constant input-output coefficients*. Fortunately, there is absolutely no requirement in any aspect of input-output theory that we have used that coefficients should be constant. Rather, modeling their changes can be an important challenge.

What has been said here of the *A* matrix applies with even greater force to the H and T matrices which divide A between domestic and imported parts. At least in the case of A there is likely to be some technological reason for stability of the coefficients, but with H and T substitution of imports for domestic goods or vice versa can be rapid. Modeling of changes in these matrices is crucial.

In short, *constancy of input-output coefficients plays no role in an IM model*. The question is not whether they change but how they change.

9. Indirect taxes in an IM model

Changes in indirect taxes have played a prominent role in recent European economic policy, and a Value Added Tax (VAT) is frequently discussed in the United States. It will illustrate nicely how the structure of an IM model can be exploited to look in some detail at the treatment of the indirect taxes. We will see that correct treatment of three types, the excise, the *ad valorem* tax, and the European-style VAT are quite different.

In order to deal with these, we must be aware about their location in the I/O table. Usually, indirect taxes are recorded in the value added sector, VA, of the I/O table. Both accounting definition and economic meaning of such flows can be understood considering the content of each flow in a table with three industries, one value added row (net of indirect taxes), VA, one final demand vector, c, and the "totals"; we, now, distinguish two kinds of indirect taxes: excise and ad valorem taxes; their input-output location is presented in the next two sub-sections; in the third subsection the indirect taxes model in an Inforum price side model is presented.

9.1 Excise taxes in the I/O table

We consider the case of an I/O table where an excise tax burden is added to each intermediate consumption and final demand flow; tax flows are, then, located as in the following table

$q_{11}p_1+s_{11}$	$q_{12}p_1+s_{12}$	$q_{13}p_1+s_{13}$	$c_1p_1+s^1$	$q_1p_1+s_{11}+s_{12}+s_{13}+s^1$
$q_{21}p_2+s_{21}$	$q_{22}p_2+s_{22}$	$q_{23}p_2+s_{23}$	$c_2p_2+s^2$	$q_2p_2+s_{21}+s_{22}+s_{23}+s^2$
$q_{31}p_3+s_{31}$	$q_{32}p_3+s_{32}$	$q_{33}p_3+s_{33}$	$c_3p_3+s^3$	$q_3p_3+s_{31}+s_{32}+s_{33}+s^3$
VA_1	VA_2	VA_3		
q_1p_1	q_2p_2	q_3p_3		
II_1	II_2	II_3		
qq_1	qq_2	qq_3		

where

$$II_j = s_{j1}+s_{j2}+s_{j3}+s^j \qquad and \qquad qq_i = q_ip_i + II_i$$

First of all, in order to work out an excise tax model to be merged with the price equation, tax burdens (s_{ij}'s and s^j's) from the table have to be removed; of course, after the removal the intermediate consumption and final demand flows turn out to be net of these taxes, but the I/O table is consequently submitted to a clear modification; excise tax flows will no longer affect intermediate consumption and final demand flows, but excise tax flows will still be in the I/O table. In fact, the excise taxes removal can be interpreted like brushing them down to the value added area where the "new" flows, IA_j and IA^c , contain now the column sums of the excise taxes “removed” from intermediate consumption and final demand flows

$q_{11}p_1$	$q_{12}p_1$	$q_{13}p_1$	c_1p_1	q_1p_1
$q_{21}p_2$	$q_{22}p_2$	$q_{23}p_2$	c_2p_2	q_2p_2
$q_{31}p_3$	$q_{32}p_3$	$q_{33}p_3$	c_3p_3	q_3p_3
VA_1	VA_2	VA_3		
IA_1	IA_2	IA_3	IA^c	
q_1p_1	q_2p_2	q_3p_3		

where

$$IA_j = s_{1j}+s_{2j}+s_{3j} \quad and \quad IA^c = s^1+s^2+s^3 \ .$$

The excise tax flows, IA_i's, are now correctly computed and located among the costs of production.

9.2 VAT in a I/O table

The *ad valorem* tax considered in this section is the EC value added tax (VAT). The working mechanism of this tax is such that firms act as tax collectors and the consumer represents the actual taxpayer; hence, VAT turns out to be a tax on final consumption as it is generally considered in the economic literature. It is a matter of fact that specific tax rules introduce the so called "impurities" which make this *ad valorem* tax acting like any other tax burden on production (see Bardazzi *et al.*, 1991).
VAT burden in the I/O table can be considered by adding to each flow the product of it by the tax rate (whereas the impurities make VAT non-deductible); assuming without any loss of generality that the tax rate, t_i, is constant along the row, the VAT tax burdens may be represented as in the following table where it is assumed that the "impurities" make VAT non-deductible for three flows out of six in the intermediate consumption; VAT is largely charged on final demand and, for sake of simplicity, only one component, c_i, is considered

$q_{11}p_1$	$q_{12}p_1$	$q_{13}p_1$	$c_1p_1(1+t_1)$	$q_1p_1+VATRS_1$
$q_{21}p_2$	$q_{22}p_2$	$q_{23}p_2(1+t_2)$	$c_2p_2(1+t_2)$	$q_2p_2+VATRS_2$
$q_{31}p_3$	$q_{32}p_3(1+t_3)$	$q_{33}p_3(1+t_3)$	$c_3p_3(1+t_3)$	$q_3p_3+VATRS_3$
VA_1	VA_2	VA_3		
q_1p_1	q_2p_2	q_3p_3		
$VATRS_1$	$VATRS_2$	$VATRS_3$		
TOT_1	TOT_2	TOT_3		

Sector 2 and sector 3 are affected by non-deductible VAT, $VATRS_i$ represents the VAT charged along the *i-th* row, and $TOT_i = q_i p_i + VATRS_i$. By brushing away VAT flows from the table, that is to say, deleting the following terms

$$VATRS_1 = c_1 p_1 t_1$$
$$VATRS_2 = c_2 p_2 t_2 + q_{23} p_2 t_2$$
$$VATRS_3 = c_3 p_3 t_3 + q_{32} p_3 t_3 + q_{33} p_{32} t_3$$

as a consequence of that, *VATRS* row disappears and a new VAT row will take its place; it will contain

$$VAT_1 = 0$$
$$VAT_2 = q_{32} p_3 t_3$$
$$VAT_3 = q_{23} p_2 t_2 + q_{33} p_3 t_3$$
$$VAT^c = c_1 p_1 t_1 + c_2 p_2 t_2 + c_3 p_3 t_3$$

The first three terms are located among the value added components while the forth term falls out of the table; in fact, VAT^c is the VAT yield which is specifically recorded in the accounts related to the Institutions. The I/O table is now

$$\begin{array}{ccccc} q_{11}p_1 & q_{12}p_1 & q_{13}p_1 & c_1p_1 & q_1p_1 \\ q_{21}p_2 & q_{22}p_2 & q_{23}p_2 & c_2p_2 & q_2p_2 \\ q_{31}p_3 & q_{32}p_3 & q_{33}p_3 & c_3p_3 & q_3p_3 \\ VA_1 & VA_2 & VA_3 & & \\ VAT_1 & VAT_2 & VAT_3 & VAT^c & \\ q_1p_1 & q_2p_2 & q_3p_3. & & \end{array}$$

The removal of VAT and excises taxes is now complete and the price equation with indirect taxes can be defined.

9.3 Indirect taxes in the price equation

If the basic I/O table has no tax burdens added to intermediate consumption as well as to final demand flows, from the price equation

$$\begin{array}{l} a_{11}p_1 + a_{21}p_2 + a_{31}p_3 + v_1 = p_1 \\ a_{12}p_1 + a_{22}p_2 + a_{32}p_3 + v_2 = p_2 \\ a_{13}p_1 + a_{23}p_2 + a_{33}p_3 + v_3 = p_3 \end{array}$$

we get the price vector labeled **basic** (ignoring the time index)

$$\begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}_{basic} = \begin{bmatrix} 1-a_{11} & -a_{21} & -a_{31} \\ -a_{12} & 1-a_{22} & -a_{32} \\ -a_{13} & -a_{23} & 1-a_{33} \end{bmatrix}^{-1} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

Here we assume that $s_{ij}=q_{ij}\alpha_i$, where α_i is the excise rate for the i-th good; we are aware that because of the composition and policy discrimination among industries, tax rate on the i-th good usually differs sector by sector; assuming excise tax rate constant along the row makes the notation easier without seriously compromising the understanding of the excise tax role in price determination.
The amount of this kind of indirect tax per unit of output is given by $a_{ij}\alpha_i$ and the price equation will get the following structure

$$\begin{aligned}
a_{11}p_1 + a_{21}p_2 + a_{31}p_3 + a_{11}\alpha_1 + a_{21}\alpha_2 + a_{31}\alpha_3 + v_1 &= p_1\\
a_{12}p_1 + a_{22}p_2 + a_{32}p_3 + a_{12}\alpha_1 + a_{22}\alpha_2 + a_{32}\alpha_3 + v_2 &= p_2\\
a_{13}p_1 + a_{23}p_2 + a_{33}p_3 + a_{13}\alpha_1 + a_{23}\alpha_2 + a_{33}\alpha_3 + v3 &= p_3
\end{aligned}$$

from which we see the relationship between basic price and price including the effect of excise taxes (here labeled **excise**)

$$\begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}_{excise} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}_{basic} + \begin{bmatrix} 1-a_{11} & -a_{21} & -a_{31} \\ -a_{12} & 1-a_{22} & -a_{32} \\ -a_{13} & -a_{23} & 1-a_{33} \end{bmatrix}^{-1} \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix}$$

This equation makes clear how to deal with excise taxes in price determination and shows that in the multisectoral framework indirect taxes produce an additive component of the basic prices.

The presence of non-deductible VAT on intermediate consumption flows leads to the following price equation

$$\begin{aligned}
a_{11}p_1 + a_{21}p_2 + a_{31}p_3 + v_3 &= p_1\\
a_{12}p_1 + a_{22}p_2 + a_{32}p_3(1+t_3) + v_2 &= p_2\\
a_{13}p_1 + a_{23}p_2(1+t_2) + a_{33}p_3(1+t_3) + v_3 &= p_3
\end{aligned}$$

As a cost component, VAT is equal to zero in the first equation, equal to $a_{32}p_3t_3$ in the second equation and equal to ($a_{23}p_2t_2+a_{33}p_3t_3$) in the third equation; even if non-deductible VAT is located in a small number of flows, its influence is widespread over the three prices according to a non linear function represented by the solution

$$\begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}_{vat} = \begin{bmatrix} 1-a_{11} & -a_{21} & -a_{31} \\ -a_{12} & 1-a_{22} & -a_{32}(1+t_3) \\ -a_{13} & -a_{23}(1+t_2) & 1-a_{33}(1+t_3) \end{bmatrix}^{-1} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

In the case of the "ideal VAT" (that is to say, when non-deductible VAT is not present in any intermediate consumption flow), p_{vat} is equal to p_{basic}; the presence of non-deductible VAT makes p_{vat} different from p_{basic} but it is not possible to represent one in term of the other like in the case of excise taxes.

After the introduction of indirect taxes (excise and *ad valorem*), the price equation for the j-th sector is restated as

$$p_j = \sum_{i=1}^{n} a_{ij} p_i (1+t_{ij}) + \sum_{i=1}^{n} a_{ij} \alpha_{ij} + v_j$$

where v_j is computed as above.

10. Final remarks

We have seen that from the accounting identities it is possible to obtain a very simple model for the real and price sides of an input-output model, that is to say

$$q = g(f) \qquad and \qquad p = f(v)$$

An Inforum model provides the endogenization of many final demand and value added components; these are gathered into vectors F and d; the primitive real and price sides take now the form

$$q = Aq + f(q,p,z_R) \qquad and \qquad p = Hp + Tp^{m} + v(p,q,z_P)$$

where z_R and z_P are respectively the exogenous variables in the real and price sides of the model. Now, we can see that having modeled final demand and value added components the dependence of both of them on total output and prices is established; then, the Inforum model has prices and quantities fully integrated.

This review has concentrated on the part of the model which involves its multi-sectoral structure. An IM model must also include a number of macroeconomic equations. Various types of income -- wages, depreciation, profits, and so on -- originates in industries and is then summed over the industries to give totals of these types of income. They are allocated among various “institutions” such as families, business, and governments. Taxes are then “collected” from the families at the aggregate level, without regard to the industry in which the wages were paid. Likewise, subsidies are paid at the aggregate level. The personal savings rate is also established and total household expenditure is derived. There may — or may not — be further equations for a detailed construction of all of the flows in the institutional accounts of the Standard National Accounts.

Other variables, such as the overall unemployment rate or interest rates may be determined in the macroeconomic part of the IM model. Thus, the Inforum models completely integrate the sectoral and aggregate aspects of the model. There is no macro-economic driver model, and no need for one. In so far as possible, the Inforum models build from bottom up and only use aggregate equations where it would make no sense to have sectoral equations, as for example the personal savings rate.

Given the attention which has centered in recent years on "expectations", it is perhaps important to note that the Interdyme software allows the use of future values of any variable as well as the more traditional current and lagged values of variables. It is thus possible to use "model-consistent" expectations, which it was once fashionable to call "rational" expectations. In fact, one ancestor of today's Inforum models [Almon 1963] stressed that it employed such "consistent" forecasting techniques. Most models currently in use, however, employ mainly adaptive expectations because of the more plausible forecasts which they yield.

Finally, it should be clear that builders of Inforum models take data and behavioral equations seriously. In contrast to models with casually chosen parameters that have been "calibrated" to only one year of data and produce only comparative static results, the Inforum models can be tested over several years of past data and used to forecast specific future years. These forecasts may, of course, prove wrong. But they offer the possibility of learning from mistakes, something you cannot do with models that cannot make mistakes.

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