Interdyme Report #2: Output/Final Demand Discrepancies

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Introduction

The logic for the calculation of output in the Seidel() function with a simple linear import function is:

$$Q_i = F_i + \sum_{j \neq i} a_{ij} Q_j + a_{ii} Q_i - M_i$$

or:

$$Q_i = F_i + \sum_{j \neq i} a_{ij} Q_j + a_{ii} Q_i - (b_{1i} + b_{2i}(sum + a_{ii} Q_i))$$

or finally:

$$Q_i = \frac{F_i + \sum_{j \neq i} a_{ij} Q_j - (b_{1i} + b_{2i} sum)}{(1 - a_{ii}(1 - b_{2i}))}$$

This report suggests how the above calculation may be altered when one needs to insert a discrepancy into the input-output identity.

Output or Final Demand Discrepancies?

When balanced benchmark tables are not available for each year, published data must be used to extrapolate input-output tables and final demand vectors beyond the benchmark year. In this case, in the last year of available data, the fundamental output identity:

$$\mathbf{q} = \mathbf{Aq} + \mathbf{f} - \mathbf{m}$$

probably will not hold. However, we can append a discrepancy vector, $\boldsymbol{\varepsilon}$, to the equation:

$$\mathbf{q} = \mathbf{Aq} + \mathbf{f} - \mathbf{m} + \boldsymbol{\varepsilon}$$

If the output solution is calculated as described in the previous section, then how should the discrepancy be applied in the forecast periods of the model? A deceptively simple (but wrong!) way would be to allow the Seidel algorithm to converge on a solution, and then add the discrepancies to the resulting output vector. This is however, not correct, as we are not making use of all available information as Seidel runs. Specifically, each sector's output calculation does not have the benefit of using the discrepancy-adjusted output values for all the other industries.

A better alternative is to adjust the output calculation of each sector within the Seidel loop, after it has been calculated as in the previous section. In other words, form output as:

$$Q_i = \frac{F_i + \sum_{j \neq i} a_{ij} Q_j - (b_{1i} + b_{2i} sum)}{(1 - a_{ii}(1 - b_{2i}))} + \varepsilon_i$$

This method has the advantage of using the discrepancy-adjusted output of the other industries within each industry output calculation of the Seidel loop, and generally leads to a smooth output forecast. An obvious problem is that when discrepancies are large, as they often are with U.S. data, large percentage changes in final demand can lead to small percentage changes for calculated output, since the discrepancy is constant. For better or worse, this is the discrepancy adjustment technique used in INFORUM models LIFT and *Iliad*.

However, there is a subtle problem with this version of discrepancy adjustment. It assumes that the discrepancy is an *output discrepancy*. In other words, we believe there is a true output, say $\bar{\mathbf{q}}$, for which the output identity holds, but that we can only observe calculated or estimated output, $\hat{\mathbf{q}}$. Assuming that we know the true **A** matrix and the vectors **f** and **m**, we have the relationship for the true $\bar{\mathbf{q}}$:

$$\bar{\mathbf{q}} = \mathbf{A}\bar{\mathbf{q}} + \mathbf{f} - \mathbf{m}$$

Now, assume that the difference between $\hat{\mathbf{q}}$ and $\bar{\mathbf{q}}$ is δ, so that we have:

$$\bar{\mathbf{q}} = \hat{\mathbf{q}} + \boldsymbol{\delta}$$

Suppose we write the discrepancy calculated using $\hat{\mathbf{q}}$ as $\hat{\boldsymbol{\varepsilon}}$. Then we get

$$\bar{\mathbf{q}} - \boldsymbol{\delta} = \mathbf{A}(\bar{\mathbf{q}} - \boldsymbol{\delta}) + \mathbf{f} - \mathbf{m} + \hat{\boldsymbol{\varepsilon}}$$

In the last period of historical output data, when the discrepancy is calculated,

$$\hat{\boldsymbol{\varepsilon}} = (\bar{\mathbf{q}} - \boldsymbol{\delta}) - (\mathbf{I} - \mathbf{A})^{-1}(\mathbf{f} - \mathbf{m})$$

There are two conclusions that can be drawn from this exercise. First, the measured discrepancy is not the "true" discrepancy, but rather will always differ by an unknown δ. Second, by appending the discrepancy as a constant term adjustment *after* the Seidel calculation for each sector, we are only using information about the **A** matrix in the last year of data. If the matrix is projected to change over time, the discrepancy cannot take this into account, since it is a constant..

What if we interpret the discrepancy as a *demand* discrepancy instead of an output discrepancy? In other words, we assume the discrepancy exists either in **f**, **m** or **A**. Then we can write the input-output identity as:

$$\bar{\mathbf{q}} = \hat{\mathbf{A}}\bar{\mathbf{q}} + \hat{\mathbf{f}} - \hat{\mathbf{m}} + \hat{\boldsymbol{\varepsilon}}$$

The apparent form of this discrepancy is equivalent to the form with the output discrepancy. However, the logical way to apply the discrepancy in the model forecast is different. With a final demand discrepancy, the output calculation can be expressed as:

$$\mathbf{q} = (\mathbf{I} - \hat{\mathbf{A}})^{-1}(\hat{\mathbf{f}} - \hat{\mathbf{m}} + \hat{\boldsymbol{\varepsilon}})$$

In terms of the detailed Seidel calculation:

$$Q_i = \frac{F_i + \sum_{j \neq i} a_{ij} Q_j - (b_{1i} + b_{2i} sum) + \varepsilon_i}{(1 - a_{ii}(1 - b_{2i}))}$$

This version of the discrepancy application is at least internally consistent, in that the discrepancy we are trying to measure is also the discrepancy we actually use. This is the preferred method for discrepancies in the Seidel solution for Interdyme models. However, we recognize that there are still many problems in its use. The best solution to the discrepancy problem would appear to be to get rid of the discrepancy in the historical data by modifying final demand, output or the **A** matrix. But if we are not able or prepared to make this decision, then we are forced to live with discrepancies.