

Horizontal Integration in the Dutch Financial Sector

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Abstract

In this paper, the consequences of cross-shareholding in an n -firm industry are analyzed. Our attention is focused on the case where firms have silent interest in each other. These interests can be direct or indirect. We analyze the effects of cross-shareholding on the price-cost margins in a Cournot and a Bertrand setting. The model is mathematically equivalent to an Input-Output model. In all cases, competition is reduced due to shareholding interlocks. As an example, the Dutch financial sector is used. Comparing the case of shareholding with the case of no-shareholding, the price cost margins are found to be 2% higher in a Bertrand-market, and 13% higher in a Cournot-market.

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1 Introduction

Silent financial interests between firms through cross-shareholding are of relevance to these firms. Since the profit functions of the firms are linked via cross-shareholding, the firms may increase their individual and joint profits. When maximizing their profits, the firms take account of the effects their actions have on their competitors, customers, etc. Silent interests thus induce collusive behavior and cartelizing effects, in the sense that competition reduces which leads to higher prices.

Several types of interests through shareholding can be distinguished in the literature. Vertical integration through financial links can be thought of as ‘cooperation’ between producers in the same production column (see e.g. Flath, 1989). Horizontal integration occurs when rivals implicitly collude due to their financial interests. Reynolds and Snapp (1986) and Bresnahan and Salop (1986) focus on joint ventures which is a very stringent form of horizontal integration, since firms have direct control over each other’s production policies. In the case of silent financial interests firms hold shares in their rivals but cannot control the output or price of any of its rivals. Flath (1991, 1992a) proposes a model for analyzing horizontal integration through silent interests. In contrast to the framework in Reynolds and Snapp (1986), Flath’s model also takes the effects of indirect shareholding into account. This occurs, for example, when firm A owns shares in firm B , which owns shares in firm C . Although A has no direct interests in B , it does have indirect interests in C , namely through B .

In this paper the formulation of Flath (1992a) is adopted. Our theoretical analysis distinguishes between an industry that is characterized by n Cournot oligopolists and an industry consisting of n Bertrand competitors. In the Cournot case, an upper and lower bound is obtained for the uniform price-cost margin in the presence of shareholding. It is further proved that increased shareholding leads to an increase in the price-cost margin, and hence in the industry’s price. For Bertrand oligopolists, it is shown that shareholding raises the individual price-cost margin above the monopolist’s margin.

Although the theoretical effects of shareholding have received considerable attention in the literature (see also Martin, 1993), empirical analyses are extremely rare due to a lack of data. An exception is Flath (1992b, 1993) who analyzes the extent of cross-shareholding for the six major keiretsu groups in Japan. Due to data limitations, however, a quantification of the effects of shareholding seems not possible. In this paper we present data on cross-shareholding between the four largest financial conglomerates in the Netherlands. The major findings of our empirical analysis are as follows. The uniform price-cost margin increases by at least 13% if the Cournot model applies. In the Bertrand case, the individual price-cost margins also increase, but rather marginally by one to two percent.

The next section presents the general framework for the model with shareholding. The theoretical results are obtained in Section 3 for Cournot oligopolists and for Bertrand oligopolists in Section 4. The empirical application for the Dutch financial sector is presented in Section 5.

2 General Framework

Consider an industry with n firms and suppose the industry is characterized by horizontal integration through cross-shareholding. Firms have silent financial interests in each other, which means that they cannot control the policies (with respect to e.g. output or prices) of other firms. Each firm i 's objective therefore is to maximize its own profit, π_i , which includes the returns on the shares that firm i holds in its rivals. The profit for firm i can be written as

$$\pi_i = (p_i - c_i)q_i + \sum_{j \neq i} d_{ij} \pi_j. \quad (1)$$

The first term indicates the operating earnings. p_i is the price of the product of firm i , c_i are its unit costs of production, and q_i is the output. The second term gives the returns of equity holding by firm i in any of the other firms j ($\neq i$). $d_{ij} \geq 0$ denotes the fraction of the shares of firm j that is held by firm i . In matrix notation equation (1) can be rewritten as¹

$$\boldsymbol{\pi} = (\hat{\mathbf{p}} - \hat{\mathbf{c}})\mathbf{q} + \mathbf{D}\boldsymbol{\pi} \quad (2)$$

The matrix \mathbf{D} is the direct shareholding matrix. By definition its diagonal elements d_{ii} are all equal to zero. Also by definition, the column sums of \mathbf{D} (i.e. $\sum_i d_{ij}$) cannot exceed one. We assume that each column sum is smaller than one, i.e. $\sum_i d_{ij} < 1 \forall j$. This is sufficient (see e.g. Takayama, 1985, Ch. 4) to guarantee that the matrix $(\mathbf{I} - \mathbf{D})$ is nonsingular.² The solution of equation (2) is given as $\boldsymbol{\pi} = (\mathbf{I} - \mathbf{D})^{-1}(\hat{\mathbf{p}} - \hat{\mathbf{c}})\mathbf{q}$, where \mathbf{I} is the identity matrix. Define $\mathbf{L} = (\mathbf{I} - \mathbf{D})^{-1}$, then

$$\boldsymbol{\pi} = \mathbf{L}(\hat{\mathbf{p}} - \hat{\mathbf{c}})\mathbf{q}. \quad (3)$$

Similar to the Leontief inverse in input-output analysis (see e.g. Miller and Blair, 1985), the matrix \mathbf{L} can be expressed as a power series.

$$\mathbf{L} = \mathbf{I} + \mathbf{D} + \mathbf{D}^2 + \mathbf{D}^3 + \dots \quad (4)$$

In interpreting equations (3) and (4), it is useful to note that π_i reflects the market value of firm i 's assets, provided that the stock market is efficient (see Flath, 1989). So the profits of firm i consist of three parts. First, its own operating earnings, reflected by the i th element of $(\hat{\mathbf{p}} - \hat{\mathbf{c}})\mathbf{q}$. Second,

¹Vectors and matrices are in bold letters, scalars are italicized. \hat{p} denotes the diagonal matrix with the elements p_i on its main diagonal and all other entries zero. We adopt the following notation for inequalities between vectors and matrices. $\mathbf{x} \leq \mathbf{y}$ means $x_i \leq y_i \forall i$, $\mathbf{x} \ll \mathbf{y}$ means $x_i < y_i \forall i$, and $\mathbf{x} < \mathbf{y}$ means $\mathbf{x} \leq \mathbf{y}$ but $\mathbf{x} \neq \mathbf{y}$.

²From a mathematical point of view, the assumption of all column sums being smaller than one can be relaxed (see e.g. Fisher, 1965). From an economic point of view, however, the case where some column sums are allowed to equal one is rather exceptional.

firm i 's shares in the operating earnings of the other firms j , reflected by the i th element of the vector $\mathbf{D}(\hat{\mathbf{p}} - \hat{\mathbf{c}})\mathbf{q}$. This second term indicates the effect of direct shareholding. The third term gives the effects of indirect shareholding and is, for firm i , reflected by the i th element of the vector

$$(\mathbf{D}^2 + \mathbf{D}^3 + \dots)(\hat{\mathbf{p}} - \hat{\mathbf{c}})\mathbf{q}.$$

For example, if element (i, j) of matrix \mathbf{D}^2 is positive it must be true that d_{ik} and d_{kj} are positive for some k . It may thus happen that, indirectly, firm i 's market value of its stock is based on the operating earnings of firm j , whereas firm i may hold no shares in j (i.e. $d_{ij} = 0$). However, since firm i holds shares in firm k , which in turn holds shares in firm j , there is an indirect interest of i in j . The matrices \mathbf{D}^3 , \mathbf{D}^4 , etc. can be interpreted in a similar way. All effects of indirect shareholding are obtained by summing over the separate effects so as to yield $(\mathbf{D}^2 + \mathbf{D}^3 + \dots)(\hat{\mathbf{p}} - \hat{\mathbf{c}})\mathbf{q}$.³

Note that it follows from (4) that $\mathbf{L} \geq \mathbf{I}$ and when for example the elements $d_{ij} > 0 \forall i \neq j$, we find $\mathbf{L} \gg \mathbf{I}$. As a consequence $\pi_i > (p_i - c_i)q_i$ for all i , in that case the total profits exceed the total operating earnings of the industry. Although this may seem strange at first glance, it should be borne in mind that the part of the total profits that is for external shareholders equals⁴

$$\iota^0(\mathbf{I} - \mathbf{D})\boldsymbol{\pi} = \iota^0(\hat{\mathbf{p}} - \hat{\mathbf{c}})\mathbf{q}$$

which equals the total operating earnings of the industry.⁵

3 Cournot Oligopolists

In this section the behavior of n firms who have silent interests in each other is analyzed in a Cournot-setting. In particular we are interested in the effects of cross-shareholding on the price-cost margin. It is assumed that the firms produce homogeneous goods, all face a constant returns to scale production function and all have the same constant unit cost of production. So, $p_i = p$ and $c_i = c$ for all i . The profit of firm i then becomes

$$\pi_i = (p - c) \sum_k l_{ik} q_k$$

The firms have silent interests in each other, this implies that firms can only control their own level of output. Firms take the amount of cross-shareholding as given. The inverse market demand function

³Reynolds and Snapp (1986) analyze a model that takes account only of direct shareholding. In the present notation, the model would be given as $\mathbf{L} = (\mathbf{I} + \mathbf{D})(\hat{\mathbf{p}} - \hat{\mathbf{c}})\mathbf{q}$, see also (Flath, 1992a). The results for both models coincide only in the exceptional case when indirect shareholding is absent, i.e. when $\mathbf{D}^k = \mathbf{0} \forall k \geq 2$. In general, $\mathbf{L} > (\mathbf{I} + \mathbf{D})$.

⁴ ι^0 denotes the summation vector, consisting of ones, as a column. An accent is used to denote transposition. So, $\iota^0 = (\mathbf{1}, \dots, \mathbf{1})$.

⁵As Flath (1992b) points out, equation (3) gives a relation between capitalization and operating earnings. If \mathbf{D} increases also \mathbf{L} will increase and market capitalization will rise, given constant operating earnings.

is $p = f(Q)$, where $Q = \sum_k q_k$ is the aggregate demand. Assume that the price-elasticity of the demand for product k , that is $\varepsilon_k = (dq_k/dp)(p/q_k)$, is the same for all k (i.e. $\varepsilon_k = \varepsilon \forall k$). Then

$$\begin{aligned}\frac{\partial p}{\partial q_k} &= \frac{dp}{dQ} \frac{\partial Q}{\partial q_k} = \frac{dp}{dQ} = \left(\frac{dQ}{dp}\right)^{-1} = \left(\sum_k \frac{dq_k}{dp}\right)^{-1} \\ &= \left(\sum_k \varepsilon_k \frac{q_k}{p}\right)^{-1} = \left(\varepsilon \sum_k \frac{q_k}{p}\right)^{-1} = \left(\varepsilon \frac{Q}{p}\right)^{-1} = \frac{p}{\varepsilon Q}\end{aligned}$$

The first order condition of the maximization problem of the firm is

$$\begin{aligned}\frac{\partial \pi_i}{\partial q_i} &= \frac{\partial p}{\partial q_i} \sum_k l_{ik} q_k + (p - c) l_{ii} \\ &= \frac{p}{\varepsilon Q} \sum_k l_{ik} q_k + (p - c) l_{ii} \\ &= 0\end{aligned}$$

Define the output share of firm k as $s_k \equiv q_k/Q$. The first order condition for firm i can then be rewritten as

$$\sum_k l_{ik} s_k = -\varepsilon \frac{p - c}{p} l_{ii}$$

The simultaneous market solution can easily be expressed as

$$\mathbf{Ls} = -\varepsilon \frac{p - c}{p} \mathbf{d}_L$$

where \mathbf{d}_L denotes the column vector with diagonal elements of matrix \mathbf{L} , i.e. $\mathbf{d}'_L = (l_{11}, \dots, l_{nn})$. The output shares are obtained as

$$\mathbf{s} = -\varepsilon \frac{p - c}{p} (\mathbf{I} - \mathbf{D}) \mathbf{d}_L \tag{5}$$

The sum of the output shares is 1 of course, or in matrix notation $\mathbf{1}'\mathbf{s} = 1$. Now we can derive the expression for the price-cost margin

$$m = \frac{p - c}{p} = -\frac{1}{\varepsilon \mathbf{1}'(\mathbf{I} - \mathbf{D}) \mathbf{d}_L} \tag{6}$$

The price-cost margin in the industry depends on the elasticity of demand, and on the matrix \mathbf{D} , which represents the cross-shareholding.

Theorem 1. Assume $\iota' \mathbf{D} \ll \iota'$. Then

$$-\frac{1}{n\varepsilon} \leq m < -\frac{1}{\varepsilon}$$

Proof. See appendix.

Note that the lower bound for the price-cost margin is obtained in the absence of shareholding; i.e. $\mathbf{D} = \mathbf{0}$ implies $m = -1/n\varepsilon$. The upper bound is the price-cost margin in the case of a pure monopolist. It is also the limiting case for n identical firms that hold a share of $1/(n-1)$ in each other. This is the case of a perfect cartel in Flath (1992a).

Example (perfect cartel). Let $d_{ii} = 0 \forall i$ and $d_{ij} = d \forall i, j$ with $i \neq j$. Then

$$l_{ii} = \frac{1}{1+d} \frac{1-(n-2)d}{1-(n-1)d} \text{ and } l_{ij} = \frac{1}{1+d} \frac{d}{1-(n-1)d}$$

Hence

$$\frac{p-c}{p} = -\frac{1}{\varepsilon \iota' (\mathbf{I} - \mathbf{D}) \mathbf{d}_L} = -\frac{1}{n\varepsilon} \frac{1+d}{1-(n-2)d}$$

If $d \rightarrow 1/(n-1)$, then $(p-c)/p \rightarrow -1/\varepsilon$.

Next, we consider the sensitivity of the price-cost margin and the shares with respect to increases in shareholding. Theorem 2 states that when firm i increases the shares in firm j , the margin must also increase.

Theorem 2. Let $\bar{d}_{ij} > d_{ij}$ and assume that $\iota' \mathbf{D} \ll \iota'$, then $\bar{m} > m$.

Proof. See appendix.

Observe that the same result holds for the price, since $\bar{m} > m$ if and only if $\bar{p} > p$.

It should be emphasized that the output shares in equation (5) may become negative. Note that the output shares are obtained from equations (5) and (6) as

$$\mathbf{s} = (\mathbf{I} - \mathbf{D}) \mathbf{d}_L / \left[\iota' (\mathbf{I} - \mathbf{D}) \mathbf{d}_L \right].$$

If firm i holds large amounts of shares in other firms, its output share tends to be small and may even become negative. A convenient example to illustrate this is when \mathbf{D} is an upper-triangular matrix (i.e.

$d_{ij} = 0 \forall i \geq j$). In that case $\mathbf{d}_L = \boldsymbol{\iota}$ and the output shares are given as $(\mathbf{I} - \mathbf{D}) \boldsymbol{\iota} / [\boldsymbol{\iota}' (\mathbf{I} - \mathbf{D}) \boldsymbol{\iota}]$. Now if the total amount of shares that firm i holds in other firms is larger than 1, the output share of firm i becomes negative. In this example it is also easily seen that when firm i increases its share in some other firm, s_i will fall and the output shares of the other firms will slightly increase with the same percentage.

The results for the general case are extremely complex and depend upon the specific values of the elements d_{ij} . Yet, the results from the stylized upper-triangular example seem to carry over to the general case. That is, if firm i increases its share in some other firm, the output share s_i tends to fall. Also, if the total amount of shares as held by firm i becomes large, s_i may become negative. On the one hand, since negative output shares as an outcome of the model are unacceptable, these findings limit the applicability of the model to shareholding matrices with relatively small elements. On the other hand, in analyzing *silent* interests one typically expects the shares to be fairly small, so that this restriction does not seem to be very binding.

4 Bertrand Oligopolists

In this section we analyze the consequences of cross-shareholding in a Bertrand-setting. The n firms produce heterogeneous goods and all have constant returns to scale production functions, although their unit costs may differ. The market demand function is $\mathbf{q} = F(\mathbf{p})$. For firm i the profits equal

$$\pi_i = \sum_k l_{ik}(p_k - c_k)q_k$$

Each firm chooses a price to maximize its profits, given the amount of cross-shareholding. The first order conditions $\partial\pi_i/\partial p_i = 0$ yield

$$\frac{\partial\pi_i}{\partial p_i} = \sum_k l_{ik}(p_k - c_k) \frac{\partial q_k}{\partial p_i} + l_{ii}q_i = 0 \quad (7)$$

Throughout the rest of this section it is assumed that the market demand functions have constant elasticities. That is

$$\varepsilon_{ki} \equiv \frac{\partial q_k}{\partial p_i} \frac{p_i}{q_k} = \text{constant} \quad (8)$$

It is furthermore assumed that own-price elasticities are larger than one in absolute value, i.e. $\varepsilon_{ii} \leq -1$. Using $\partial q_k/\partial p_i = \varepsilon_{ki}q_k/p_i$ in (7) gives

$$\sum_k l_{ik}(p_k - c_k)\varepsilon_{ki}q_k = -l_{ii}p_iq_i \quad (9)$$

Write \mathbf{m} for the vector of price-cost margins, i.e. $m_i = (p_i - c_i)/p_i$, and write \mathbf{v} for the vector of output values, i.e. $v_i = p_i q_i$. Then (9) can be rewritten as⁶

$$(\mathbf{L} \circ \boldsymbol{\varepsilon}') \hat{\mathbf{v}} \mathbf{m} = -\hat{\mathbf{d}}_L \mathbf{v} \quad (10)$$

The price-cost margins are now readily obtained as

$$\mathbf{m} = -\hat{\mathbf{v}}^{-1} (\mathbf{L} \circ \boldsymbol{\varepsilon}')^{-1} \hat{\mathbf{d}}_L \mathbf{v}. \quad (11)$$

When shareholding is absent, we have $\mathbf{L} = \mathbf{I}$ and $\mathbf{L} \circ \boldsymbol{\varepsilon}' = \hat{\mathbf{d}}_\varepsilon$, this yields

$$m_i = -1/\varepsilon_{ii}, \quad (12)$$

which is the monopolist's price-cost margin. The next theorem asserts that this monopolist's margin is a lower bound for the margin in the case of shareholding.

Theorem 3. If $\varepsilon_{ii} \leq -1$ and $\varepsilon_{ij} \geq 0 \forall j$, then $m_i \geq -1/\varepsilon_{ii}$ holds for all i .

Proof. See appendix.

5 Application

In the literature, empirical evidence of horizontal shareholding interlocks is rare. Apart from the Japanese *keiretsu*'s, a few examples have been reported in the US, but these ran foul of antitrust laws. Although Flath (1991) argues that in a Bertrand-industry with imperfect substitutes it is rational for firms to acquire interests in their rivals, empirical examples are almost nonexistent.

In this section we present and analyze data on horizontal shareholding interlocks for the Dutch financial sector. According to a new law on shareholding (Wet Melding Zeggenschap 1992) financial interests have to be reported. That is, in the Netherlands every person or institution that acquires 5 percent or more of the stocks (as traded on the market) of a firm, has to announce this interest. Therefore it now has become possible to carry out empirical research on shareholding interlocks.⁷ We have chosen to investigate the financial sector in the Netherlands. In this sector, a few large firms cover almost the entire market. The firms have interests in each other and indirect shareholding exists. Finally, in this sector there are only silent interests, that is firms do not control the output (in the Cournot-case) or the price (in the Bertrand-case) of their rivals. The data on direct shareholding are given in Table 1.

⁶ \circ denotes the Hadamard product of elementwise multiplication. That is, element (i, j) of matrix $\mathbf{A} \circ \mathbf{B}$ is equal to $a_{ij} b_{ij}$.

⁷This implies a restriction to the cases in which the direct interests exceed 5%, however. If the direct interests are below 5% they can still be of relevance. In fact by taking indirect shareholding into account, interests of more than 5% can be obtained, whereas direct interests are below 5% implying that they are not being announced.

Table 1: Direct shareholding in the Dutch financial sector (percentages of the total shares in 1995)

	1	2	3	4
1 ABN AMRO Holding N.V.	-	-	-	-
2 AEGON N.V.	12.85	-	6.25	-
3 ING Groep N.V.	16.57	5.31	-	-
4 Rabobank Nederland	5.64	-	-	-

Source: Annual financial reports for 1995 of the firms.

The table shows the direct share d_{ij} that firm i holds in its rival j (measured as a percentage of the total shares of firm j). For example, AEGON N.V. holds 6.25% of the shares of ING Groep N.V., so $d_{2,3} = 0.0625$. The matrix \mathbf{D} consists of the elements d_{ij} . The inverse matrix $\mathbf{L} = (\mathbf{I} - \mathbf{D})^{-1}$ for the Dutch financial sector yields

$$\mathbf{L} = (\mathbf{I} - \mathbf{D})^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0.1393 & 1.0033 & 0.0627 & 0 \\ 0.1731 & 0.0533 & 1.0033 & 0 \\ 0.0564 & 0 & 0 & 1 \end{pmatrix}.$$

This matrix represents the direct and indirect shareholding interests across the Dutch financial sectors. Indirect shareholding is rather limited, which might be expected given the typical structure of direct interests. Matrix \mathbf{D} is an example of a reducible matrix.⁸ Note that only firms 2 (AEGON) and 3 (ING) have direct interests in *each other*. The only other direct interests are in firm 1 (ABN AMRO). The sum of all indirect shareholding interests is given by

$$\mathbf{L} - (\mathbf{I} + \mathbf{D}) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0.0108 & 0.0033 & 0.0002 & 0 \\ 0.0074 & 0.0002 & 0.0033 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Next we analyze the effects of shareholding by comparing the results for the case of shareholding with those for the case of no shareholding (i.e. $\mathbf{D} = \mathbf{0}$). First, we assume that the Dutch financial

⁸A matrix is defined to be reducible if it can be written by a suitable renumbering of firms (i.e. permutation of rows and columns) as

$$\mathbf{D} = \begin{bmatrix} \mathbf{D}_1 & \mathbf{0} \\ \mathbf{A} & \mathbf{D}_2 \end{bmatrix}$$

where \mathbf{D}_1 and \mathbf{D}_2 are square submatrices.

Table 2: Percentage shares of output (

Table 3: Profits, output and price-cost margins

	π_i	v_i	m_i
1 ABN AMRO Holding N.V.	2368	16205	0.1461
2 AEGON N.V.	1317	20979	0.0405
3 ING Groep N.V.	2603	41203	0.0520
4 Rabobank Nederland	1427	8609	0.1502

Source: Annual financial reports for 1995 of the firms.

$v_i = p_i q_i$ and the profits π_i , allow for calculating price-cost margins m_i for each firm separately. If the Dutch financial sector would be in its Cournot-optimum the price-cost margins should be equal to each other ($m_i = m \forall i$) and firm i 's share in the total output value should be equal to s_i . The additional data and the results are given in Table 3.

The price-cost margins in the Dutch financial sector are clearly not uniform. This indicates that this sector does not follow a Cournot-model, and that a Bertrand-model is probably more appropriate.

Next, we empirically examine the effects of shareholding, assuming that the Dutch financial sector follows a Bertrand-model. Let m_i denote the price-cost margin in the case of shareholding, as given in equation (11). The price-cost margin for the case without shareholding is denoted by m_i^0 and equals $-1/\varepsilon_{ii}$, see equation (12). The ratio of the two price-cost margins yields

$$m_i/m_i^0 = -\varepsilon_{ii}m_i \quad (13)$$

or, in matrix terms

$$\mathbf{m}(\hat{\mathbf{m}}^0)^{-1} = \hat{\mathbf{v}}^{-1} \hat{\mathbf{d}}_\varepsilon (\mathbf{L} \circ \varepsilon')^{-1} \hat{\mathbf{d}}_L \mathbf{v}. \quad (14)$$

The data for the Dutch banking sector are such that, next to \mathbf{D} and \mathbf{L} , also output values v_i and price-cost margins m_i are known (see Table 3). The only difficulty in calculating the ratios between the price-cost margins in (13) is that the matrix ε of elasticities is unknown. Under certain assumptions, however, these elasticities can be deduced from available data.

First, assume that the matrix ε of elasticities takes the following form

$$\varepsilon = \begin{bmatrix} -\varepsilon_{11} & & \delta \\ & \ddots & \\ \delta & & -\varepsilon_{nn} \end{bmatrix} \quad (15)$$

Table 4: ‘Revealed’ own-price elasticities in the Dutch financial sector

ABN AMRO	-6.84
AEGON	-25.22
ING	-19.46
Rabo	-6.76

implying that all cross-price elasticities are the same and equal to $\delta > 0$.

Note that $\varepsilon = \delta \mathbf{E} - (\delta \mathbf{I} + \hat{\mathbf{d}}_\varepsilon)$ where \mathbf{E} is the matrix with all entries equal to one. Equation (11) can be rewritten as $(\mathbf{L} \circ \varepsilon') \hat{\mathbf{v}} \mathbf{m} = -\hat{\mathbf{d}}_L \mathbf{v}$. Observe that $\mathbf{L} \circ \varepsilon' = \delta \mathbf{L} - (\delta \hat{\mathbf{d}}_L + \hat{\mathbf{d}}_\varepsilon \hat{\mathbf{d}}_L)$, which yields

$$\delta \mathbf{L} \hat{\mathbf{v}} \mathbf{m} - \delta \hat{\mathbf{d}}_L \hat{\mathbf{v}} \mathbf{m} - \hat{\mathbf{d}}_\varepsilon \hat{\mathbf{d}}_L \hat{\mathbf{v}} \mathbf{m} = -\hat{\mathbf{d}}_L \mathbf{v}.$$

Solving for \mathbf{d}_ε gives the following expression

$$\mathbf{d}_\varepsilon = \delta \hat{\mathbf{v}}^{-1} \hat{\mathbf{m}}^{-1} \hat{\mathbf{d}}_L^{-1} \mathbf{L} \hat{\mathbf{v}} \mathbf{m} - \delta \boldsymbol{\iota} + \hat{\mathbf{m}}^{-1} \boldsymbol{\iota} \quad (16)$$

Next, it should be noted that the ratio between the price-cost margins in (13) is insensitive to scalar multiplication. That is, define $\bar{\varepsilon} = \lambda \varepsilon$, then $\hat{\mathbf{d}}_\varepsilon = \lambda \hat{\mathbf{d}}_\varepsilon$ and $(\mathbf{L} \circ \bar{\varepsilon}')^{-1} = \frac{1}{\lambda} (\mathbf{L} \circ \varepsilon^0)^{-1}$ so that $\mathbf{m}(\hat{\mathbf{m}}^0)^{-1}$ in (14) is unaffected. This implies that, for the purpose of calculating the ratios in (13) or (14), we may choose $\delta = 1$.

Given data for \mathbf{L} , \mathbf{v} and \mathbf{m} , assuming that the banking sector is in a Bertrand optimum and assuming that the matrix ε of elasticities takes the form in (15), equation (16) with $\delta = 1$ may be used to ‘reveal’ the own-price elasticities. The results are given in Table 4.

Two major conclusions can be drawn from the results in Table 4. First, the own-price elasticities are substantially larger (in absolute value) than the cross-price elasticities (which have been set at one). Second, the own-price elasticities of two firms (AEGON and ING) are three to four times as large as those of the other two firms (Rabobank and ABN AMRO). The first observation may be explained by the ‘sticky’ behavior of the customers. Relatively few customers react upon price changes by leaving their ‘own’ bank for another bank. In competing their rivals it might well be that other, non-price factors are a more effective tool than prices. The second observation may be explained by the fact that for both firms with the large elasticities, insurances account for a major part of their services. The other two firms, with the lower elasticities, are basically confined to banking services. The results indicate that the market for insurances is characterized by customers which are very ‘sticky’.

Given the observations for m_i and given the calculated own-price elasticities ε_{ii} , the ratio between the price-cost margins is $-m_i \varepsilon_{ii}$. The results are given in Table 5. They show that if the Dutch financial sector is in its Bertrand-optimum, shareholding has increased the price-cost margins of the firms,

Table 5: Ratio of the price-cost margins

ABN AMRO	1.000
AEGON	1.022
ING	1.011
Rabo	1.016

but only marginally. Note that the price-cost margin for ABN AMRO is not affected by shareholding. This is again explained by the fact that this firm holds no shares in its rivals. The following theorem gives the general result.

Theorem 5. If firm i holds no interest in any other firm, its price-cost margin remains constant at $-1/\varepsilon_{ii}$.

Proof See appendix.

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Proof of Theorem 1

(i) $m \geq -1/n\varepsilon$. Consider $\iota'(\mathbf{I} - \mathbf{D})\mathbf{d}_L$ and observe that $\iota'(\mathbf{I} - \mathbf{D}) \gg 0$ and $\mathbf{d}_L \leq \mathbf{L}\iota$. Then $\iota'(\mathbf{I} - \mathbf{D})\mathbf{d}_L \leq \iota'(\mathbf{I} - \mathbf{D})\mathbf{L}\iota = \iota^0\iota = n$.

(ii) $m < -1/\varepsilon$. Metzler's theorem⁹ states that $l_{ij} < l_{ii}$ for all j and i , when $\iota^0\mathbf{D} \ll \iota^0$. Hence $\mathbf{L}\iota < n\mathbf{d}_L$. Thus $\iota'(\mathbf{I} - \mathbf{D})\mathbf{d}_L > (1/n)\iota'(\mathbf{I} - \mathbf{D})\mathbf{L}\iota = 1$.

Proof of Theorem 2

Let $\bar{d}_{ij} = d_{ij} + \delta$ or in matrix notation, $\bar{\mathbf{D}} = \mathbf{D} + \delta\mathbf{e}_i\mathbf{e}_j'$, where \mathbf{e}_j is the j th unit vector, i.e. $\mathbf{e}_j' = (0, \dots, 0, 1, 0, \dots, 0)$. Then, see e.g. Henderson and Searle (1981),

$$\bar{\mathbf{L}} = (\mathbf{I} - \bar{\mathbf{D}})^{-1} = \mathbf{L} + \frac{\delta}{1 - \delta l_{ji}} \mathbf{L}\mathbf{e}_i\mathbf{e}_j' \mathbf{L}$$

which implies for the diagonal elements

$$\bar{l}_{hh} = l_{hh} + \frac{\delta}{1 - \delta l_{ji}} l_{hi}l_{jh} = l_{hh} + \Delta l_{hi}l_{jh}$$

where $\Delta = \delta/(1 - \delta l_{ji})$. Consider the elements of the vector $\bar{\varphi} = (\mathbf{I} - \bar{\mathbf{D}}) \mathbf{d}_L$. For notational convenience let $\bar{\mathbf{T}} = (\mathbf{I} - \bar{\mathbf{D}})$, so $\bar{\varphi} = \bar{\mathbf{T}}\mathbf{d}_L$. Distinguish between $\bar{\varphi}_k$ with $k \neq i$ and $\bar{\varphi}_i$.

$$\begin{aligned} \bar{\varphi}_k &= \sum_h \bar{t}_{kh} \bar{l}_{hh} = \sum_h t_{kh} \bar{l}_{hh} \\ &= \sum_h t_{kh} (l_{hh} + \Delta l_{hi}l_{jh}) \\ &= \varphi_k + \Delta \sum_h t_{kh} l_{hi}l_{jh} \end{aligned}$$

$$\begin{aligned} \bar{\varphi}_i &= \sum_h \bar{t}_{ih} \bar{l}_{hh} = \sum_h t_{ih} \bar{l}_{hh} - \delta \bar{l}_{jj} \\ &= \varphi_i + \Delta \sum_h t_{ih} l_{hi}l_{jh} - \delta (l_{jj} + \Delta l_{ji}l_{jj}) \\ &= \varphi_i + \Delta \sum_h t_{ih} l_{hi}l_{jh} - \Delta l_{jj} \end{aligned}$$

⁹See e.g. Metzler (1945, 1951), or Seneta (1973), Berman and Plemmons (1979), Sierksma (1979) and Dietzenbacher (1997).

Taking the summation yields an expression for $\iota' (\mathbf{I} - \overline{\mathbf{D}}) \mathbf{d}_L$, that is

$$\sum_k \overline{\varphi}_k = \sum_k \varphi_k + \Delta \sum_k \sum_h t_{kh} l_{hi} l_{jh} - \Delta l_{jj}$$

Note that $\Delta \sum_k \sum_h t_{kh} l_{hi} l_{jh} = \Delta \sum_h (\sum_k t_{kh}) l_{hi} l_{jh}$. By assumption $\sum_k t_{kh} > 0 \forall h$ and $l_{jh} < l_{jj} \forall h$ according to Metzler's theorem. Hence $\Delta \sum_h (\sum_k t_{kh}) l_{hi} l_{jh} < \Delta l_{jj} \sum_h \sum_k t_{kh} l_{hi} = \Delta l_{jj} \sum_k \sum_h t_{kh} l_{hi}$. Note that $\sum_h t_{kh} l_{hi}$ equals the element (k, i) of $\mathbf{T}\mathbf{L} = \mathbf{I}$. Thus $\sum_k \sum_h t_{kh} l_{hi} = 1$. This yields $\sum_k \overline{\varphi}_k < \sum_k \varphi_k$ or $\iota' \overline{\varphi} < \iota' \varphi$. Using $m = -1 / (\varepsilon \iota' \varphi)$, this proves the result.

Proof of Theorem 3

Equation (9) can be rewritten as $\sum_k l_{ik} m_k \varepsilon_{ki} v_k = -l_{ii} v_i$. This yields

$$-v_i = \sum_k (l_{ik}/l_{ii}) m_k \varepsilon_{ki} v_k = m_i \varepsilon_{ii} v_i + \sum_{k \neq i} (l_{ik}/l_{ii}) m_k \varepsilon_{ki} v_k \geq m_i \varepsilon_{ii} v_i. \text{ Hence } -m_i \varepsilon_{ii} \geq 1 \text{ or } m_i \geq -1/\varepsilon_{ii}.$$

Proof of Theorem 5

The price-cost margin without shareholding is $m_i^0 = -1/\varepsilon_{ii}$. The price-cost margins in the case of shareholding are given by $\mathbf{m} = -\hat{\mathbf{v}}^{-1} (\mathbf{L} \circ \varepsilon^0)^{-1} \hat{\mathbf{d}}_L \mathbf{v}$. If $d_{ij} = 0 \forall j$, then $l_{ij} = 0 \forall j \neq i$ and $l_{ii} = 1$. Thus element (i, j) of $(\mathbf{L} \circ \varepsilon^0)$ is zero $\forall j \neq i$ which implies that also element (i, j) of $(\mathbf{L} \circ \varepsilon^0)^{-1}$ equals zero $\forall j \neq i$. Element (i, i) of $(\mathbf{L} \circ \varepsilon^0)$ equals ε_{ii} and element (i, i) of $(\mathbf{L} \circ \varepsilon^0)^{-1}$ yields $1/\varepsilon_{ii}$. Substitution into the expression for \mathbf{m} gives $m_i = -1/\varepsilon_{ii}$.