MODELING PERSONAL CONSUMPTION AND GOVERNMENT TRANSFERS IN A LONG-RUN MACROECONOMETRIC FORECASTING MODEL

by

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Dissertation submitted to the Faculty of the Graduate School
of the University of Maryland in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
1996

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ABSTRACT

Title of Dissertation: Modeling Personal Consumption and Government Transfers in a Long-run Macroeconometric Forecasting Model

Jeffry John Janoska, Doctor of Philosophy, 1996

Dissertation directed by: Professor Clopper Almon
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This study presents the specification and estimation of an eighty commodity system of aggregate Personal Consumption Expenditure Equations that treat the Medicare program as a Price subsidy. Previous work treated Medicare as an income transfer. The system of equations show that when Medicare is treated as an income transfer, any change in the size of Medicare will affect the categories with large income elasticities, but when Medicare is treated as a price subsidy, the categories most affected are medical goods and services.

Because the distribution of income affects consumer expenditures, a functional form of the Lorenz curve was found that not only allowed for an exact fit of the known data, but also allowed the smooth forecasting of the Lorenz curve. The forecast of the Lorenz curve was possible through the use of previously unknown properties of Lorenz curves.

Demographic factors also play an important role in determining consumer expenditures. As part of this study, a model was constructed that gave consistency between population forecasts and various indirect-age demographic characteristics of the population.

Chapter 1 gives a brief introduction to the study. Chapter 2 reviews systems of consumer demand equations and ranks them on their suitability for a long-term

forecasting model. Chapter 3 describes the system of equations selected. In chapter 4, previously unknown properties of Lorenz curves are discussed and a model is constructed that forecasts the Lorenz curve. Chapter 5 describes the new treatment of Medicare as a price subsidy. Chapter 6 describes the Demographic Projections Model that forecasts the size and age structure of the population as well as various indirect-age demographic variables. Chapter 7 presents several simulations that highlight the importance of both treating Medicare as a price subsidy and consistently forecasting population and indirect-age demographics. Chapter 8 contains concluding remarks.

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ACKNOWLEDGEMENTS

I would like to thank Professor Clopper Almon for not only being my dissertation advisor, but also for the opportunity to be a part of the INFORUM Project at the University of Maryland. The opportunity to work with him and the staff at the project has been invaluable in my growth as an economist. I am grateful to everyone at INFORUM who have all provided me with assistance, a sympathetic ear and patience. In particular, I would like to thank: Dr. Ralph Monaco for being in the office next to mine where I could always find him; Dr. Lorraine Sullivan Monaco for a conversation one night that saved much more than just this dissertation; and Amy Carr for being a sounding-board for ideas. I would like to thank everyone in my family for not asking when I would finish. I would also like to acknowledge and thank the Health Care Financing Administration for funding much of this work, though I bear the responsibility for any opinions or errors expressed here. Finally, I would like to thank Christine Wnuk for being extremely supportive for the last several months and for baking many batches of cookies that always lifted my spirits when I was at my lowest.

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CHAPTER 1

INTRODUCTION

Personal consumption expenditures account for over two-thirds of U.S. Gross Domestic Product. Since changes in the size and composition of consumption expenditures alter the size and composition of industry outputs, employment and investment, any comprehensive economic forecasting model must include a forecast of personal consumption expenditures. Reasonable forecasts of consumption should be guided by economic theory of the consumption behavior of individuals.

Economic theory not only suggests that income and relative prices determine consumption, but also provides a guide to the relationship that exists between income, prices and consumption. For example, well-known consumer theory states that, under very specific conditions, both the "adding-up" and the "homogeneity" constraints must hold for aggregate demand functions. Nearly all models of personal consumption make use of these axioms of consumer theory when estimating expenditure equations.

Typically, these models use disposable income as defined by the National Income and Product Accounts (NIPA) as their income variable - where disposable income equals personal income less taxes, mandatory social insurance contributions and non-tax payments to the government. One part of this study will show that NIPA-style disposable income is an inappropriate choice for the income variable in these models. This is because disposable income, as defined by NIPA, includes many items that are neither income nor disposable.

The starting point for my work is the system of personal consumption expenditure functions used by INFORUM's LIFT, a long-term, inter-industry macroeconometric model.[1] The basis of the LIFT model's personal consumption expenditure functions is the Almon system of symmetric consumption equations (Almon 1978). Previous work on the Almon system incorporated modifications that allow income distribution and demographic variables to influence the composition of consumption (Devine 1983, Chao 1991, Janoska 1994a).[2] This work modifies the LIFT system to better account for the various types of government transfers to persons that are typically included in disposable income.

As mentioned above, the system allows demographic variables and the distribution of income to influence personal consumption expenditures. This implies that an accurate forecast of consumption using the Almon system also must include accurate forecasts of demographic variables and the income distribution. As part of my work, I have constructed an endogenous income distribution model. Similarly, I have also constructed a simple model that forecasts the required demographic variables.

The income distribution model forecasts the distribution of personal taxable income and then assesses personal taxes based on tax rates and the forecasted income distribution. We then convert this after-tax distribution of income into a distribution of spendable, disposable income by adding appropriate components of non-taxed personal income.

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[1]LIFT, the Long-term Inter-industry Forecasting Tool, was developed under the guidance of Clopper Almon at the University of Maryland. McCarthy (1991) provides an excellent overview of LIFT.

[2]The Almon system as modified by the later work will be referred to as the LIFT system of equations.

The system of consumption functions used by LIFT allows forecasters to evaluate the macroeconomic impacts of changes in certain demographic variables. Examples of these demographic variables include:

> The age distribution of the population: Strong evidence indicates that consumption patterns differ across individuals based on age. Our system of equations allows for these differences through the use of a set of Adult Equivalency Weights (AEWs). Thus, the system captures the effects of events such as the birth and aging of the baby-boom generation.
>
> Region: Differences in climate and taste lead to differences in spending patterns across households. For example, households in the Northeast spend more on heating oil than households in the South. Our model controls for these differences and includes their effects when forecasting personal consumption.
>
> Household Size: Large households can capitalize on economies-of-scale. Large households tend to have lower spending per capita on durables and services than smaller households.

These demographic variables are all interrelated. For example, we would expect an increase in the number of children to affect the various demographic variables. If we wanted to forecast the effects of an increase in the fertility rate, we would see an immediate effect of an increase in the number of children in the population compared to the number of children in a simulation without an increase in fertility.

However, children do not form new households. Thus, we should see an increase in the average size of households and very little effect, if any, on the number of households. As these children reach adulthood, we should see a fall in the average size of households and an increase in the number of households. Later, as these adults marry and have children of their own, we again should see effects on the demographic variables.

Until the work reported here, LIFT lacked a link between the age structure of the population and the number of households in each household size. As part of this study, I construct a model that forecasts the various non-age demographic variables.[3] This demographic model allows us to evaluate in a consistent manner the macro and inter-industry effects of various immigration policies as well as the effects of increased fertility or higher death rates among the elderly.

In chapter 2, I review the existing literature on demand equations and establish the criteria used in selecting a system of demand equations. Chapter 3 describes the current system used in LIFT. Chapter 4 describes the new income distribution model. Chapter 5 describes the new treatment of the Medicare program. Chapter 6 describes the non-age demographic model. In chapter 7, I present the results of several simulations that highlight the work described in the previous chapters. Chapter 8 contains concluding remarks.

[3]See Chapter 3 for a complete list of these variables.

CHAPTER 2

A REVIEW OF THE LITERATURE

There has been a long search for a functional form for an aggregate system of consumer demand that simultaneously satisfies the axioms of economic theory and the properties required for applied work. The search has concentrated on satisfying the theoretical constraints of consumer theory and has tended to ignore the demands of applied economics. A functional forms that rests on a sound theoretically foundation, yet cannot duplicate the results we see in the real world, is of little value. One can easily see why the literature concentrates on meeting theoretical constraints; there is a strong consensus among the profession regarding the implications of consumer theory. Consensus is lacking, however, as to the empirical properties that the demand system should possess. Any functional form lacks generality, for it makes something -- its parameters -- constant, but there is no consensus on what is, in fact, constant. This lack of consensus is understandable, since the criteria for determining the appropriate applied properties are entirely subjective. Like beauty, the relevance of applied properties is in the eye of the beholder.

As I later demonstrate, much of the effort towards finding a functional form that satisfies the constraints implied by the theory of the individual consumer can be categorized as nonproductive since the results of the theory of the single consumer may not apply to an aggregate demand system. Only under very restrictive assumptions are all the results for the single consumer valid for an aggregate demand function, and since empirical work has rejected these assumptions, the search for an appropriate functional form loses objective criteria and is instead left with the subjective criteria of "good" applied properties.[4] This reliance, however, requires a definition of good or useful empirical properties and there is no consensus on this.

One can define a good empirical property as any property that ensures consistency with known empirical facts. Thus, a functional form should allow for a range of *ex-ante* parameters that are consistent with the empirical facts. This definition is vague and of little help since economists acknowledge few such facts (Lau 1986).[5] There are, however, a few such facts. For example, one generally-acknowledged and consistently confirmed fact is Engel's Law, which says that the demand for food has an income elasticity of less than unity (Houthakker 1957, 1965). A functional form derived from a homothetic direct or indirect utility function would violate Engel's Law since homotheticity implies that all goods have an income elasticity of unity (Lau 1986).

When defining what constitutes a useful property, one must also distinguish between *ex-ante* and *ex-post* properties of a functional form. For example, one might reject a functional form if, by assumption, the form required that all goods are substitutes. However, if the functional form allowed goods to be either complements or substitutes, then one might be perfectly satisfied with the form - even if the estimated parameters of the system indicated that all goods were substitutes. In the first example, universal substitutability of all goods is imposed *ex-ante* and the

[4]For example, the assumption that individual expenditure functions are of the PIGLOG (Price Independent General Linear) family implies that the aggregate demand system must exhibit Slutsky symmetry (Muellbauer 1975, 1976; Deaton and Muellbauer 1980). Since the estimated coefficients reject the hypothesis of Symmetry in the aggregate demand function, one must also reject the underlying assumption of PIGLOG expenditure functions.

[5]Following Leamer (1983), I define a "fact" as an opinion held "by all, or at least by a set of people you regard to be a close approximation to all (Leamer, 1983, pp.37)."

functional form is considered inappropriate. In the second example, universal substitutability of all goods is an empirical result, not an assumption. In general, when determining whether a functional form is useful, one should be concerned with the *ex-ante* properties of the model only.

The luckless modeler who seeks to satisfy these subjective *ex-ante* properties quickly realizes that properties which please one audience will displease a different audience. Furthermore, properties that please an audience at one level of aggregation (for example, a simple three-sector consumption model), may displease the same audience with a different level of aggregation (for example, an eight hundred-sector consumption model). A good example is provided by the Linear Expenditure System. In this system:

$$p_i q_i = \alpha_i p_i + \beta_i \left(M - \sum_j \alpha_j p_j\right) \qquad (2.1);$$

where:

p_i = the price of good i;

q_i = the quantity purchased of good i;

M = income;

α_i, β_i = the estimated parameters.

Since the sum of expenditures cannot exceed total expenditures, the β_i are constrained such that $\sum_i \beta_i = 1$. Since consumption must be positive, all of the β_i are further constrained as non-negative. Thus, the existence of inferior goods is ruled out by assumption. Additionally, the parameter α_i indicates whether the i^{th} good is a universal complement or universal substitute. That is to say, the good will substitute for all other goods or it will complement all other goods. By assumption, the system given in (2.1) eliminates the possibility that a good is a substitute in some instances and a complement in other instances. However, the usefulness of (2.1) is dependent on many factors -- all subjective.

For example, if the demand system only included three goods -- services, durables, and non-durables -- then (2.1) might avoid rejection. For example, since few believe that any of these three broad categories of consumption are inferior, the prohibition against inferior goods in (2.1) does not pose a serious problem. However, if the demand system had five hundred sectors, the assumption that no goods are inferior is more objectionable.

Similarly, one might accept the idea that in a three-sector (durables, non-durables and services) system of demand, a given commodity is either a substitute or complement with all other goods. However, in a large multi-commodity demand system, one would expect a good to complement some goods and substitute for others.

Thus, when choosing the "correct" functional form for the aggregate demand system, the modeler must base his choice on a set of subjective criteria that themselves are based on subjective criteria. All of this illustrates the difficult choices that a modeler faces. Since no single functional form satisfies all economists, there can be no iron-clad, detailed criteria for determining the correctness of a functional form. While economists do agree on three very broad criteria for any demand system, these criteria -- consistency with theory, ease of estimation, and good empirical qualities, including good fit and predictive performance (Barten 1993) -- are defined so broadly that they are of little use in determining the correct functional form.[6]

[6]Barten (1977), Almon (1979) and Lau (1986) address the issue of subjective criteria.

This chapter is divided into three sections. The first section of this chapter reviews the theory of consumer choice as well as the constraints the theory places on the functional form for any one individual. The first section also discusses why these constraints might be invalid for a functional form of the aggregate consumer demand system. The second section of this chapter presents the subjective criteria I use in the current work. These criteria, like any subjective criteria, are not universal and may inspire criticism. However, unlike many authors, I present my criteria for examination and do not rely on unstated subjective decisions when I choose my functional form. The third section reviews many of the more well-known systems of demand equations and lists the areas in which they fail to satisfy the demands I place on a functional form.

Theoretical Consistency and a System of Aggregate Demand Equations

At the foundation of every econometric model is a theory. Sometimes the theory is well-developed and formalized - for example, the Theory of Utility Maximization - and sometimes the theory is nothing more than a strong belief that a particular variable should be included in the model. Theory provides the framework on which to hang empirical work.[7] Systems of consumer demand equations rely on the theory of utility maximization (also known as consumer theory) as their theoretical guide.

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[7]"The role of the theory is to support empirical analysis (Barten, 1977, pp.25)." See Leamer (1983) for an example of how *a-priori* beliefs influence econometric analysis.

Any functional form claiming to represent the demand system of a consumer must possess certain properties to satisfy constraints from consumer theory.[8] These constraints are well-known and generally are accepted as facts. Consumer theory by itself, however, provides no constraints on the functional form for the system of demand of an aggregation of individuals. That is to say, when we discuss the demands of a single consumer, then consumer theory is a valid guide, but when we discuss market demand, consumer theory is a less-valuable tool. Only under additional assumptions regarding consumer preferences and the distribution of income does consumer theory provide constraints on the aggregate demand system.

This section consists of two sub-sections. The first sub-section briefly reviews the theoretical constraints for any individual's system of demand equations. The second sub-section discusses when these constraints are invalid and when they are valid.

A. The Individual Consumer's System of Demand Equations

Let q_i be an n-vector of quantities of commodities and services (hereafter goods) for the i^{th} consumer, m_i equals total expenditures by the i^{th} consumer, and p is the n-vector of the prices per-unit of those goods. Then, one can write:

[8]For a derivation of Consumer Theory, see Varian (1984), Deaton (1986) or Deaton and Muellbauer (1988).

$$q_i = q_i(m_i, p) \qquad (2.2);$$

$$m_i = p' q \qquad (2.3).$$

Equations (2.2) and (2.3) represent a complete set of demand functions for any individual consumer, where the subscript, i, denotes the i^{th} agent. If one assumes these equations are differentiable with respect to m and p, one can write:[9]

$$dq_i = q_{i,m}\, dm_i + Q_{i,p}\, dp \qquad (2.4);$$

where:

$q_{i,m}$ is the n-vector of derivatives with respect to income for the i^{th} consumer;

$Q_{i,p}$ is the n x n matrix of derivatives of demand with respect to p for the i^{th} consumer.

Using these four equations and the standard axioms of the theory of utility maximization, one can derive the set of theoretical constraints that equation (2.2) must meet.

A1. Additivity: The sum of the consumer's spending as given by the system identically equals total expenditure. This is written:

$$p' q_{i,m} = 1 \text{ (The Engel Aggregation)} \qquad (2.5);$$

$$p' Q_{i,p} + q_i' = 0 \text{ (The Cournot Aggregation)} \qquad (2.6).$$

The Engel Aggregation says that if income increases, the sum of the change in spending on all goods must equal the change in income.[10] The Cournot Aggregation says that if the price of the i^{th} good changes, then total spending is unaffected.

[9]It is a standard assumption in utility theory that these demand functions are twice differentiable with respect to m and p.

[10]Saving, since it is used for future consumption, is considered a good.

A2. Homogeneity: There can be no money illusion. If all prices and the consumer's income are multiplied by some scaler, α, then demand is unchanged. This is written:

$$q_i = q_i(\alpha m_i, \alpha p) \qquad (2.7).$$

Using Euler's Theorem, (2.6) can be written:

$$q_{i,m}\, m_i + Q_{i,p}\, p = 0 \qquad (2.8).$$

A3. Slutsky Symmetry: The derivative of the consumer's income-compensated partial derivative of the demand for good X with respect to the price of good Y must equal the consumer's income-compensated partial derivative of the demand for good Y with respect to the price of good Y. This can be written:

$$S^i = Q_{i,p} + q_{i,m}\, q_i' = S^{i\prime} \qquad (2.9).$$

where S_i is an n x n matrix particular to the i^{th} consumer. S is often called the substitution matrix and, as is shown in equation (2.9), the matrix is symmetric: the element of the matrix in the j^{th} column and the k^{th} row (s_{jk}) must equal the element in the k^{th} column and the j^{th} row (s_{kj}). This is written $s_{jk} = s_{kj}, \forall\, k,j$.

B. The Aggregate Demand System

Under standard theory, the constraints given in subsection A always apply for an individual's demand system. While A1 and A2 apply to the system that results from the aggregation across all individuals, only under strict aggregation conditions will A3 apply to the this system. If these aggregation conditions are unsatisfied, then Slutsky symmetry will not apply. Furthermore, beyond these broad constraints, consumer theory provides little guidance in determining the desirable properties of a functional form.

An aggregate demand system is the summation across all of the demand systems for all consumers. If we let q_{agg} equal the n-vector of aggregate demands and M equal aggregate expenditures, then, for an aggregate demand system, equations (2.2) and (2.3) are written:

$$q_{agg} = \sum_i q_i = \sum_i q_i(m_i, p) \quad (2.10);$$

$$M = \sum_i m_i = \sum_i p' q_i \quad (2.11).$$

Equations (2.10) and (2.11) show that the aggregate demand system is simply the summation of the individual systems of demand. It is relatively easy to show that the additivity and homogeneity hold if one constructs the aggregate system of demand equations by summing all of the individual demands. Thus, if the aggregate demand system is the summation of all of the individual demands, the macro system exhibits the same properties as the micro.

The empirical construction of such a system, however, is impossible. The system requires time-series consumption and income data for every individual in the economy. These data do not exist. Typically, the only time-series of consumption and income data available are aggregate consumption and income values.[11,12] Thus, one must determine the conditions under which additivity, homogeneity and Slutsky symmetry are valid if the system is constructed using aggregate consumption and disposable income.

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[11]While the aggregate consumption data for any good is often constructed by the summation across all - actually a sample - purchases of the good, the data do not record the income of the individual making the purchase (U.S. Department of Commerce, 1990). Thus, the detailed data is of little-value except when aggregated.

[12]Typically, per-capita data are also available, but there is little difference between constructing the system with aggregate versus per-capita data.

That these constraints might be invalid for aggregate demand functions is well-known (Muellbauer 1975, 1976; Barten 1977, 1993; Almon 1979; Deaton and Muellbauer 1980). Establishing the conditions under which these constraints hold has been a goal of the literature. Work by others (Gorman 1961; Muellbauer 1975, 1976; Barten 1977) has shown that the homogeneity and Slutsky symmetry constraints are valid only if one of the following are true:

> B1. Representative Agent: All persons in the economy must have identical preferences and identical income. This income equals the average, or per-capita level of income and each agent consumes the average, or per-capita, level of each and every good in the economy (Muellbauer 1975, 1976; Barten 1977, 1993);
>
> B2. PIGLOG Cost Functions: If each consumer has identical preferences and these preferences give rise to individual cost functions that are members of the "Price Independent Generalized Linear" (PIGLOG) family of cost functions, then use of average consumption and income data insures that the three constraints are valid (Muellbauer 1975, 1976; Deaton and Muellbauer 1980).
>
> B3. Construction by Summation of Demand Across All Individuals: If one constructs the aggregate demand system by a summation across the individual demand systems, then the three constraints from utility theory are valid. However, in one sense, the aggregate demand system does not exist. Instead, one has many individual demand systems that, when summed, create something called aggregate demand. If, instead, the system is constructed from aggregate data, Slutsky symmetry will not hold since there is no such thing as an aggregate compensated demand function unless conditions B1 or B2 hold.

Thus, except in the case where individual data is available for the entire economy, homogeneity and symmetry are invalid constraints unless one assumes that consumers have identical preferences. Additionally, one must assume that either all consumers have identical income - a very strong assumption that clearly is invalidated by the observed data, or one must assume that consumers' preferences are of a specific form so as to generate cost functions of the PIGLOG family - an equally strong assumption that empirical evidence seems to invalidate (Deaton and Muellbauer 1980).

Other authors have shown that "... there is, in general, no reason why aggregated market data should obey the same rules as the micro data ...." (Deaton, 1975 pp.525) (see also Barten [1977, 1993], Almon [1979], Deaton [1986] and Janoska [1994c]). However, much of the recent literature forces macro systems to possess homogeneity and symmetry (Alley *et al.* 1992; Van Heeswijk *et al.* 1993; Brenton 1994; Hunt-McCool *et al.* 1994) under all circumstances, when instead, these constraints are called for only if the one of the three conditions (B1, B2, B3) are satisfied.[13,14]

It is easy to illustrate why using aggregate data renders these two constraints invalid. Let q_{agg} equal the n-vector of the aggregate demand for goods, m_{agg} equal aggregate income, and p equal the n-vector of prices per-unit of those goods. Then, the aggregate demand system is written:

$$q_{agg} = q_{agg}(m_{agg}, p) \qquad (2.12);$$

$$m_{agg} = p' q_{agg} \qquad (2.13).$$

Given equations (2.12) and (2.13), Slutsky symmetry will not hold. This result is well known (Deaton 1975, 1976; Barten 1977, 1993; Almon 1979; Chambers 1990). Even though the individual demand curves possess Slutsky symmetry, aggregation over individuals, who consume different quantities of any good and who have different marginal propensities to consume any good, is virtually certain to give market demand functions that do not possess Slutsky symmetry (Almon 1979; Deaton 1986).[15] Given

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[13]This not a recent development. Much of the older literature in this field incorrectly maintained that consumer theory demanded these constraints (Stone 1954; Houthakker 1960; Sato 1972, Christensen *et al.* 1975). Actually, it was the implicit assumption of a representative agent that gave these constraints their validity.

[14]For homogeneity, it is sufficient that all prices and everyone's income increase in the same proportion. However, since such an event is doubtful unless there is a representative consumer, this case is ignored.

[15]Except in the cases specified above.

that Slutsky symmetry is an irrelevant constraint, except under very specific conditions that cannot be true, one is forced to conclude that absolute Slutsky symmetry is neither necessary nor particularly desirable for an aggregate demand system.

It is less well-recognized that homogeneity may not be a desirable property for an aggregate demand system. Homogeneity always is desirable, if the problem is one of units. That is to say, demand should be identical when we measure our variables in dollars or in lire, provided the only change instituted is a change in units. A second instance where homogeneity is desirable in the aggregate demand system is the case when everyone's income and all prices double. In this second case, we expect aggregated demand to be unchanged, however, it is unlikely that this doubling of aggregate income and all prices will be caused by a doubling of every consumer's income.

For example, consider a policy change that doubles all prices as well as aggregate income and alters the distribution of aggregate income in some manner.[16] Homogeneity of the aggregate demand function implies that demand is unchanged by the policy change, despite the shift in the distribution of income. If demand is unchanged by this scenario, then one must either assume that all consumer's have the same marginal propensity to consume out of income, or that the distributional change was of the correct form to induce homogeneity.

The first conclusion implies that consumer's have homothetic preferences -- something the literature has rejected (Barten 1977, 1993; Almon 1979; Lau 1986;

[16]There are many ways in which aggregate income and prices may double and, besides a measurement change, only a few of these is the case where the income of every single individual doubles.

Deaton 1986). The second conclusion, while unlikely for every possible change in the income distribution, is still possible. Though given the dubious nature of the second assumption, one easily could take the view that homogeneity is an invalid and undesirable condition for the aggregate demand functions.

One runs into a theoretical problem, however, if the homogeneity condition is dismissed completely. As already mentioned above, we know that there is at least one case when a doubling of aggregate income and all prices should leave aggregate demand unchanged and that is the case when every consumer's income doubles. In this case, the demand functions must possess homogeneity since neither relative price or the income of any consumer has changed. This means that in at least one case, we know that homogeneity applies. However, it is unclear whether the case where there are equi-proportional changes to all prices and every consumer's income can occur. The relevance of homogeneity depends entirely on the manner in which aggregate income doubles. To solve the paradox, one should incorporate distributional effects into the functional form (Barten 1977).

Some (Deaton 1986) claim there is little need to explicitly incorporate the income distribution into the aggregate demand system since they find little evidence that the income distribution has a significant impact on aggregate demand.[17] Thus, these authors acknowledge that the income distribution does influence demand, but that the influence is so slight that an approximation of the distributional effects is justified. While the evidence is mixed as to whether one should model the effects of the income

[17] According to Deaton, "This position seems defensible in light of the many studies which, ..., have failed to find strong influence of the income distribution on consumer behaviour (Deaton, 1986, pp.1821)."

distribution, one could make an argument that functional forms lacking homogeneity do not violate theoretical consistency and that these forms should not be rejected simply because they lack homogeneity.

There is little debate that additivity is a valid constraint for an aggregate demand system. The truth of this is easily illustrated, since, by definition, total aggregate consumption is identically equal to the summation of consumption by good over all consumers. What is less clear, however, is whether the parameters in the system should be constrained to force this identity, or whether one can make use of an *ex-post* scaling factor to insure that the system satisfies additivity. Ideally, the estimated coefficients from the system are such that additivity always is satisfied; however, given the well-known problems with aggregation across goods (Muellbauer, 1975), one should not reject, *a-priori*, a demand system that fails to satisfy additivity automatically.

Additivity, however, creates difficulty in the estimation of the system. In fact, it is the additivity constraint that makes the collection of demand equations into a system of demand equations. Additivity imposes a constraint across all the equations, thus forcing a joint, or simultaneous estimation of the system. In general, imposing the additivity constraint is relatively simple. However, the estimation of a demand system with more than a handful of commodities can be difficult since the number of price parameters in a demand system without some form of symmetry equals the square of the number of commodities in the system.[18] Thus, symmetry may be useful, if not theoretically correct.

The additivity constraint imposed by the theory of utility maximization is the only theoretical constraint that must be imposed on aggregate demand systems. Since one may impose additivity on the system without deriving the system from either an explicit or implicit utility function, there is no reason why a system must be derived from a utility function. In fact, there are serious problems with deriving the system from an explicit utility function since, in general, demand systems derived from explicit utility functions are empirically uninteresting (Almon 1979; Barten 1993). Take for example, the Cobb-Douglas utility function. The Cobb-Douglas utility function implies a set of demand equations with all income elasticities equal to one and all own-price elasticities equal to negative one.

Unfortunately, a demand system that satisfies additivity but does not mimic the patterns of consumption we observe in the real world and whose only worthwhile property is additivity, is of little use to the modeler. Thus, a modeler must develop his own criteria for determining the usefulness of an aggregate demand system (Almon 1979).[19] The introduction to this chapter gives three broad criteria for an ideal demand system - consistency with theory, ease in estimation, and good empirical properties.[20] Narrowing the scope of these criteria is the area to which I now turn.

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[18]Thus, a demand system with twenty goods would have over 400 jointly estimated parameters. 400 price parameters, 20 income parameters as well as intercepts, stock variables, etc.

[19]Barten (1977) recognizes that simply satisfying the demands of theory does not create an ideal demand system. "....other criteria are used, like convenience, generality, and theoretical relevance (Barten, 1977, p.24)."

[20]See Lau (1986) for a second broadly defined set of qualities.

Useful Empirical Properties of An Aggregate Demand System

Since theory provides little guidance in a search for a functional form that represents the aggregate demand functions, one must find guidance elsewhere. Most authors use theoretical consistency and empirical properties as their criteria. With theory providing little guidance, the usefulness of a functional form should rest on the *a-priori* empirical properties of the functional form.[21] Unfortunately, the usefulness of an empirical property is a subjective criteria and there is little in the way of consensus on which properties are necessary for a functional form and which properties are rubbish.

For example, some authors maintain that for proper welfare analysis there must be consistency between the micro relationships given in the previous section and the macro data (Muellbauer 1975, 1976; Deaton 1986; Barten 1993; Brenton 1994). Thus, one empirical property might be whether the functional form forces the macro data to mimic the micro relationships. For the most part, modelers agree that having the macro data possess the same theoretical properties as the micro data is a "good" thing, the only disagreement is whether this similarity should be enforced at the cost of losing flexibility or making unreasonable assumptions. Other authors consider the flexibility of the functional form as the primary empirical goal and structure their functional form so that it satisfies their notions of how the world works (Almon 1979; Deaton 1986). Neither school of thought is wrong and the opinion of most authors lies between these

[21]By empirical property, I mean that certain desired statistical results should not be proscribed mathematically by the functional form. For example, economists believe that substitution and complementarity are both seen in the real world. The mathematics of the functional form should not restrict the estimated coefficients so that statistical results from the functional form allow for only substitution or only complementarity between goods.

two extremes. In short, the modeler has his own preferences and faces a trade-off between his desire for consistency in the micro-macro relationships and his desire for a functional form that replicates his view of the world.[22]

For example, the Rotterdam model of Theil (1965) and Barten (1969) claims perfect consistency between the micro relationships and macro data as well as flexibility.[23] However, as shown by Yoshihara (1969), achieving this consistency requires imposing constraints on the system so that the form is no longer flexible. According to Deaton (1986), Yoshihara's results do not imply that the Rotterdam model should be rejected, but instead implies "that it is not sensible to impose the restriction; it (relaxing continuity) does not affect the usefulness ... for the approximation and study of the true demands via the approximation ... (Deaton, 1986, pp.1789)." Thus, even among functional forms that claim a consistency between the micro relationships and the macro data is essential, there is room to relax some of the micro relationships.[24]

Given that the "true" function representing aggregate demand can never be known, one must approach the choice of a functional form with the realization that whatever form is chosen can only be an approximation of the true form (Deaton 1986). This realization makes the choice of functional form a less daunting task since the

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[22]See Monaco (1991) for an in-depth discussion on a related topic.

[23]Here, I am using the Diewert's (1971) definition of flexibility. That is, there must be at least one parameter estimated for the measurement of each effect of interest.

[24]Lau (1986) gives a list of criteria that a functional form should satisfy. He then shows that simultaneously satisfying all of the criteria is not probable. He then goes on to show that failure to satisfy any of his criteria is not advisable. This leaves one in a quandary, since it is inadvisable to violate his criteria and improbable that one can satisfy all of these criteria simultaneously.

search is no longer for the truth, but instead is for something near the truth. Since all functional forms are only approximations, one quickly realizes that approximation A may be more appropriate than approximation B under one set of circumstances and that given a different set of circumstances, the order could be reversed.

Given that a unanimous decision on the proper trade-off between consistency in the micro-macro relationships and empirical properties is lacking, and given that there are only a few stylized facts which the functional form must replicate, one is virtually free to suggest any form. Or rather, one is free to suggest a form that does not stray too far from the desired micro-macro consistency and the stylized facts.

The stylized facts of demand analysis are few. For example, I have already mentioned Engel's Law - the income demand elasticity for food consumed at home should be less than one. A second stylized fact is that the marginal propensity to consume out of income varies between goods. A third stylized fact of demand analysis is that both substitution and complementarity are exhibited between goods. Finally, most economists believe that each good has a different own-price elasticity. These are the four generally accepted empirical facts of demand analysis. Thus, our functional form should allow the statistical work to replicate them. That is to say, one should not impose these facts on the functional form, but instead, allow for their possibility.

Almon (1979) has developed a set of requirements that satisfy these four stylized facts and that also account for the demands of theory. Using the Almon criteria as a guide, I suggest using the following criteria when evaluating a functional form:

1 It (the functional form) should allow for the possibility of substitution/complementarity between goods.

2 It should allow a variation in own-price elasticities for each good.

3 It should allow for a unique marginal propensity to consume out of income for each good in the system.

4 It should allow for variation in the complementarity/substitutability of one good for a second good. That is to say, it should allow a good to be a complement for some goods and a substitute for other goods.

5 Price changes should alter the effect of income and non-income determinants of demand - such as stocks, interest rates, or time trends- in approximately equal proportions.

6 It should possess Additivity.

7 It should be easy to estimate.

8 Including the effects of variables other than prices and income should be easy. These variables can include interest rates, stocks of durables and time trends.

9 The number of parameters in the system must be manageable. A simple way of reducing the number of parameters in the system is assuming some form of Slutsky symmetry.

10 As income increases, the asymptotic budget shares should depend on prices, or at least this dependence should not be ruled out *a-priori*. Additionally, the marginal propensity to consume of income should depend on a price in a way that is estimated and not specified.

11 In the absence of any attempt to model income distribution effects, it should possess homogeneity in prices and income.

Almon specifically developed these criteria for determining the suitability of using a functional form in a long-term forecasting model. Depending on the time-horizon of the model, income can grow more than three-fold during the forecast. Therefore, as the criteria above show, Almon paid special attention to the effects of prices on the marginal propensity to consume each good as income increased (requirement 10 above). Similarly, one must examine the impact of prices on consumption as income increases. A functional form in which prices cease to influence the marginal propensity to consume as income grows large is of little use to the long-range forecaster.

The rationale for the remaining Almon criteria are easy to identify. Requirements 1 through 4 reflect the general beliefs among economists regarding the actual relationships between goods and consumption. Criteria 6 is the lone theoretical constraint on the system. Requirements 7, 8 and 9 are born from a desire to develop a functional form that is simple enough to be readily used in a wide-range of applications.

The equi-proportional requirement (criteria 5) is equivalent to the proposition that the price elasticities of demand should be equal at different levels of income. Consumer theory places no such constraint on the functional form. This criteria reflects the view that the substitutability of one good for another is the same for a man making $300,000 as for the man making $30,000. Almon gives several examples of the dangers of restricting price effects solely to the income coefficients, or alternatively, the non-income coefficients.[25]

Given the discussion in the first section if this chapter, criteria 9 and 11 might seem odd criteria for a functional form. For example, since Slutsky symmetry almost certainly does not hold for the aggregate data, imposing symmetry on the functional form might appear inappropriate. However, without some form of price-effect symmetry, the number of price parameters in the system can be exceptionally large. Thus, the symmetry assumption serves to reduce the number of parameters in the

[25]See Almon (1979), page 87-89.

system. For example, a demand system with eighty goods and no symmetry would have 6400 price parameters. Assuming symmetry of some type reduces the price parameters to 3240.[26]

Homogeneity is a criteria because of the problems discussed earlier. If the functional form does not account for distributional effects, then it is unclear whether homogeneity should be imposed on the functional form. That is to say, there is no over-riding rational for requiring that the demand system leave demand unchanged if all prices and aggregate income double. Since I know that homogeneity will always occur in at least one not improbable instance, and since I cannot rule out the possibility that homogeneity will occur for any possible change in the income distribution, I choose to require homogeneity.

Armed with these criteria, I can now evaluate some of the well-known systems of aggregate demand.

A Review of Aggregate Demand Systems[27]

There is a long and rich history of systems of demand analysis. The number of functional forms that may approximate the true demand functions is beyond limit and a large number of these forms have been used in demand analysis. Since many of these forms are rarely used by anyone other than their discoverer, I have chosen to restrict my review of systems of aggregate demand to the well-known systems. I

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[26]Without symmetry, the number of price parameters equals: N^2 (N equals the number of goods in the system). With symmetry imposed, the number of price parameters equals: $N + (N^2 + N)/2$.

[27]This section draws heavily on Almon (1979).

review each system in the context of the eleven criteria presented in the previous section.

Systems of aggregate demand can be divided into three categories:

A. Systems derived from either additive explicit utility functions or additive implicit utility functions. These include the linear expenditure system (Stone 1954), the indirect log-additive system (Houthakker 1960), Powell's system (1966) and the double-log system (Sato 1972).

B. Systems derived from an implicit or indirect utility function. These include the Rotterdam model (Theil 1965; Barten 1969), the Translog model (Christensen, Joregenson and Lau 1975), the Almost Ideal Demand System (AIDS) (Deaton and Muellbauer 1980) and the CBS model (Keller and Van Driel 1985).

C. Systems which are not derived from either an explicit or implicit utility function. This group includes the Almon system (Almon 1979), the modified Almon system (Devine 1982; Chao 1991) and the generalized Logit equation model (Tyrrell and Mount 1982).

With the exception of the Almon and modified Almon models, all of these models assume either the existence of a representative agent or that preferences may be represented by a cost function of the PIGLOG family.

One cannot universally reject the use of any of these models for forecasting aggregate demand. All that a modeler can do is to reject the model based on his own criteria. The review below is not intended to disparage the work of the economists who developed the following systems. It is intended to highlight how these functional forms fail to satisfy the criteria given above.

A. Additive Preferences

This branch of the literature is the oldest among the various types of demand systems. The "father" of these systems is the Linear Expenditure System (Stone 1954) with the indirect log-additive system (Houthakker 1960) launching the literature on systems derived from additive indirect or implicit utility functions. These models had many applications before their popularity waned for the reasons given below.[28]

Stone's system serves as an example of why these models lost their popularity. Let q_i equal purchases of the i^{th} good, m equal total income, p_i equal the price of the i^{th} good and there are n number of goods in the economy. Purchases of the i^{th} good are given by:

$$p_i\, q_i = \alpha_i\, p_i + \beta_i\, (m - \textstyle\sum_j \alpha_j\, p_j) \qquad (2.14);$$

$$\textstyle\sum_{k=1}^{n} \beta_k = 1 \qquad (2.15a);$$

$$\beta_k \geq 0 \;\forall\; k \qquad (2.15b);$$

where m equals income, p_i is the price of the i^{th} good and the α's and the β's are the estimated parameters.

Equation (2.15b) constrains consumption of the good to be positive as income increases. Thus, the system does not allow inferior goods. Equation (2.15b) also forces a good to be either a complement with all goods or a substitute with all goods -

[28]A partial listing of these models includes Pollak and Wales (1969), Parks (1971) and Lluch (1973).

a violation of criteria 4.[29,30] There is no need to impose symmetry of any kind on the system since the number of parameters in the system is relatively small.

In fact, imposing Slutsky symmetry on the demand system causes the system to violate criteria 1, since symmetry eliminates the possibility of complementarity between goods (Stone 1954). As shown by Deaton (1974), the imposition of symmetry implies

> ... approximate linear relationships between own-price and income elasticities; under direct additivity the ratio of own-price to income elasticity is approximately constant, while under indirect additivity the sum is approximately constant. (Deaton, 1974, pp.338)

That is to say, the system is completely determined by the set of income elasticities and one of the own-price elasticities. There is little empirical evidence for such a result nor is there a commonsense rationale to impose such a result on the system. Thus, one must conclude, that assuming symmetry in the linear system is improper.

Since the Linear Expenditure System fails to satisfy the criteria and since Houthakker (1960) shows that any demand system based on additive preferences violates criteria 4, I am forced to reject systems of demand equations derived from utility functions with additive preferences.

The system is designed, however to pass criteria 6, additivity. The first term, $\alpha_i p_i$, can be thought of as a first guess of how much will be spent on the good. The $(m - \sum_j \alpha_j p_j)$ in the second term is the difference between income and the first guess at

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[29]The cross-price elasticity is given by:

$$\epsilon_{i,j} = \frac{\beta_i \alpha_j P_2}{p_i q_i} \quad \forall i,j$$

With the exception of the α_j term, all of the terms in the elasticity are positive. Thus, the j_{th} good will either be a substitute or a complement with all goods.

[30]Houthakker (1960) shows that any demand system based on additive preferences will reject criteria 3.

expenditures. The second term can be thought of as a "spreader" and it spreads this difference over all the goods. The trick is clever and, as shown below, is used in various forms by other authors.

B. Demand Systems Derived From Indirect Utility Functions

This area of the literature is the most active. All of these models are derived from expenditure functions and the principles of duality. In short, these models derive the demand equations by inverting the specified expenditure function. While an innovative approach, the demand systems that result fail to satisfy the criteria given in the previous section.

Rotterdam Models

The Rotterdam model is typically attributed to Theil (1965) and Barten (1969). Models of this type have been extensively used and can be summarized:[31]

$$dq_i = (\beta_i \, P/p_i) \, d(m/P) + (m/p_i) \, \Sigma_j \, c_{ij} \, d\,(\ln p_j) \qquad (2.16);$$

$$\Sigma_i \, \beta_i = 1 \qquad (2.17a);$$

$$\Sigma_j \, c_{ij} = 0 \qquad (2.17b);$$

$$c_{ij} = c_{ji} \qquad (2.17c).$$

where dq_i equals the derivative of the i_{th} good, m equals income, P equals some price index and the β's and c's are the estimated parameters.

Given (2.16), the marginal propensity to consume good i out of real income, $\beta_i(P/p_i)$, is assumed to have an elasticity of -1.0 with respect to p_i, ignoring the effect

[31] In addition to the considerable literature by Theil and Barten, a partial bibliography includes O'Riordan (1975), Conniffe and Hegarty (1980), Murty (1980) and Borooah (1985).

of p_i on P. This is equivalent to saying, if, at current prices, a \$100 increase in income causes demand for good i to increase by \$10, then should the price of good i double, the \$100 income increase causes demand for good i to increase by exactly \$5. Thus the functional form imposes an answer, *a-priori*, on one of the questions that the empirical investigation attempts to answer: how do prices affect the marginal propensity to consume out of income? Thus, the Rotterdam class of demand systems fails criteria 10.

Translog Models

The Translog model of Christensen, Joregenson and Lau (1975) has also been used in estimating aggregate demand systems.[32] The Translog model is written:

$$(p_i q_i/m) = (\alpha_i + \Sigma_j \beta_{ij} \ln(p_j/m))/(\Sigma_k \alpha_k + \Sigma_k \Sigma_j \beta_{kj} \ln(p_j/m)) \qquad (2.18);$$

where the α's are the estimated parameters.

Given (2.18), one can easily show that no matter what prices may be, the asymptotic budget share of good i equals $\Sigma_j \beta_{ij}/ \Sigma_k \Sigma_j \beta_{kj}$. Thus, these budget shares do not depend upon price. Alternatively, one can say that the own-price elasticity goes to -1 as income increases. Thus the model fails criteria 10. Given that the demand system will be used in a long-term forecast, this property is unsatisfactory.[33]

[32]A partial list of these models includes Christensen and Manser (1977), McLaren (1982), Kim (1988).

[33]See Almon for additional problems with this functional form. Lutton and LeBlanc (1984) show that Translog model frequently may generate negative expenditures on a good.

Almost Ideal Demand System

The Almost Ideal Demand (AID) System of Deaton and Muellbauer (1980) perhaps is the most popular of the recent systems of demand.[34] The AID system is written:

$$(p_i\, q_i/m) = \alpha_i + \Sigma_j\, \gamma_{ij} \ln p_j + \beta_i \ln(m/P) \qquad (2.19);$$

$$\gamma_{ij} = \gamma_{ji} \qquad (2.20);$$

$$\Sigma_i\, \alpha_i = 0 \qquad (2.21a);$$

$$\Sigma_i\, \beta_i = 0 \qquad (2.21b);$$

$$\Sigma_j\, \gamma_{ij} = \Sigma_i\, \gamma_{ij} = 0 \qquad (2.22);$$

$$\ln p = \alpha_0 + \Sigma_k\, \alpha_k \ln p_k + \tfrac{1}{2}\Sigma_j\, \Sigma_k\, \gamma_{kj} \ln p_k \ln p_j \qquad (2.23);$$

where the α's, the γ's and the β's are the estimated parameters.

While possessing excellent flexibility and ease of estimation, the asymptotic properties of the AID function is undesirable. Given (2.21b), either the income coefficients must all equal zero (a violation of criteria 3) or at least one of the β_i is less than zero. In this latter case, spending on the good is driven towards zero as income increases and, given a large increase in income, may become negative. As discussed earlier, the desirability of having demand fall to zero as income rises depends on the level of detail at which one estimates the consumption functions. However, it is not desirable to have consumption driven to less than zero. Additionally, the AID system imposes this property *a-priori* on the system, rather than having this result occur through the empirical properties of the model.

[34] A very partial bibliography includes: Chalfant (1987), L. Fulponi (1989), Alley, Ferguson and Stewart (1992), Kaboudan (1992), Pashardes (1993), Van Heeswijk, De Boer and Harkema (1993) and Brenton (1994). In fact, the AID system has supplanted nearly all other systems.

The AID function, however, is of the PIGLOG family - giving rise to equations that must possess absolute Slutsky symmetry (Muellbauer 1975, 1976). As discussed earlier, possessing this symmetry has been deemed desirable by many economists. The statistical evidence, however, consistently rejects the proposition that the AID system possesses absolute symmetry (see footnote 29 for sources). Thus, the AID system can be rejected because it fails to satisfy one of its own theoretical constraints.

CBS Model

The CBS Model of Keller and Van Driel (1985) is an attempt to combine the Rotterdam and AID system.[35] The CBS system uses the income coefficient from the AID system and the price terms from the Rotterdam System and is written:

$$w_i \{d(\ln q_i) - d(\ln Q)\} = b_i\, d(\ln(m/P)) + \Sigma_j\, c_{ij}\, d\ln(p_j) \qquad (2.24);$$

$$d(\ln Q) = d(\ln m) - d(\ln P) \qquad (2.25a);$$

$$b_i = \beta_i - w_i \qquad (2.25b);$$

where the c's and the β's are the estimated parameters.

The system can be simplified:

$$dq_i = \beta_i\, (P/p_i)\, d(m/P) + (m/p_i)\Sigma_j\, c_{ij}\, d\ln(p_j) \qquad (2.26).$$

As can be seen in equation (2.26), the CBS function dictates the relationship between the marginal propensity to consume out of real income and prices. Thus, the functional form fails criteria 10.

It must be acknowledged that the CBS system is an advance over the Rotterdam model in that the CBS functions are of the PIGLOG family of cost functions. Thus,

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[35]The system is named in honor of the Netherlands Central Statistics Bureau.

when using the CBS system, the problems of aggregation across individuals is reduced. Unfortunately, Keller and Van Driel do not test whether their unrestricted parameters possess absolute symmetry as required under the assumption of a PIGLOG cost function.

NBR System

Recently, Neves (1994) has proposed the NBR system. Like the CBS system, the NBR system is an attempt to combine the AID system and the Rotterdam system. The NBR system combines the Rotterdam income coefficients and the AID price coefficients and is written:

$$w_i \{d(\ln q_i) + d(\ln p_i) - d(\ln P) = \beta_i \, d(\ln (m/P) + \Sigma_j \, \zeta_{ij} \, d \, (\ln p_j) \qquad (2.27a);$$

$$\zeta_{ij} = \gamma_{ij} - w_i w_j + w_i \delta_{ij} \qquad (2.27b);$$

where δ_{ij} is the Kronecker delta and the β's and the γ's are the estimated coefficients.

As shown in (2.28), the NBR system imposes a relationship between the marginal propensity to consume out of real income and price - failing to satisfy criteria 10.

$$dq_i = \beta_i(P/p_i) \, d(m/P) + (m/p_i)\Sigma_j \, \zeta_{ij} \, d \, (\ln p_j) + q_i \, \{ \, d \, (\ln \, (P/p_i))\} \qquad (2.28).$$

As in the case of the Rotterdam, the AID and the CBS systems, the relationship between the marginal propensity to consume and price is specified by the functional form. Thus the system fails criteria 10.

C. Other Systems

Compared to the other two types of systems, this area of the literature has been neglected. This neglect can be attributed to the desire among economists to derive the system of demand equations from either an explicit or an implicit utility function. It has been shown in the previous sections of this chapter that theory does not require such a derivation. The two systems I discuss in this subsection are the generalized Logit system (Logit) and the Almon system.

The Generalized Logit (Logit) System

The generalized Logit (Logit) system of Tyrrell and Mount (1982) is derived from neither an explicit or an implicit utility function.[36] The Logit system is an attempt at a system that is flexible, easily estimated and satisfies criteria 6, additivity. The system is written:

$$w_i = \frac{e^{f_i}}{\sum_{j=1} e^{f_j}} \qquad (2.29);$$

$$f_i = \alpha_i + \beta_i (\ln m - \ln P^m) + \Sigma_j c_{ij} (\ln p_j - \ln P) \qquad (2.30);$$

where P^m is an index and the α's, the β's and the c's are the estimated parameters. In principle, any function can be chosen to represent the f_i's. Though in practice, the f_i's are of the form shown in (2.30) (Tyrrell and Mount 1982; Considine and Mount 1984; Considine 1990; Dumagan and Mount 1992).

Equation (2.30) is used in the literature because it simplifies the estimation of the system. By selecting good N as the base good and dividing all of the other shares by the share of good N, (2.29) is transformed:

$$\ln (w_i/w_N) = (\alpha_i - \alpha_N) + (\beta_i - \beta_N) \ln m + ... \qquad (2.29a);$$

[36]Examples of the use of the these model include Considine and Mount (1984), Considine (1990) and Dumagan and Mount (1992).

Using equation (2.29a), the N-1 equations can be estimated jointly by least squares. If the regressors in each equation are identical and no constraints are imposed across equations the system can be estimated using OLS, otherwise GLS is the preferred technique (Tyrrell and Mount 1982).

$$dq_i = (q_i/m)\ (\beta_i + 1 - \Sigma_j\ w_j\ \beta_j)\ dm \tag{2.31}$$

Equation (2.31) shows that price influences the marginal propensity to consume out of income - criteria 3. Since the shares, w_j, are determined by price, then the marginal propensity to consume out of income also depends on prices.

It is easily shown that the function specified satisfies criteria 6, additivity. However, the way in which the system satisfies additivity is less than perfect. As income increases, the function will "squeeze" income out of the goods with the smallest β's. As income continues to increase, the budget share of the good with the largest β will move towards 1 and the budget share of all other goods will fall to zero. Thus, the function probably should not be used in a long-term forecasting model.

The Almon System

Like the Logit system, the Almon system (Almon 1979) is derived from neither an explicit or implicit utility function.[37] The Almon system is written:

[37]A partial bibliography of the Almon system includes (Devine 1982; Chao 1991, Janoska 1994c). These models expand upon the Almon model and are reviewed in a later chapter.

$$q_i = \{ a_i(d) + \beta_i (\frac{m}{P}) \} \prod_j p_k^{c_{ik}} \qquad (2.32);$$

Where:

$$\Sigma_j \beta_k = 1 \qquad (2.32a);$$

$$\Sigma_k c_{ik} = 0 \qquad (2.32b);$$

$$P = \prod_k p_k^{s_k^o} \qquad (2.32c);$$

$$c_{ij} = \lambda_{ij} s_j^o \qquad (2.32d);$$

where $a_i(d)$ represents a constant term and other non-income, non-price factors and the β's and the c's are the estimated parameters.

Given (2.31), it is easily shown that the marginal propensity to consume out of income depends on price:

$$dq_i = (\frac{\beta_i}{P}) \prod_j p_k^{c_{ik}} dm \qquad (2.32)$$

The system does not possess additivity at all locations. Given (2.32c) and (2.32d) the system does possess additivity at a fixed base set of prices. As prices move from that fixed point, a spreader is used to insure additivity. The difference between total expenditures and the sum of expenditures is allocated to the various goods based on the basis of the income coefficients.

The Almon system can be faulted for lacking automatic additivity, as one would hope that the functional form used would insure, *a-priori*, that the sum of expenditures equaled total expenditure. In defense of the Almon system, experience has shown that the magnitude of the scaling is typically in the range of two to three percent of total expenditures (Almon 1979; Devine 1982; Chao 1991; Janoska 1994c). Thus, the use of an *ex-ante* income spreader does not seriously alter the forecast of the system.

A common criticism of the Almon system is that the system lacks absolute Slutsky symmetry. The lack of absolute symmetry is a valid criticism of the system if one is estimating a time-series of cross-section demand functions. That is to say, one would not use the Almon system if the data used in the estimation were two or more years of cross-sectional data on the same group of individuals. Almon's system, however, represents an aggregate system of demand. As shown above, the imposition of Slutsky symmetry on an aggregate demand system is neither required nor always desired. Consequently, as long as one limits the function to representations of aggregate demand, the criticism lacks validity.[38]

D. The Optimal System

As I have often stated in this chapter, no functional form can be all things to all modelers in all situations. When choosing a system, one must take into account the applications for which the system will be used. Since I will use the system to forecast aggregate consumption by good over the long-term, my primary measures of

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[38]Almon's system does possess base point symmetry - an assumption used to reduce the number of parameters in the system. As mentioned earlier, this assumption is not dictated by theory, but is a useful simplifying assumption.

"goodness" are the long-term properties of the model as well as whether the system is better suited for micro or aggregate forecasting.

Table 1 shows how the various systems presented here satisfy some of the criteria given earlier in the chapter.

TABLE 1

Satisfying the Criteria

	Complements & Substitutes	Flexible	Est. Effect of Price on MPC?	Asy. Budget Share Depend on P?	Additivity by Spreading?
Linear	No	No	Yes	No	Yes
Rotterdam	Yes	No	No	Yes	No
Translog	Yes	Yes	No	No	No
AID	Yes	Yes	Yes	No	No
CBS	Yes	Yes	Yes	No	No
NBR	Yes	Yes	No	Yes	No
Logit	Yes	Yes	Yes	Yes	No
Almon	Yes	Yes	Yes	Yes	Yes

The second column of the table indicates whether the functional form allows both complementarity and substitutability between goods. The third column shows whether the functional form is flexible. The fourth column shows whether the marginal propensity to consume out of income is affected by price and whether the effect is determined by the parameters of the system. The fifth column indicates whether the asymptotic budget shares depend on price. The final column indicates whether the functional form achieves additivity without driving categories with low income elasticities to zero spending.

As can be seen in the table, the evidence clearly indicates that the family of systems derived from additive preferences are inadequate - failing to simultaneously allow complementarity and substitution between goods. The long-run properties of the family of systems derived from utility functions indicate that these systems should be avoided as well. The two systems yet to be rejected are the Logit and the Almon system. Both systems are valuable additions to the literature, however, the long-run properties of the Logit system are undesirable. This leaves the Almon demand system as the remaining choice.

There is some evidence supporting the choice of the Almon system over the other demand systems. With the exception of the Logit model, Gauyacq (1985) investigated the usefulness of each of these types of models. Gauyacq examined each model for several characteristics. These characteristics included: the theoretical foundation of the model; possible methods of econometric estimation; the ease that one can apply the model to various levels of aggregation; and results of an application the model to French data for the period 1959 to 1979. Having investigated all of these models, Gauyacq concluded that " ... only the Almon model is from a practical point of view, convenient for determination of disaggregated demand functions (Gauyacq, page 119)." This is a strong endorsement of the Almon system and confirms the decision to use the Almon system in the present study.

With the selection of which system of demand to estimate completed, our attention can now turn to a more detailed discussion of the Almon system.

CHAPTER 3

THE LIFT SYSTEM OF EQUATIONS

The LIFT system of equations is based on the Almon system (Almon 1979) described in the chapter 2. Devine (1983) modified the Almon system by introducing income distribution and non-age demographic effects. Chao (1991) further modified the system by introducing a new method of forecasting durable goods expenditures. The foundations of both modifications are sets of cross-section expenditure functions that estimate the effects of changes in the income distribution, age structure of the population and several non-age demographic variables. The cross-section coefficients are used with time-series data on these variables to create a time-series variable (the cross-section effect variable, or C^*) that captures the effects of changes in these variables. Data on relative prices and C^* are used in estimating the system of demand equations.

This chapter reviews the work to provide the necessary background for the original parts of the study. Readers familiar with the background may proceed to the next chapter.

Cross-Section Estimation

It is a long-established tradition that age and other demographic characteristics play a key role in determining household PCE.[39] Any system of demand used in a forecasting model should account for these demographic effects, particularly if one is

[39]The most recent work has been done by Heien (1972), Denton and Spence (1976), Devine (1983), Deaton *et al.* (1989) and Chao (1991).

interested in studying the economic effects of changes in various demographic variables. The LIFT system accounts for this effects by the use of a two-tier system of estimation. The foundation of the LIFT system is the estimation of a set of cross-section expenditure functions that capture the effects of age structure, non-age demographics and the income distribution.

The goal of the cross-section analysis is to identify the effects of income, age and demographic factors in household consumption expenditures. The cross-section functions are based upon the combination of a non-linear Engel curve and adult equivalency weights (AEW) originally used by Devine (1982) and later expanded upon by Chao (1991). Both Devine and Chao used data from the Consumer Expenditure Survey (CEX). Devine used data from the 1972 CEX and Chao used the 1980 CEX.

A. The Devine Cross-Section Function

The concept of an AEW was first used by Sydenstricker and King (1921), and was later used by Prais and Houthaker (1955), who popularized the idea. The technique was extended to be compatible with any form of an Engel curve by Singh and Nagar (1973).

The most general method of forecasting PCE by household is to relate per-capita household expenditures on the i^{th} good, $C_{i,h}$, to per-capita household income in the form:

$$C_{i,h} = f_i\,(Y_h) * N_h \qquad (3.1);$$

Where:

Y_h = per-capita household income of household h;
N_h = the number of individuals in household h.

Equation (3.1) does not incorporate the effects of age and demographic structure on per-capita household expenditures. Implicitly, it assumes that a household containing one adult and two children will have the same spending habits as a household with three adults. This is imposed *ex-ante* and, given the form of equation (3.1), is not subject to empirical testing. As described in chapter 2, imposing such a solution on a functional form should be avoided unless a strong argument can be made in support of the *ex-ante* solution. By introducing the concept of an Adult Equivalency Weight (AEW), we allow for this *ex-ante* solution to occur *ex-post* while allowing empirical testing of the hypothesis.

The concept of an AEW allows household spending patterns to differ based upon the age and demographic structure of the household. Thus, a household with children will have different spending habits from an all-adult household.

Devine's functional form is based upon equation (3.1) but allows one to make full use of the information that can be garnered from the age and demographic structure of a household. The form of Devine's cross-section equation is given by:

$$C_i = (a + \sum_{j=1}^{K} b_j Y_j + \sum_{j=1}^{L} d_j D_j) * (\sum_{g=1}^{G} w_g n_g) \qquad (3.2);$$

Where:

C_i	=	Household consumption expenditures on good i;
Y_j	=	The amount of per capita household "income" within income category j (see below);
D_j	=	A zero/one dummy variable used to show membership in the j_{th} demographic group;
n_g	=	The number of household members in age category g;
K	=	The number of "income" groups;
L	=	The number of demographic categories;
G	=	The number of age groups.
a,b,d,w	=	Parameters to be estimated for each commodity.

The above function breaks household consumption expenditures into two components: consumption expenditure per "person" and the "size" of the household. Household per-capita income and demographic characteristics determine the value of the term in the parenthesis. The size of the household is determined by the term in the second parenthesis. The size of a household, for the purposes of the cross-section work, does not equal the number of people in the household, but is a function of the ages of the household members and the commodity under examination

The "size" of the household is variable because it depends on the "adult equivalency weights" of each age cohort. The weights vary by both age and commodity because, for some goods, each age group will "count" differently. An additional infant in a household will not significantly increase the expenditures on alcohol by the household, but adding a person in their mid-twenties will increase household alcohol expenditures. Similarly, an additional twenty-year-old in the household will not increase the expenditures by the household on infant formula, but a child will.

An examination of equation (3.2) shows that there is no role for price effects in the equation. Since the equation is estimated for a single year, there are no differences in relative prices over the sample. The effect of relative price movements is determined when the time-series analysis is undertaken. Equation (3.2) has three features that make it an attractive equation: the treatment of the household age-structure; the treatment of the effect of the demographic variables; and the treatment of the income-consumption relationship. One factor determining household consumption expenditures is the age structure of the household. The examples already given show how consumption expenditures on alcohol and baby food would be influenced by the age structure of the household. A simple model like equation (3.1) ignores the information that is contained within the household age structure because equation (3.1) imposes the constraint that all AEWs equal 1.0.

In order to use the information that can be gained by estimating equation (3.2) instead of equation (3.1), one must relax the constraint that the AEWs all equal 1.0.

This allows the "size" of the household to vary according to the commodity under investigation. The size of a household is defined as:

$$N_i = \sum_{g=1}^{G} w_{ig} * n_g \qquad (3.3);$$

Where:

N_i = The weighted household size for commodity i;
G = The number of distinct age groups;
N_g = The number of household members in the g^{th} age group;
w_{ig} = The weight of the g^{th} age group in the consumption of commodity i.

The weight of the age group, W_{ig}, depends upon the relative importance of the age group in determining consumption expenditures for the commodity. When determining consumption expenditures on alcohol, households with a two adults and two children should have smaller "sizes" than households with four legal drinking age adults. Table 2a shows some hypothetical AEWs for three age groups and table 2b shows three weighted household sizes for alcohol and clothing. (Please note that specific values of the AEW's are conjecture at this point). All three households contain six individuals, but have dissimilar age structures. This is why the households have such different weighted sizes. The households range in "size" from 4.5 to 18.0 for clothing with the "smallest" household for clothing being the "largest" household for alcohol. As the number of age groups increases, the amount of information that can be extracted from the household age structure increases.

TABLE 2a

Adult Equivalency Weights for Alcohol and Clothing

Commodity	Young	Adult	Elderly
Alcohol	0.1	1.0	1.5
Clothing	4.0	1.0	0.5

TABLE 2b

Weighted Household Sizes for a Sample of Hypothetical Families

	Composition of Household			Weighted Household Size	
Family	Young	Adults	Elderly	Alcohol	Clothing
A	4	2	0	2.4	18.0
B	0	3	3	7.5	4.5
C	3	1	2	4.3	14

This AEW approach becomes more attractive when one considers the time- series analysis that will be done. Given the above weighting scheme, one can incorporate this information into the time-series estimation so that the increase in the number of infants in the population associated with the baby boom will be an important variable in explaining changes in expenditure patterns. As the baby boomers become adults, we should see the consumption of infant-related products fall and consumption of alcohol rise. The system of AEWs will automatically adjust for changes in consumption patterns caused by these changes in age structure.

I have repeatedly referred to adult equivalency weights without defining the term "adult". Devine defined an adult as individual between the ages of thirty and forty. These thirtysomethings were then used as the measuring stick to compare all other age groups. It is necessary to pre-determine one of the AEWs because equation (3.2) is not uniquely identified. If we double all of the parameters within the income portion of the equation (the a's, b's and d's) and halve the weights (the w's), the product of the left hand parenthesis and the right hand parenthesis remains unchanged. By defining the thirtysomething age bracket as our "adults," the equation can be estimated.

Individuals are placed into eight population categories (GPOPs) depending on their age. Table 3 shows these age classifications.

TABLE 3

Age Classifications

Age	Classification (GPOP)
0 to 5 years old	Group 1
5 to 15 years old	Group 2
15 to 20 years old	Group 3
20 to 30 years old	Group 4
30 to 40 years old	Group 5
40 to 50 years old	Group 6
50 to 65 years old	Group 7
65 years and older	Group 8

The most general method for allowing demographic variables to affect consumption expenditures is the estimation of a consumption function for each demographic group. This lets each group have separate intercepts and marginal propensities to consume out of income. This method, while attractive, is unworkable for the current study. Estimating a consumption function for each demographic group would require the estimation of a large number of equations with data that does not have the necessary information.

An added complication of estimating these multiple consumption functions arises when one moves to the time-series analysis. To estimate each of these demographic expenditure functions, one would need the historical time-series data on the age and income distribution of each demographic group. While one might find the historical time-series data, one would need to forecast each of these age and income distributions before the equations could be used in a forecasting model. The difficulty in developing reliable forecasts of these variables at an aggregated level (for example, households in the west or the number of households with 2 or more persons) is considerable. Developing reliable forecasts at a lower level of aggregation (for example, the number of households in the west with 2 or more persons with a household income of more than $10,000) is much more difficult and is outside the scope of this study.

Equation (3.2) allows only the intercept to differ between demographic groups. The marginal propensity to consume out of income for any commodity is the same for all demographic groups. A zero-one dummy variable, D_j, is created to show membership in any demographic group. To prevent collinearity and to allow us to estimate the equations, one demographic group per characteristic does not have a D_j. The households for which all D_js equal zero are the reference group and are represented by the intercept.

The non-age demographic effects used in the study include:

Region: Regions were defined as North East, North Central, South and West. The reference group was for a household in the North East.

Working Spouse: If spouse employed, dummy equaled one.

Family Size: Size variables were defined as: one person, two person, three or four person, and five or more person households. The reference household was defined as a three or four member household.

Education: One variable to identify households headed by a college-educated individual.

Age of Household Head: This characteristic was defined as households with heads: under thirty-five, between thirty-five and fifty-five; and over fifty-five. The reference household was defined as the middle group.

The region a household is located is important in determining the consumption patterns of the household. Households located in the Sun Belt should consume a greater amount of electricity (due to air conditioning) than will a household in the Rust Belt. Similarly, households in the northern regions should consume utilities that are for heating in greater amounts than will households located in warmer climates. Purchases of new automobiles should be higher in the rust belt, again because of the climate. The amount of snow received by the region will determine the amount of salt put down on the road. The heavy use of salt will cause automobiles to rust quicker, thus needing replacement sooner.

Households with two earners will probably need to spend a greater amount on day care and domestic services than households with only one earner.

In order to capture economies of scale, a variable for household size is used in the cross-section estimation. For example, assume a household purchases a washing machine for $500. A two-person household will have per-capita expenditures of $250 on the washing machine. A five-person household will only have $100 of per-capita expenditures. The larger household can spend a smaller per-capita amount on a good because of an economy of scale.

The age of household head variable is an attempt to capture the life-cycle of the household. For example, new households should purchase greater quantities of appliances and furniture than older households.

The above system is additive. There are no interaction terms between the demographic variables or, for that matter, between the demographic variables and the age structure. While this is probably not the case in the real world -- a working spouse household with young children should consume more day care services than a working spouse with no children or a nonworking spouse -- these effects have been ignored. The assumption of additive effects is not overly restrictive and the benefits from relaxing this assumption would be small. If one were to relax this assumption and allow the full range of interaction between the demographic groups one would face two daunting problems: the need to estimate a large number of additional parameters in an already complicated system; and the need to forecast these very specific demographic series. For example, to undertake the time-series analysis, one would need to know the percentage of households with three or four people in the south with working spouses that had college educated heads under thirty-five years of age. While one might be able to find a time-series of such data, one would still need to forecast these series. By assuming that the demographic variables have an additive effect, it is much easier to obtain the needed historical data as well as forecasts of the demographic and variables.

I have repeatedly referred to "household income" and "per-capita household income". These terms need to be defined. For the cross-section work, household income equals total expenditures as reported by the household. Thus, per-capita household income for the cross-section work equals total household expenditures divided by the number of people in the household.[40] So, when a I refer to a household with income of $20,000, I am saying that the household had total expenditures on PCE of $20,000. If the household contained four persons, the per-capita household income equals $5000. If the household contained a single person, the per-capita household income would naturally equal $2000. Once per-capita household income has been determined, it is allocated to the various Y_j's by the process described below and in chapter 4.

Expenditures, rather than income, are used to avoid the problem of identifying permanent versus transitory income effects. In the cross-section household data, wealth and income are positively, but not perfectly, correlated. It is, therefore, impossible to separate permanent versus transitory income effects. Since a consumer's consumption habits will depend more heavily on his wealth (which is a better measure of permanent income) than upon his annual income, the use of current annual income in the cross-sectional analysis would be inappropriate. Since the effect of current and past incomes, as well as wealth, jointly determine a consumer's expenditures, using expenditures makes it possible to automatically capture the effects of permanent and transitory income.

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[40]Please note, this is the actual number of individuals living in the household and is **not** the weighted size of the household.

Perhaps the most important factor determining household consumption expenditures is household income. Various methods have been put forward to explain the relationship between income and consumption. Brown and Deaton (1972) explored the use of various forms of the Engel curves in explaining consumption. They recommend the use of specific forms of equations based upon the income elasticity of the good under investigation.[41] This seems to suggest that a single form of the Engel curve is unable to match the particular characteristics of all goods that might be investigated. However, one functional form of the Engel curve adapts itself to any shape. It is known as the Piecewise Linear Engel Curve (PLEC). Figure 1 shows a PLEC.

[41]The semi-logarithmic form is suggested for use with goods that are income inelastic; the linear form for goods that have an income elasticity close to unity; the double log form for income elastic goods (but can not be used if the observed value of the dependent variable is sometimes zero) and a log-reciprocal form for goods that approach a saturation level.

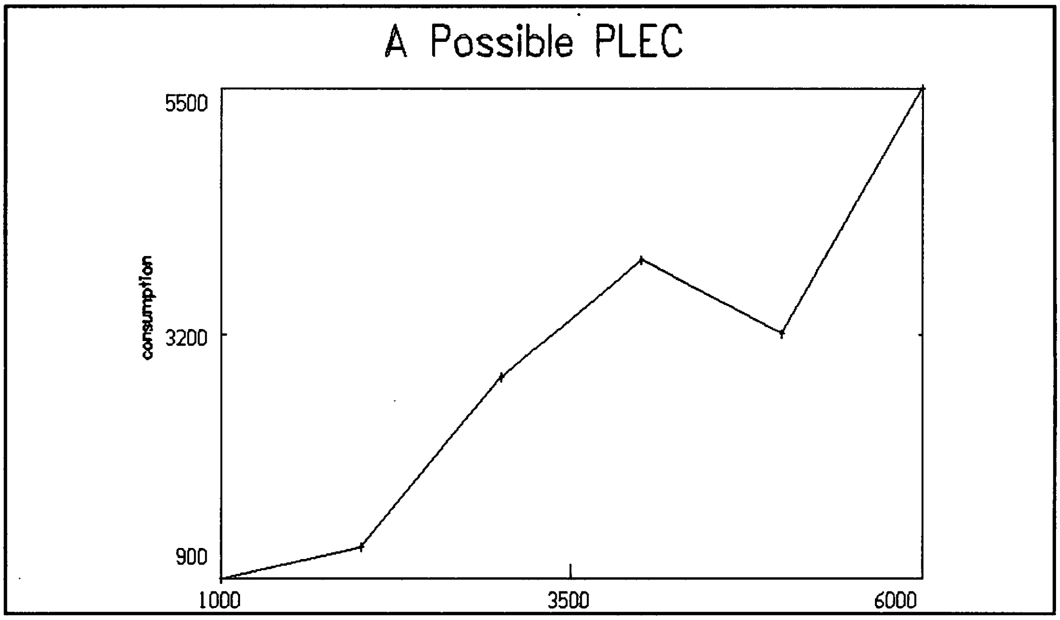

Figure 1

Piecewise Linear Engel Curve

As can be seen from the figure, the PLEC consists of linear segments with the slope of each segment unconstrained. The only constraint placed upon the PLEC is the requirement that the curve be continuous. Figure 1 depicts the case where income is broken into five brackets. For luxuries, the slopes of the line segments are greater than one. For inferior goods, the slopes of the line segments are less than zero. A more typical PLEC, however, will show that the good is a luxury over some income ranges and a necessity over the remaining income ranges. In figure 1, the good is a necessity over the first and third income brackets, a luxury over the second and fifth income brackets, and inferior over the fourth income bracket. Here is the beauty of the PLEC -- its ability to adapt to any shape so that it may represent nearly any income-consumption relationship.

The income brackets in figure 1 are evenly spaced at \$1000, \$2000, and so on. A household with a per-capita income of less than \$1000 would have all of its income attributed to the first income bracket. A household with a per-capita income of \$2500 would have the first \$1000 of per-capita income allocated to the first income bracket; the second \$1000 of per-capita income allocated to the second income bracket; and the last \$500 of per-capita income allocated to the third income bracket. The income in each bracket becomes the Y_j used in equation 3.2 as the income variables. The income brackets are defined so that each bracket contains one-fifth of the households in the cross-section data.

Algebraically, the allocation of income to each Y_j can be represented by:

$$Y_j = \begin{cases} B_j - B_{j-1} & \text{if } B_j \le Y \\ Y - B_{j-1} & \text{if } B_{j-1} \le Y < B_j \\ 0 & \text{if } Y \le B_{j-1} \end{cases} \qquad (3.4);$$

where:

Y = Household per-capita income;
K = The number of income brackets;
B_0 = Zero;
B_k = Infinity.

Assume the boundaries are: $B_0 = 0$, $B_1 = \$1000$, $B_2 = \$2100$, $B_3 = \$2315$, $B_4 = \$4500$, B_5 = infinity. Table 4 shows how a set of hypothetical per-capita incomes are allocated to the various income brackets, Y_j.

TABLE 4

Sample Income Brackets

Household Income	Y_1 \$0 - \$1000	Y_2 \$1001 - \$2100	Y_3 \$2101 - \$2315	Y_4 \$2316 - \$4500	Y_5 Above \$4501
\$1200	\$1000	\$200	\$0	\$0	\$0
\$2100	\$1000	\$1100	\$10	\$0	\$0
\$3900	\$1000	\$1100	\$215	\$1585	\$0
\$10000	\$1000	\$1100	\$215	\$2185	\$5500

As can be seen in the table, income is allocated to the first bracket until the upper boundary of the bracket is reached or income is exhausted. If income remains, unallocated income is allocated to the second bracket until income is exhausted or the upper income boundary of the second income bracket is reached. This process continues until income is allocated to the final income bracket, where all remaining income is then allocated.

The slope of the PLEC over any income bracket, Y_j, corresponds to the parameter b_j in equation (3.2). The b_j can also be interpreted as the marginal propensity to consume (MPC) out of an additional dollar of income that lies within the income bracket. This MPC is not universal. Since b_j's are estimated for all commodities, each commodity will have a **specific** propensity to consume out of an additional dollar of income in each income bracket.

One potential problem with the above method is caused by the presence of durable consumption items. For some commodities, estimating a cross-section expenditure function like equation (3.2) may lead to cross-section function parameters that are biased and inconsistent. It was this problem that the Chao study attempted to eliminate and towards which we now turn our attention.

B. The Chao Cross-Section Function

One problem with Devine's method is caused by the presence of durable consumption items. These goods provide flows of consumption over several years and are purchased at irregular intervals. For example, if consumption were smooth over the lifetime of a durable item, then the consumption of a $10,000 automobile with a lifespan of 10 years is $1,000 per year. The expenditures on new cars, however, would be $10,000 in the first year and $0 in the next nine years. Depending upon the year in which we undertake our cross-section survey, the household will either have $0 or $10,000 of expenditures.

Because of the presence of a durable items, our dependent variable is censored. The variable is censored because the lowest level of expenditures possible for any household equals zero. Households wishing to have negative expenditures for commodities are recorded as having zero expenditures. Since there is information missing on the dependent variable, but the corresponding information for the independent variables is known, our dependent variable is censored. The ordinary least-squares estimator will give us biased and inconsistent estimates of our parameters (Kmenta, 1971, pp 561).

In an attempt to correct this problem, Chao modified the original Devine cross-section work using a probit analysis. Chao used a probit model, rather than the more conventional Tobit model, because "the factors determining whether or not a family buys a particular good may be quite different from those that determine the amount spent, given that the family buys" (Chao pp. 41). The decision to purchase an engagement ring is a function of one's age (those in their twenties tend to get engaged more often than those in their eighties), but once the decision has been made to purchase the ring, per-capita income of the household will determine the amount spent.

Chao's model allows each variable to carry the appropriate "weight" in the two separate decisions: (1) purchase or not purchase; and (2) spend a large amount or spend a small amount. The Tobit analysis does not allow the variables to carry separate weights. As can be seen in the above example, we might not want to include a variable in the equation explaining the amount purchased, but we might want to include that variable in the equation explaining whether a purchase was made at all.

Chao's model is straight forward. The value of Y_{i1} determines whether a household purchases the good. The probability of a purchase by a household is the estimate of Y_{i1} which we denote as $\hat{Y}_{i1}$. The probability of any household making a purchase is determined by:

$$y_{i1} = f_{i1}(x_{i1}\beta_{i1}) + u_{i1} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x_{i1}\beta_{i1}} e^{-\frac{t^2}{2}} dt + u_{i1} \qquad (3.5);$$

Where:

y_{i1} = 1 if household purchased, 0 else;

x_{i1} = A row vector of variables that determine the yes-no purchasing decision;

β_{i1} = A column vector containing the weights of the determining variables;

u_{i1} = A disturbance term.

The integral is the normal cumulative density function.

For those who purchased the good, the amount purchased, Y_{i2} is determined by:

$$f(x_{i2}) + u_{i2} \qquad (3.6);$$

Where:
$f(x_{i2})$ = The normal PLEC function or equation (3.2);
u_{i2} = A disturbance term.

The expected expenditure for any household, Y_{i3}, is calculated as the product of the probability a household will purchase the good and the amount spent if the household buys. Algebraically this is:

$$y_{i3} = f_{i1}(x_{i1}\beta_{i1}) * f(x_{i2}) \qquad (3.7).$$

Equation (3.7) is a non-linear function of our independent variables. A problem arises because of this non-linearity when we consider the time-series analysis we wish to undertake.

The time-series analysis uses the coefficients estimated from the cross-section estimation to construct a cross-section expenditure variable, C^*_t. C^*_t is designed to capture the effects of the cross-section variables on expenditures. C^*_t equals expenditure per AEW in period t, if expenditures were solely a function of the income and demographic variables in the cross-section estimation. For any commodity,

$$C^* = \sum_{i}^{Households} \left(\sum_{j} b_j Y_{ij} + \sum_{j} d_j D_{ij} \right) \qquad (3.8a);$$

This allows us to construct C^*_t so that equation (3.8a) can be rewritten:

$$C_t^* = \sum_{j} b_j Y_j + \sum_{j} d_j \, D_j \qquad (3.8b);$$

Where:

Y_j = The aggregate expenditure variables;
D_j = The population totals. in period t.

Because equation (3.7) is non-linear, it is no longer possible to use the Devine formulation to construct the C^*_t variable needed for the time-series analysis from the available data on the distribution of expenditures and the demographic variables.

Chao dealt with this problem by estimating an equation of the normal form (3.2) using, as the dependent variable, not the actual expenditures of the household, but the expected expenditure, Y_{i3}. The equation is written:

expected household expenditures, Y_{i3}. This equation is written:

$$y_{i3} = f(x_{i2}) + u_{i3} \qquad (3.9);$$

where:

u_{i3} = the disturbance term.

This procedure was used when estimating "big ticket" items (such as durable goods), as well as commodities where a large number of households had no expenditures.[42]

C. Estimation Technique

The multiplicative nature of both equations (3.3) and (3.9) makes them non-linear in the parameters to be estimated. However within each of the two component pieces, the equation is linear. Thus the problem can be solved in an iterative process by first assuming that the parameters of one component (A) are known and estimating the parameters of the other component (B) using ordinary least squares. The parameters in B are then fixed at their estimated values and the parameters of A are estimated. This procedure continues until the parameter values converge. Like all non-linear estimations, the solution may depend on the initial values used in evaluating the function and there is no guarantee that the maximum found by the process is global and not local. The only defense that can be offered is the standard defense of any non-linear estimation; that the initial values are the standard assumption and that a numerical search for starting values is intractable.

[42]According to Chao, the data for the 1980 - 1981 CEX show that approximately 45% of households did not purchase cigarettes and 40% of households did not purchase alcohol.

D. Cross-Section Parameters Used in Time-Series

The ultimate goal of this study is the incorporation of the demand system into the LIFT model. Ultimate control of LIFT belongs to those who use the model on a regular basis. There is little point in estimating a system of demand equations that will not be used in LIFT. Such is the case with the Chao cross-section function. While the theoretical foundation of the Chao function is stronger than the Devine foundation, the estimated parameters are implausible because of data limitations.

Because the CEX only records out-of-pocket expenditures, only a portion of the spending in categories such as medical services is recorded. This would not be a problem if the distribution of unrecorded expenditures across age groups was approximately proportional to the distribution of spending across age groups. Unfortunately, this is not the case. It appears that most of the unrecorded spending is done by persons below the age of sixteen or above nineteen. Consequently, the cross-section parameters of the Chao function indicate that spending on hospitals by the sixteen to nineteen age cohort is five times the spending of the sixty-five and older population. This pattern of spending simply cannot be defended and users of the forecasting model prefer the cross-section parameters estimated Devine.

The LIFT System of Demand Equations

The cross-section estimates identify the extent that household consumption is affected by demographic, age and income distribution factors. However, since the cross-section estimation is for a single year, it is incapable of identifying the degree that relative price changes effect consumption spending. The time-series estimation combines the results from the cross-section work with a modified-version of the system of demand equations suggested by Almon (1979).

The cross-section effects are incorporated in the time-series through two avenues. For each commodity, the non-age demographic and income distribution effects are combined into a single cross-section effect variable called C^*. The age effects are incorporated into a single variable for each commodity called known as the weighted population for that commodity. These two variables are then used in estimated the LIFT system of equations.

In this section I describe the construction of the weighted population and C^* variables. The LIFT system of equations is described in greater detail and a list of the commodities in the system is given.

A. Constructing the Weighted Population Variables

The weighted population variables incorporate the effects of changes in consumption expenditures that are caused by changes in the age structure of the population. The parameters are estimated in the cross-section and the weighted population variables for each year are and commodity are calculated by:

$$N_{i,t} = \sum_{g=1}^{G} w_{ig} * n_{g,t} \qquad (3.10);$$

Where:

N_{it} = The weighted population for commodity i, year t;
G = The number of distinct age groups;
N_{gt} = The number of persons in the g^{th} age group, year t;
w_{ig} = The weight of the g^{th} age group in the consumption of commodity i.

Equation (3.10) is similar to equation (3.3), but instead of calculating the weighted population of a single household, equation (3.10) gives the aggregate weighted population for the i^{th} commodity. Age data is gathered from the P-25 series of published U.S. Bureau of the Census reports.

B. Constructing the Cross-Section Effect Variable

The cross-section effect variable, C^*, captures the non-age demographic and income distribution effects. Like the age-weighted population, the parameters are estimated in the cross-section. The equation for C^* is:

$$C_t^* = \sum_j b_j Y_j + \sum_j d_j D_j \qquad (3.8b);$$

Where:

Y_j = The aggregate expenditure variables;
D_j = The population totals. in period t.

Data on the demographic variables is gathered from the P-20 series of published U.S. Bureau of the Census Reports. The income distribution work is described in Chapter 4.

C. The System of Aggregate Demand Equations

The system of symmetric consumption functions used in the LIFT model is based on a system originally developed by Clopper Almon (1979). The LIFT consumption system is estimated for 76 categories of PCE. These commodities are listed in table 5. Commodities in bold are not part of the system of equations but are included in total PCE.

TABLE 5

List of Commodities in the LIFT System

1 New cars
2 Used cars
3 New & used trucks
4 Tires & tubes
5 Auto accessories & parts
6 Furniture mattresses bedsprings
7 Kitchen household appliances
8 China glassware tableware utensils
9 Radio, tv, records & musical instruments
10 Floor coverings
11 Durable housefurnishings, NEC
12 Writing equipment
13 Hand tools
14 Jewelry
15 Ophthalmic & orthopedic appliances
16 Books & maps
17 Wheel goods & durable toys
18 Boats, recreational vehicles. & Aircraft
19 Food, off premise
20 Food on premise
21 Alcohol, off premise
22 Alcohol, on premise
23 Shoes & footwear
24 Women's clothing
25 Men's clothing
26 Luggage
27 Gasoline & oil
28 Fuel oil & coal
29 Tobacco
30 Semidurable housefurnishings
31 Drug preparations & sundries
32 Toilet articles & preparations
33 Stationery & writing supplies
34 Nondurable toys & sport supplies
35 Flowers seeds potted plants
36 Lighting supplies
37 Cleaning preparations
38 Household paper products
39 Magazines & newspaper
41 Owner occupied space rent
42 Tenant occupied housing
43 Hotels and motels
44 Other housing
45 Electricity
46 Natural gas
47 Water & other sanitary services
48 Telephone & telegraph
49 Domestic services
50 Household insurance
51 Other household operations:repair
52 Postage
53 Auto repair
54 Bridge, tolls, etc.
55 Auto insurance
56 Taxicabs
57 Local public transport
58 Intercity railroad
59 Intercity buses
60 Airlines
61 Travel agents & other transportation services
62 Laundries & shoe repair
63 Barbershops & beauty shops
64 Physicians
65 Dentists & other professional services
66 Hospitals & sanitariums
67 Health insurance
68 Brokerage & investment counseling
69 Bank service charges & services w/o payment
70 Life insurance
71 Legal services
72 Funeral expenses other personal business
73 Radio & tv repair
74 Movies, theater & spectator sports
75 Other recreational services
76 Education
77 Religious & welfare services
78 Foreign travel by US residents
79 Expenditures in US by foreigners
80 Nursing homes

The Almon system is written:

$$q_i = \{ a_i(d) + \beta_i (\frac{m}{P}) \} \prod_j p_k^{c_{ik}} \qquad (3.11);$$

Where:

$$\Sigma_j \beta_k = 1 \qquad (3.12a);$$

$$\Sigma_k c_{ik} = 0 \qquad (3.12b);$$

$$P = \prod_k p_k^{s_k^o} \qquad (3.12c);$$

$$c_{ij} = \lambda_{ij} s_j^o \qquad (3.12d);$$

$$\Sigma_j \alpha_k = 0 \qquad (3.12e);$$

Where:

$a_i(d)$ = A constant term and other non-income, non-price factors;
s_j^o = The base year share of total expenditures for commodity i;
β,c = The estimated parameters.

Equation (3.12b) imposes homogeneity of degree zero on the system. The system does not possess a global adding-up property. Equations (3.12a) and (3.12e) insure constant-price adding-up.[43] For consumption to exhaust income and prices other than the base price, a spreader is employed. The spreader adjusts expenditure in each category and is based on the estimated income elasticities and the difference between total expenditures and the sum of expenditures across all commodities.

With 76 commodities in the system, there are 5700 price parameters. If we were to estimate the system without additional assumptions, we would find that we would

[43]If relative prices are unchanged and income changes, the system possesses adding-up.

exhaust our available degrees of freedom. For this reason, approximate Slutsky symmetry is imposed on the system to reduce the number of parameters. Specifically, we assume:

$$\frac{c_{i,k}}{s_j^o} = \frac{c_{k,i}}{s_i^o} \qquad (3.13).$$

Equation (3.13) and equation (3.12d) give us the symmetry condition:

$$\lambda_{ij} = \lambda_{ji} \qquad (3.14).$$

This reduces the number of price parameters by half, leaving 2850 price parameters in the system. While the number of observations in the time-series allows for the estimation of this number of parameters, there is no easy method of checking whether the estimated price parameters are sensible. Since the ultimate objective of this work is for the system to be used in a forecasting model, one needs to check the estimated coefficients for the nebulous quality of reasonableness. Thus, the number of price parameters should further be reduced.

The reduction is done by combining commodities into narrowly-defined groupings (or sub-groups). Each of these groupings is then placed into broadly-defined groups. The criteria for placing a commodity in a group and sub-group is based on *a-priori* beliefs as to whether the goods are strong/moderate complements. The system is designed with the idea that: weak price effects occur between categories in different groups; moderate price effects occur between categories in different sub-groups within a group; (3) and strong price effects occur between categories within a sub-group. The system imposes Slutsky symmetry between each group in the system and between each sub-group within a group. Table 6 lists the commodity groups.

TABLE 6

Commodity Groups

1 Food, Alcohol and Tobacco	6 Medical Services
2 Clothing, Accessories & Personal Care	7 Personal Business Services
3 Household Durables	8 Transportation
4 Household Operation	9 Recreation and Travel
5 Housing & Household Utilities	10 Reading and Education

We introduce the following notation before providing the general equation used in the time-series estimation:

M = The number of groups;

s_i = The budget share of commodity i in total expenditures in the base year;

S_L = The sum of the budget shares of categories in group L in the base year;

SG_L = The number of sub-groups in group L;

S_K^L = The total budget share of sub-group K in group L in the base year.

The general form of the time-series equation is:

$$\frac{q_{it}}{WP_{it}} = (a_i + b_i C_{it}^{*} + c_i \Delta C_{it}^{*} + d_i TIME + e_i\ OTHER_i) \prod_{L=1}^{M} \left(\frac{P_{it}}{\bar{E}_{Lt}}\right)^{-S_L \lambda_{IL}} \prod_{K=1}^{SG_L} \left(\frac{P_{it}}{\bar{e}_{Kt}^{L}}\right)^{-SG_K^L}$$

$$q_i \notin Group\ L$$

$$q_i \notin Sub\text{-}group_K^L$$

(3.15);

Where:

q_{it} = Expenditures on category i during year t;

$$\bar{E}_L = \left(\prod_{j \in G_L} P_j^{s_j}\right)^{\frac{1}{S_L}}$$

$$\bar{e}_L = \left(\prod_{j \in SG_K^L} P_j^{s_j}\right)^{\frac{1}{S_K^L}}$$

WP_{it} = Weighted population size, good i, in year t;
C^*_{it} = Cross-section variable, good i, in year t;
P_{it} = Price of good i in year t;
$\bar{E}_{LT}$ = Average price of group L in year t (see above);
$\bar{e}^L_{Kt}$ = Average price of sub-group K, group L in year t (see above);
TIME = Trend variable with 1960 = 1;
$Other_i$ = A non-price, non-income variable affecting good i;
S_L = Share of total consumption, group L, in base year;
$a_i, b_i, c_i, \lambda_{IL}, \gamma^L_K$ = Parameters to be estimated.

The variables WP and C^* are determined from the parameters estimated in the cross-section work. Prior to my work, equation (3.15) was the general equation of each commodity in the demand system.

With the old system of equations described, we are now ready to turn our attention to the modifications of the LIFT system of demand equations that are part of this study.

CHAPTER 4

Forecasting the Income Distribution

As described in chapter 3, the system of consumption expenditure functions makes use of a Piecewise Linear Engel Curve (PLEC). The PLEC allows us to forecast changes in consumption expenditures caused by changes in the distribution of income. To estimate our system of consumption functions, we need historical data on the distribution of income. For forecasting, we need forecasts of this distribution. This chapter describes the method used in forecasting the distribution of income. Areas that are covered in this chapter include the importance of forecasting the distribution of income, various methods of forecasting the distribution of income, the previous method of forecasting the distribution of income and the work undertaken as part of this thesis.

At this point, I must reemphasis that the consumption system does not use disposable income directly when forecasting expenditures. As described in chapter 3, the system uses total expenditures to forecast expenditure by category. In this chapter, when the distribution of "income" is discussed, I mean the distribution of total expenditures across the population. Unless otherwise stated, for the remainder of this chapter, the two terms are used interchangeably. The principles involved in constructing the distribution of expenditures are identical to those used in constructing a distribution of income.

The first sections of this chapter discuss why we must forecast the income distribution, the definition of a Lorenz curve and some of its properties. Until now, the literature on functional forms representing the Lorenz curve has lacked a discussion of the general properties that these functions possess. This study presents general properties of any functional form that represents the Lorenz curve.

The later sections of this chapter discuss the functional forms that were investigated as possible representations of the Lorenz curve and how the curve is forecasted. Four major problems presented themselves as part of this chapter: choosing a functional form for years without data; estimating an equation to forecast the parameters of the functional form; finding a functional form that perfectly fitted the known data; and insuring a smooth transition between the two functional forms used.

The first problem was finding a functional form that could represent the Lorenz curve for years in which no data were available. The available data covered the period 1984 through 1993. Since the consumption system is estimated over the period 1960 to 1995 and forecasts can extend to 2050, some method of backcasting the Lorenz curve for the years 1960 to 1983 was needed. Similarly, a method of forecasting the Lorenz curve to the year 2050 was needed as well.

The second problem dealt with developing a forecasting equation that would forecast, for years after 1993, and backcast, for years before 1984, the parameters in the functional form of the Lorenz curve.

The third major problem dealt with the years 1984 to 1993; the years in which data were available. For the years 1984 to 1993 I wanted to use the historical data and not the estimate generated by the functional form used to represent the Lorenz curve. Consequently, a method of exactly fitting these known points had to be developed.

The last major difficulty dealt with developing some means of merging the Lorenz curve predicted by the functional form and the Lorenz curve fitted for the years 1984 to 1993. The properties of a Lorenz curve that were formalized in the earlier sections are employed here. Use of these properties allowed the construction of a series of coherent Lorenz curves for the years 1959 to 1983 - years where data were unavailable - and the years 1984 to 1993 - years where data were available.

The next-to-last section deals with the calculation of the expenditure variables that are used in the consumption system. The final section concludes this chapter.

Forecasting the Lorenz Curve

In chapter 3, we used the Survey of Consumer Expenditures to provide cross-section data on the relation between household expenditures and income (i.e. total expenditure). The cross-section analysis allocated income to one of five income brackets. So that we may use the information gained in the cross-section analysis when we undertake our time-series analysis, we must predict the distribution of income.

We could hold the distribution constant over time, but this would reduce significantly the usefulness of the cross-section work. The PLEC, although still improving the performance of the cross-section estimations, would lose its simulation applications. Since having the capability to model the effects of changes in the distribution of income was one of the goals of the cross-section work, holding that distribution constant during forecasts should be avoided if at all possible. One can avoid holding the distributions constant by forecasting the Lorenz curve.

Typically, the Lorenz curve plots the percentage of total income earned by various portions of the population when the population is ordered by the size of their incomes. The income is then cumulated. Thus the cumulated income of the i^{th} person is the income of all persons with less income that the i_{th} person. The cumulative income is then plotted. On the vertical, or y axis, cumulative income is represented as a fraction of total income, while on the horizontal, or x axis, the cumulative number of persons is represented as a fraction of the total number of households. The population is arranged so that as we move to the right from the origin each person, or consumer-unit, is richer than the person who proceeds him. Thus, the person at x-coordinate 0.5 is richer than fifty percent of the population and is poorer than fifty percent of the population. The y-coordinate gives the percent of income held by all persons with income less-than or equal to the person at the x-coordinate. For example, if the Lorenz curve passes through {x=.4, y=.3}, then thirty percent of total income is held by the poorest forty percent of the population.

Instead of personal income being plotted on the vertical axis, the Lorenz curve may also use household or per-capita household income. In this case, the units on the x-axis no longer would be persons, but would be households. Thus, the household with an x-coordinate of 0.5 is richer than fifty percent of households and poorer than fifty percent of households. This study forecasts Lorenz curves for per-capita household income. For example, a four-person household with total income of $40,000 has a per-capita household income of $10,000 and this four-person household would be ranked below a one-person household with total income of $11,000.

Figure 2 depicts two Lorenz curves. The curve marked A represents the egalitarian or equal distribution of income. The curve B shows a more typical, as well as more unequal, income distribution.

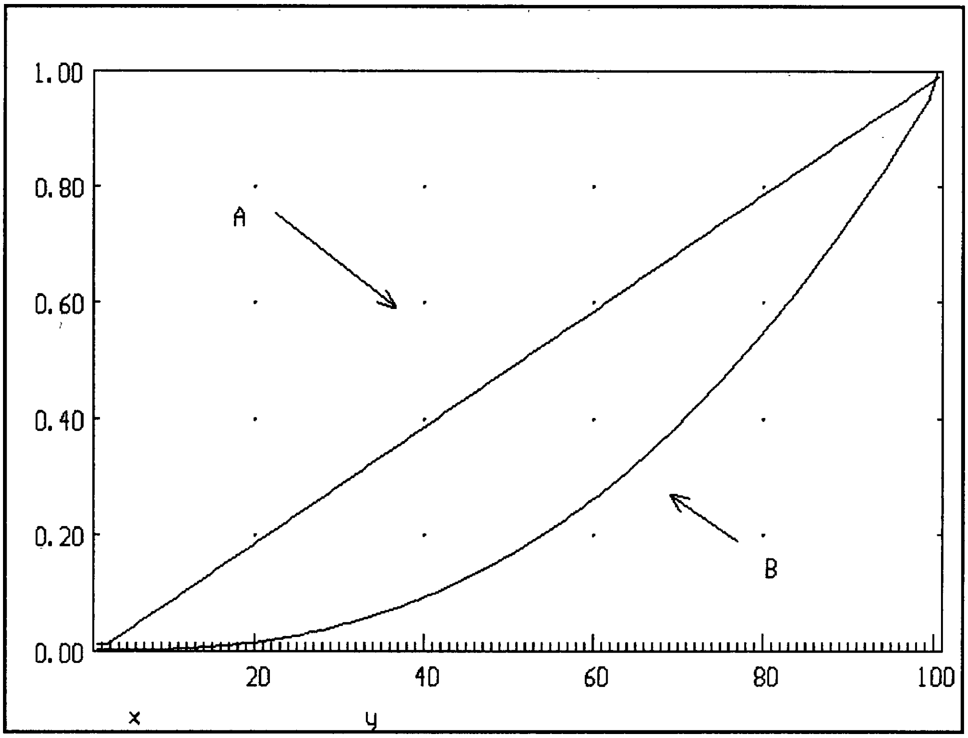

Figure 2

Sample Lorenz Curves

When specifying the Lorenz curve, one usually assumes that income, Y, is a random variable with cumulative density function x=F(Y), where x is the proportion of the population with income less than or equal to Y. Gastwirth (1971) shows that the Lorenz curve can be written as

$$y = L(x) = \frac{1}{\mu} \int_0^x F^{-1}(u)\, du \qquad (4.1);$$

where μ is mean income. The first derivative of the function L(x) is the inverse of the cumulative density function F(Y), denoted by $F^{-1}(x)$. Thus, choosing a functional form for the Lorenz curve implicitly specifies the functional form of the distribution of income (Thistle and Formby 1987).

The early literature in this area tended to specify the probability density function (Simon 1955; Metcalf 1969; Levine and Singer 1970; Thurow 1970), but since Gastwirth's 1971 article, the preponderance of articles directly specify the function describing the Lorenz curve (Kakwani and Podder 1973; Kakwani and Podder 1976; Rasche *et al.* 1980; Gupta 1984; Basmann *et al.* 1990; Ortega *et al.* 1991).

One reason the literature has concentrated on specifying functional forms for the Lorenz curve and not the probability density function is that most writers are interested in measuring the amount of inequality in the distribution of income. The most common income inequality measure used is the Gini coefficient (or index). The Gini coefficient is the ratio of the area between the Lorenz curve L(p) and the 45 line (the shaded area in figure 3) to the area under the 45° line. The 45° line is labeled A in the figure and the Lorenz curve is labeled B. The area under the 45° line always equals one-half of the total area of the box. The area between the Lorenz curve and the 45° line is called the area of concentration.

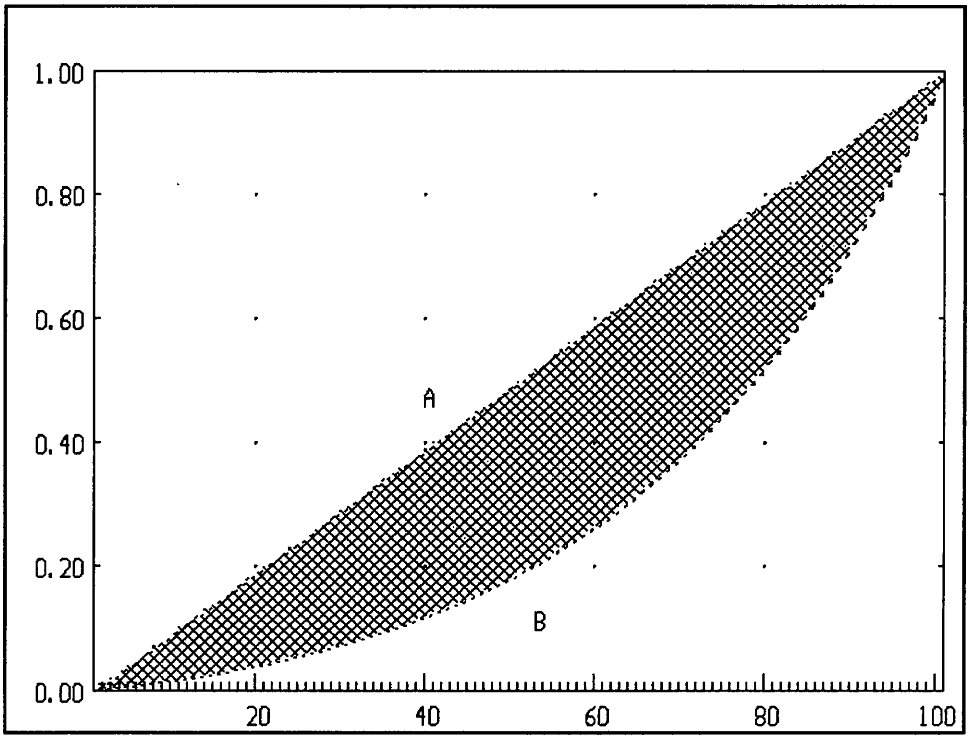

Figure 3

Area of Concentration

Since calculating the Gini coefficient requires integrating the function L(x), one often finds that specifying an integrable function L(x) is easier than specifying a probability density function that must be easily integrated twice. Since not all functions readily lend themselves to integration, researchers may feel that specifying the functional form of the Lorenz curve eases the task of calculating the Gini coefficient.

There are several other measures of inequality besides the Gini coefficient. These measures include the coefficient of variation (the standard deviation of the distribution divided by the mean of the distribution or σ/μ) and the variance of the natural log of income (Atkinson 1970). Unfortunately, these inequality measures do not always rank a set of distributions in the same way (Atkinson 1970, Slottje 1989, Levy and Murnane 1992). The differences are highlighted by applying a variant of the test suggested by Dalton (1920). The test is simple:

> If we transfer $1.00 from the X^{th} richest person to the X - 1 richest person, will the inequality measure register a decrease in inequality and, if so, will the magnitude of the decrease depend on the quintiles in which the transfer takes place?

Both Atkinson (1970) and Levy and Murnane (1992) have shown that (1) the Gini coefficient will always register a decrease in inequality, but the magnitude of the change depends on the quintile in which the transfer takes place; (2) the variance of the natural log of earnings will not always register a decrease in inequality nor is the magnitude independent of where in the distribution the transfer takes place; and (3) that the coefficient of variation will always register a decrease in inequality and the magnitude is independent of the quintile in which the transfer occurs.

Properties of the Lorenz Curve

If we assume that y = L(x), then any function describing a Lorenz curve meets the conditions below:

Condition 1	$L(0) = 0$;	
Condition 2	$L(1) = 1$;	
Condition 3	$L''(x) \geq 0$	$0 \leq x \leq 1$;
Condition 3a	$L'(x) \geq 0$	$0 \leq x \leq 1$;
Condition 3b	$x \geq L(x)$	$0 \leq x \leq 1$;

Condition 1 simply says that the poorest 0 percent of the population has no income. Condition 2 says that all income must be held by someone. Condition 3 is a result of the ordering of the population from poorest to richest and is nothing more then the definition of a quasi-convex function. The first derivative of the Lorenz curve function at any point multiplied by total income gives us the income held by the person at that point. The second derivative at the same point tells us the marginal increase in income at our selected point. Since population is ordered in terms of increasing income, the marginal increase in income held by any person must be positive or equal to zero (the egalitarian distribution). Conditions 3a and 3b follow from condition 3. Any function representing a Lorenz curve must satisfy these conditions over the range [0,1]. Some authors reject functions that do not satisfy these conditions globally (Ortega *et al.* 1991). This rejection is too harsh since the range of interest lies between $x = 0$ and $x = 1$.[44]

[44]For example, some might reject the function $L(x) = x^3$ since the function does not **globally** satisfy the condition that $L''(x) \geq 0$ - the second derivative is less than zero when x is negative. The function, however, satisfies the conditions within the range in which we are interested. Thus, we would not reject the function for failing to satisfy conditions 1 through 3.

In addition to the conditions given above, any function that represents a valid Lorenz curve has the following additional properties. If we define L as the set of all valid Lorenz curves, then:

Property 1: If $L_1(x), L(x)_2 \in L$, then $L_1(L_2(x)) \in L$;

Property 2: If $L_1(x), L_2(x) \in L$, then
$\lambda L_1(x) + (1-\lambda)L_2(x) \in L, \; 0 \leq \lambda \leq 1$;

Property 3: Let f(x) be some function where $f(1) = 1, f(x) \geq 0 \; \forall \; x \geq 0$,
$f', f'' \geq 0$ and $L(x) \in L$, then $L(x) \cdot f(x) \in L$;

Property 3A: If $L_1(x), L_2(x) \in L, L_1(x) \cdot L_2(x) \in L$;

Property 4: $L(x) = x \in L$.

Proofs of these properties are in the mathematical appendix to this chapter. The application of these properties are illustrated in the discussion of the various functional forms I selected to represent the Lorenz curve.

The Current, or Pollock, Method of Forecasting the Distribution

Pollock's model served two masters within the LIFT model: the tax model, and the consumption model. Pollock's principal area of interest was in developing a tax model for LIFT. Pollock used U.S. Internal Revenue Service (IRS) adjusted gross income (AGI) data as the basis of his estimation. The model forecasted the distribution of AGI for each year for size different household sizes. Taxes were removed from each of the distributions and the distributions were converted into a single distribution of post-tax aggregate AGI. The distribution was then converted into an aggregate distribution of disposable income (as defined by the NIPA) by means of a bridge matrix between AGI and disposable income.[45]

The columns of this matrix are the items used to reconcile AGI to disposable income and the rows of the matrix are the ventiles of the distribution. Each cell of the matrix gives the percentage of the reconciliation item that is allocated to the ventile. Thus, the first cell in the unemployment insurance column gives the percent of unemployment insurance that is allocated to the first ventile. The last cell in this column gives the percentage of unemployment insurance that is allocated to the twentieth ventile.

Pollock used a functional form for the Lorenz curve based on a form given by Kakwani and Podder (1976). Kakwani and Podder plotted the Lorenz curve using an alternate set of coordinates $\{\pi,\eta\}$. Consider figure 4:

[45]AGI excludes several items included in NIPA disposable income - mainly government transfers, income imputed by the NIPA and a statistical discrepancy. Similarly, NIPA disposable income excludes some items included in AGI - mainly personal contributions for social insurance.

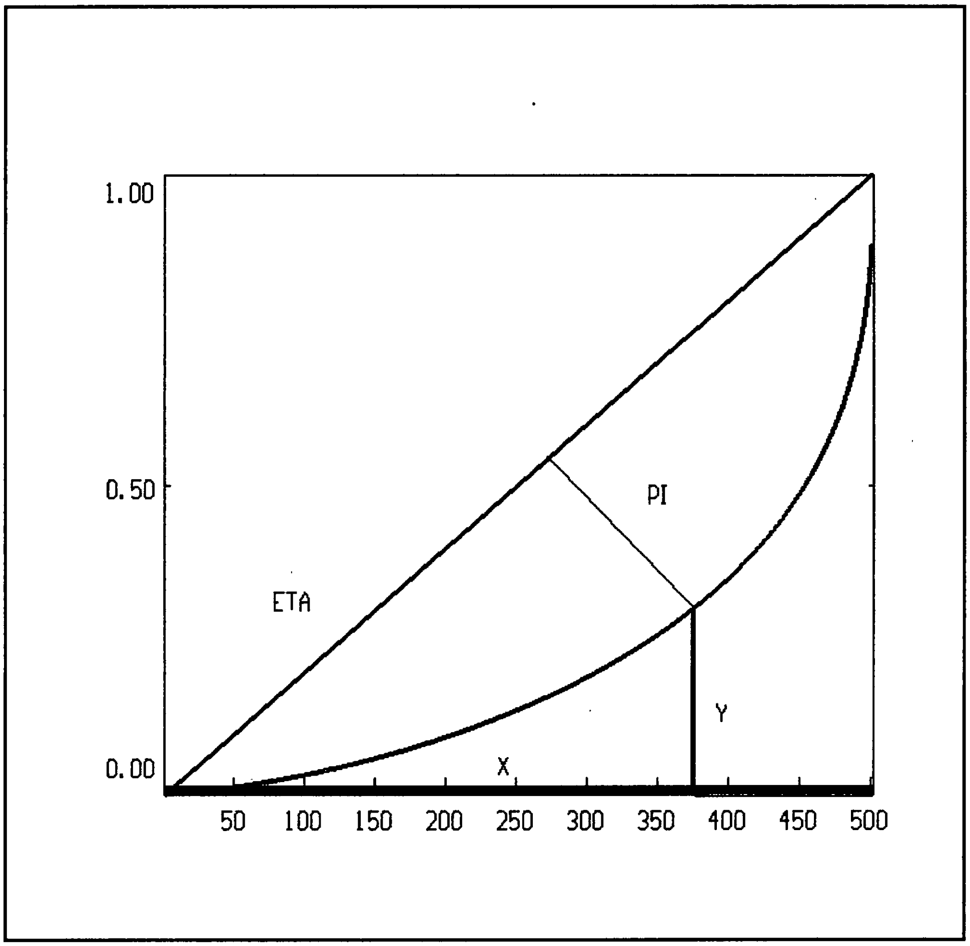

Figure 4

Alternate Coordinate System

Let P be any point on the Lorenz curve with coordinates P(x,y). The line of equal distribution is the diagonal running from O to H and has a length of $\sqrt{2}$. The angle ODP is a right angle by construction. The line segment OD is labeled S, and the line segment DP is labeled T. The lengths of S and T can be expressed in terms of x and y, and vice versa. Thus, π equals the length of T and η equals the length of S.

Kakwani and Podder's functional form is given by:[46]

$$\pi = a\eta^{\alpha} (\sqrt{2} - \eta)^{\beta} \qquad (4.2a);$$

where:

$$\eta = \frac{1}{\sqrt{2}} (x+y) \qquad (4.2b);$$

$$\pi = \frac{1}{\sqrt{2}} (x - y) \qquad (4.2c).$$

Pollock's function is given below:

$$\pi = A\ \pi_{base} + B\ (\eta^{1.5} (\sqrt{2} - \eta)^{.5}) \qquad (4.3);$$

where π_{base} equals the value of π in some base year.[47]

The function is additive in two parts. The first part uses the exact income distribution from historical observation as its base, or starting point. This part consists of the actual values of π along points of the η axis for the base year. The second part of the function is a smooth but skewed curve added to the base. The size of the coefficient, B, determines the size and direction in which income is "skewed" away from the original distribution.

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[46]Derivations of equations (4.2b) and (4.3c) are contained in the appendix to this chapter.

[47]The time and household subscripts have been deleted from the equation.

By forcing the predicted Lorenz curve in the last year of historical data to fit exactly the observed Lorenz curve, Pollock solves part of the so-called joining-on problem when the predicted Lorenz curve is used to calculate the five total expenditure variables from the cross-section work. This is a trick that we will employ when forecasting the Lorenz curve.

Use of the Kakwani-Podder coordinate system changes conditions 1 through 3. Under the new coordinate system, these conditions are written:

> Alternate Condition 1: If $\eta = 0$, $\pi = 0$;
> Alternate Condition 2: If $\eta = 1/\sqrt{2}$, $\pi = 0$;
> Alternate Condition 3: The limit of the slope of the function at $\{\eta=0, \pi=0\}$ must equal one and the limit of the slope as $\{\eta=2^{.5}, \pi=0\}$ must equal negative one (Rasche *et al.* 1980).

Rasche *et al.* show that the Kakwani and Podder function fails to satisfy condition 3 since the limit at both points is undefined. The Pollock function, being of the same family of functions as equation (4.2a), also fails to satisfy alternate condition 3. The slope of the Pollock function, 4.3 is given below:

$$\frac{d\pi}{d\eta} = A\frac{d\pi_{base}}{d\eta} + 1.5B\ \eta^{.5}\ (\sqrt{2}-\eta)^{.5} - \frac{.5B\ \eta^{1.5}}{(\sqrt{2}\ -\ \eta)^{.5}} \qquad (4.4).$$

The limit of the slope as η approaches 0 (or x=0, y=0) equals 0. The limit of the slope as η approaches $2^{.5}$ (or x=1, y =1) equals negative infinity.

Kakwani (1980) claims that these singularities at the extreme ends of the range of interest are slight and since equation (4.2a) fits most observed Lorenz curves well, one should not dismiss the function summarily.

Besides failing to satisfy the alternate conditions, Pollock's function has a second problem. In the $\{\eta,\pi\}$ space, the Lorenz curve is concave. Thus any function that represents a Lorenz curve in the $\{\eta,\pi\}$ space must be a concave function. All concave functions must have non-positive second derivatives everywhere. Except for the special case where A=1, B=0, the second derivative of the Pollock function is positive over portions of the range $0 \leq \eta \leq 2^{.5}$.[48] Consequently, it can generate only one valid Lorenz curve - the Lorenz curve for the selected base year. For this reason, the Pollock form was discarded and a new functional form was selected.

Alternate Functional Forms

After reviewing the numerous functional forms given in the literature for representing a Lorenz curve, I decided to investigate four of these. These forms are given below:

Kakwani
$$y = f(x) = x^{\alpha} e^{\beta(x-1)} ; \qquad \alpha \geq 1; \quad \beta > 0 \qquad (4.5);$$

Rasche
$$y = f(x) = [1 - (1 - x)^{\alpha}]^{1/\beta} ; \qquad 0 < \alpha \leq 1; \quad 0 < \beta \leq 1 \qquad (4.6);$$

Gupta
$$y = f(x) = xA^{(x-1)} ; \qquad A \geq 1 \qquad (4.7);$$

Ortega
$$y = f(x) = x^{\alpha} [1 - (1 - x)^{\beta}] ; \qquad \alpha \geq 0; \quad 0 < \beta \leq 1 \qquad (4.8).$$

48 The appendix to the chapter contains this proof.

Equations (4.5) is due to Kakwani and Podder (1973); (4.6) is from Rasche *et al.* (1980); (4.7) to Gupta (1984) and (4.8) to Ortega *et al.* (1991). It should be pointed out that equation (4.7) is a particular case of equation (4.5).

The Kakwani function, equation (4.5), is easily shown to be a function in the set of valid Lorenz curves, L, by using property 3 of Lorenz curves. The first term in the equation, x^{α} is a valid one-parameter Lorenz curve provided α is greater than or equal to one. The second term, $e^{\beta(x-1)}$, is a monotonically increasing function, f(x) with a positive first derivative. The value of this function equals one when x equals one and the function is positive over the range zero to one. Thus, by property 3, the product of the two terms must be an element of L.

By using property 1 of Lorenz curves, it can be easily demonstrated that the Rasche function, equation (4.6), is an element of L. The expression that is raised to the exponent $1/\beta$ is a valid one-parameter Lorenz curve. By assumption, β is less than one and, thus, the reciprocal of β is greater than one. Consequently, the function is an element of L.

The Gupta function (4.7), like the Kakwani function, can be shown to be a valid Lorenz curve by the application of property 2 of Lorenz curves. The first term, x, is a valid Lorenz curve (y=x plots the egalitarian income distribution) and the second term $A^{(x-1)}$ is a monotonically increasing function with a value of one when x equals one. Accordingly, the Gupta function is an element of L.

Using property 3A of Lorenz curves, it can be demonstrated that equation (4.8), the Ortega function, is an element of L. The first term in the function, x^{α}, is an element of L. The second term in the function also is an element of L. From property

3A, we know that the product of two elements of L is an element of L. Alternatively, we could have used property 3 of Lorenz curves to show that the Ortega function is an element of L. Since any Lorenz curve can be described as a function, f(x), that equals one when x equals one; has non-negative first and second derivatives, the product of f(x) and an element of L is also an element of L.

Data

The data used in estimating these functions is from the published Integrated Consumer Expenditure Survey (CEX) data for the years 1984 through 1993. The CEX consists of two surveys: an interview and a diary survey. In the interview survey, a household is subjected to a lengthy interview that asks whether particular items were purchased during the survey period. In the diary survey, the same household records each expenditure as it occurs or at the first opportunity. The two surveys often give different reports on the spending habits of the same household. These two surveys are then integrated into a single survey.

The aggregate CEX data for years prior to 1984 are incompatible with the aggregate data for the 1984 to 1993 period. Prior to 1979, the CEX was conducted every ten years in years ending in a two (1972, 1962 and so-on). The definitions used in determining which consumer units constitute a separate household for these earlier surveys differ from the surveys of the 1980s - thus the data from these surveys are incompatible with the 1980's surveys. In 1979, a continuous and on-going survey was initiated. While surveys for the years 1979 through 1983 use the same definitions as the 1984 through 1993 surveys, the aggregate data is incompatible. The aggregate data for the earlier period was published from either the interview data or the diary data due to budget constraints. In 1984 and all subsequent years, the aggregated data is from the integrated survey results. Consequently, the data for the years 1979 through 1983 are incompatible with the later data.

The data provide a distribution of household income for each year.[49] The information was reported by a varying numbers of income brackets. The fewest brackets published was 8 in the years 1984 through 1991 and the most brackets published was 9 in 1992 and 1993. For the purposes of constructing a distribution of expenditures, I was interested in the following data within each bracket: the average number of persons in each household, the number of households and, the average expenditures of each household. Using this data, I calculated the per-capita expenditures within each bracket. In some instances, the order of the per-capita expenditures was no longer increasing. In these instances, I re-ordered the brackets from smallest to largest per-capita expenditures. Cumulative total expenditures in each bracket, i, was divided by the sum of aggregate expenditures across all of the n-brackets to calculate the cumulative percentage of expenditures in a bracket i (or the y-coordinate):

$$y_i = \frac{\sum_{j=1}^{i} Expenditures_i}{\sum_{j=1}^{n} Expenditures_j} \qquad (4.9).$$

[49]In this instance, we do mean income and not expenditures.

The cumulative percentage of population within each re-ordered bracket (the x-coordinate) is calculated as the cumulative number of persons in a given bracket divided by the total population. This procedure gives us a set of x- and y- coordinates that describe the Lorenz curve for expenditures for each year.

For each year 1984-1993 and each functional form, I performed a non-linear estimation of equations (4.5) through (4.8) using the downhill simplex method (Nelder and Mead 1965). The method seeks to minimize the sum of the squared differences between y and predicted y. Like all non-linear techniques, the results obtained by using the downhill simplex method may depend on the initial, or starting, values used in evaluating the function. Similarly, different initial step-sizes and convergence criteria can alter the final results. To avoid mistaking a local minimum for a global minimum, a series of estimations is run using varying initial conditions and step-sizes. The results obtained for the four functions studied did not depend on initial conditions or step-size. The coefficient estimates obtained were identical within 5 or more significant digits. However, altering the initial starting values and step-sizes did alter the number of iterations required for the algorithm to converge The standard errors for each function and year are given below in table 7a:

TABLE 7a

Standard Errors of Estimation By Year and Functional Form

Year	Kakwani	Rasche	Gupta	Ortega
1984	0.00999	**0.00245**	0.01493	0.0028
1985	0.00826	**0.00316**	0.01184	0.00325
1986	0.00840	**0.00322**	0.01222	0.00329
1987	0.00700	**0.00271**	0.01033	0.00280
1988	0.00723	**0.00231**	0.00988	0.00237
1989	0.000673	**0.00280**	0.00953	0.00285
1990	0.00465	**0.00342**	0.00953	0.00351
1991	**0.00355**	0.00373	0.00730	0.00380
1992	0.00990	**0.00199**	0.01231	0.00210
1993	0.01007	**0.00193**	0.01353	0.00207

Lowest or best standard error shown in bold.

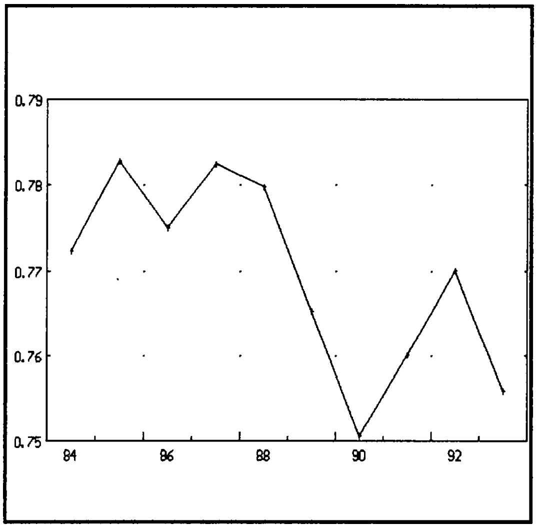
As can be seen in table 7a, the Rasche function has the best fit in nine of the ten years in the sample. For this reason, the Rasche function was selected as the best function of the four to represent the Lorenz curve. Table 7b gives the estimated coefficients and their t-values for the Rasche function for the years 1984 to 1993.

TABLE 7b

Regression Results for Rasche Function By Year

Year	β	α	SEE
1984	0.8941 (101.21)	0.7723 (100.91)	0.00245
1985	0.8707 (75.28)	0.7827 (74.52)	0.00316
1986	0.8678 (74.84)	0.7750 (72.80)	0.00322
1987	0.8754 (88.98)	0.7824 (85.34)	0.00271
1988	0.8739 (102.48)	0.7797 (96.63)	0.00231
1989	0.8802 (80.46)	0.7652 (73.27)	0.00280
1990	0.8950 (64.97)	0.7505 (57.67)	0.00342
1991	0.8898 (57.06)	0.7601 (49.54)	0.00373
1992	0.8684 (129.90)	0.7700 (125.58)	0.00199
1993	0.8882 (134.51)	0.7559 (126.70)	0.00193

Figures 5 and 6 plot the estimated values of α and β from the Rasche function.



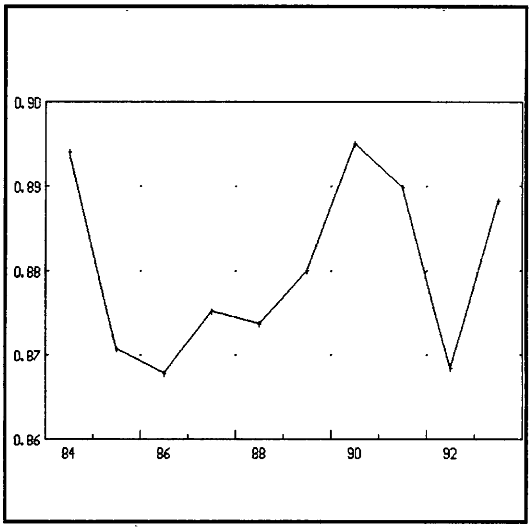


Figure 5

Estimated α

Figure 6

Estimated β

The parameters α and β have economic interpretation. As either increases *ceteris paribus*, the distribution of expenditures become more equal or egalitarian, i.e. the Gini coefficient gets smaller. If the parameters move in opposite directions, the effect on the distribution is not obvious. To know the effect of the move in opposite directions, one must calculate the Gini Coefficient or the coefficient of variation. I calculate both of these measures for the years 1983 through 1993. The Gini coefficient for the Rasche function equals:

$$Gini = 1.0 - \frac{2.0}{\alpha} B(\frac{1}{\alpha}, 1 + \frac{1}{\beta}) \qquad (4.10).$$

where B represents the Beta distribution. The coefficient of variation for the Rasche function equals:

$$CoefVar = \frac{\sigma_y}{\mu_y} = \frac{\sqrt{\int_0^1 (x - \mu_x)^2 \frac{\alpha}{\beta} (1 - x)^{\alpha - 1} [1 - (1 - x)^{\alpha}]^{\frac{1}{\beta} - 1} dx}}{\mu_y} \qquad (4.11);$$

where:

$$\mu_y = 1 - \frac{1}{\alpha} B(\frac{1}{\alpha}, 1 + \frac{1}{\beta}) \qquad (4.12).$$

B again represents the Beta distribution.[50]

The Gini coefficient is restricted to the range {0,1} with smaller Gini coefficients implying more equal distributions. The coefficient of variation ranges from .57735 (the egalitarian distribution) to zero (the completely unequal distribution). Thus, the two value of the measures, if calculated exactly, always will move in opposite directions.[51] Table 8 gives the historical values of the Gini coefficient and the coefficient of variation.[52] The fourth and fifth columns of the table list the percentage change from the previous year for the Gini coefficient and the coefficient of variation, respectively.

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[50]Proofs of equations (4.10) and (4.11) are in the appendix to this chapter.

[51]If the distribution becomes more unequal, the Gini coefficient should increase and the coefficient of variation should decrease.

[52]I calculated both statistics using programs from Press *et al.* (1986).

At this point, we should note that the calculated Gini coefficients in the table are known exactly since the value of the Beta distribution is calculated easily. Unfortunately, the coefficient of variation is an approximation since the integral in the formula for the variance does not reduce to a known form. For this reason, the value of the integral was calculated using the extended trapezoidal rule for calculating the area under a curve. This means that the coefficient of variation values given in the table contain an approximation error.

TABLE 8

Gini Coefficient and Coefficient of Variation for the Years 1984-1993

	Gini Coefficient	Coefficient of Variation	% Δ Gini	% Δ CVar
1984	0.1876	0.4775	N/A	N/A
1985	0.1948	0.4713	3.77	-1.31
1986	0.2015	0.4681	3.38	-0.68
1987	0.1922	0.4731	-4.73	1.06
1988	0.1948	0.4717	1.34	-0.30
1989	0.2005	0.4701	2.88	-0.34
1990	0.2015	0.4717	0.50	0.34
1991	0.1982	0.4724	-1.65	0.15
1992	0.2044	0.4669	3.08	0.02
1993	0.2019	0.4706	-1.23	0.79

The calculated Gini coefficients indicate that the distribution of expenditures grew more unequal during the period 1984 to 1993, but that three of the nine year-to-year movements were towards a more equitable distribution. The coefficient of variation also indicates that the distribution grew more unequal during the period 1984 to 1993. In terms of more equal versus less equal, the year-to-year movements were also in the same direction, except for the years 1990 and 1992, where the Gini coefficient indicates that equality moved in one direction and the coefficient of variation indicates the opposite.

At first glance, that the two measures give different results for 1990 and 1992, is a bit puzzling since, by the Dalton test described earlier in the chapter, if we transfer \$1.00 from the X^{th} richest person to the X - 1 richest person, the two inequality measure will register a decrease in inequality. Thus, one could conclude that the two inequality measures always should move in the same direction. However, the change in the Gini coefficient depends on the quintiles in which the transfer takes place; while the Coefficient of Variation is independent of the quintiles. Thus, one could make a series of transfers that left the Coefficient of Variation unchanged, but the Gini coefficient, which weights the transfers based on the quintiles, could change.

Table 8 shows that the distribution of income varies from year-to-year and that these fluctuations move in both directions. Since the distribution is neither constant nor changing in a constant manner, our decision to endogenize the forecasts of these distributions is justified.

Forecasting the Coefficients of the Rasche Function

Two approaches were attempted to develop equations to forecast α and β, the parameters of the Rasche function. The major objective in choosing an approach was insuring that the model possessed certain properties. Each approach was evaluated based on two criteria. The primary criteria in choosing an approach was that the reaction of the income distribution to economic variables did not contradict *a-priori* assumptions of these effects. A secondary criteria was making both α and β endogenous within the model. The *a-priori* assumptions are based on simulation experience as well as how LIFT-users want the model to react. In the case of the first criteria, an approach that gave no reaction to economic activity was preferred to an approach that gave counter-intuitive results. For example, increases in the rate of inflation are thought to benefit debtors and punish creditors. Since creditors are concentrated in higher income ventiles, one would expect that an increase in the rate of inflation would transfer income from the rich to the poor and that the income distribution would become more egalitarian. Using this criteria, any formulation that gave the result that an increase in the inflation rate makes the distribution less equal would be rejected.

The same general functional form was used in both attempts. Because both α and β must lie between zero and one, the functional form had to constrain the predicted value of α or β to lie within this range. The functional form I estimated for both equations was logit function and is:

$$\ln\left(\frac{\alpha_t}{1-\alpha_t}\right) = F(Z_t;\ \gamma_\alpha) \qquad (4.13).$$

$$\ln\left(\frac{\beta_t}{1-\beta_t}\right) = F(Z_t;\ \gamma_\beta) \qquad (4.14);$$

where:

Z_t = independent variables in year t;
$\gamma_{\alpha,\beta}$ = parameters to be estimated.

Note that $ln\ (\alpha/(1\text{-}\alpha))$ goes from $-\infty$ to $+\infty$ as α goes from 0 to 1. Thus, for any predicted value of $ln\ (\alpha/(1\text{-}\alpha))$, the value of α lies between 0 and 1. This insures that the predicted coefficients will generate a valid Lorenz curve.

The initial approach was to make both α and β endogenous. Given that there are only ten observations, the number of independent variables in the equations had to be small. The functions F_α and F_β should be a linear function of variables that are available from other parts of LIFT. Four such variables are:[53]

[53]I did not use the means-tested transfer share of personal income in any of the estimations. I excluded the means-transfer share because the direction of its effect on the distribution of income is uncertain. For example, if the increase in the transfer share is due to an increase in the generosity of these programs, either increased payments to current recipients or expanded eligibility requirements, then one expects a move towards a more egalitarian income distribution. However, if the generosity of these programs is constant then the increased share is caused by an increase in the number of persons on the welfare rolls and we expect a less egalitarian distribution. There are also questions of causality. That is to say, it is unclear whether increasing inequality affects the transfer share or whether a higher transfer share affects the distribution of income. Given these reasons, this variable was not used in estimating equations (4.13) and (4.14).

Capinc = The capital income share of total personal income. This variable is defined as the ratio of the sum of personal interest and dividend income to total personal income. Increases should imply a less equal distribution.

Inflate = The rate of inflation defined as the percent growth in the personal consumption expenditure deflator. Increases should imply a more equal distribution.

Un = The unemployment rate. Increases in this variable should imply a less equal distribution.

Indshr = The share of the sum of manufacturing, mining, transportation and utility employment in total private employment. It was felt that jobs in these four sectors represented so-called "good jobs at good wages." Increases in this variable should imply a more egalitarian distribution.

As mentioned above, the first approach made both α and β endogenous. Unfortunately, the estimated parameters for equations (4.13) and (4.14) generate equations with very poor adjusted R-squared values. The second approach tried was reestimating the Rasche function with one of the parameters set equal to its mean from the first estimation. Because the variance of α is larger than the variance of β, it was felt that holding β constant was more appropriate than holding α constant. Table 9 shows the estimated mean and variance of both α and β.

TABLE 9

Mean and Variance of α and β

	α	β
Mean	0.76951	0.88045
Estimated Variance	0.00013	0.00011

Table 10 shows the estimated values of α holding β constant at β_{mean}.

TABLE 10

Estimated Values of α with β=β_{mean}

Year	α (β=β_{mean})	α (β estimated)
1984	0.7833 (236.94)	0.7723 (100.91)
1985	0.7746 (207.91)	0.7827 (74.52)
1986	0.7643 (195.90)	0.7750 (72.80)
1987	0.7780 (245.24)	0.7824 (85.34)
1988	0.7740 (272.78)	0.7797 (96.63)
1989	0.7650 (232.26)	0.7652 (73.23)
1990	0.7635 (170.69)	0.7505 (57.67)
1991	0.7689 (162.60)	0.7601 (49.54)
1992	0.7599 (292.20)	0.7700 (125.58)
1993	0.7625 (333.46)	0.7559 (126.70)

Table 11 presents the estimation results for equation (4.13) and figure 7 plots the actual and predicted values of α when β is held constant at its mean.

TABLE 11

Estimation Results for α when $\beta = \beta_{mean}$

SEE = 0.02	RSQ = 0.8555	RHO = -0.46	Obser = 10 from 1984.000			
SEE+1 = 0.01	RBSQ = 0.7399	DW = 2.93	DoFree = 5 to 1993.000			
MAPE = 1.05						
Variable name	Reg-Coef	Mexval	t-value	Elas	NorRes	Mean
0 lac	- - - - -	- - - -	- - - -	- - - -	- - - -	1.21
1 intercept	0.95913	119.0	4.358	0.80	6.92	1.00
2 capinc	-0.04330	58.8	-2.758	-0.61	6.73	17.11
3 un	-0.00162	0.1	-0.107	-0.01	6.34	6.56
4 inflate	0.03981	40.8	2.216	0.12	5.76	3.70
5 indshr	0.03230	139.9	4.877	0.71	1.00	26.35

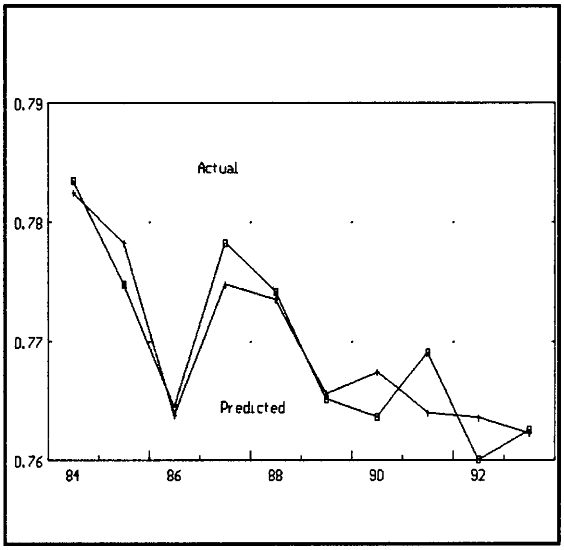

Figure 7

Actual versus Predicted α: $\beta=\beta_{mean}$

As can be seen in table 11, the equation fits well. With the exception of the unemployment variable, all of the regression coefficients are significant and have large

Mexvals.[54] The equation explains over eighty-five percent of the movement in α and the adjusted R-squared statistic is similar in magnitude to the R-squared statistic. The effects of all the variables on the income distribution matches *a-priori* beliefs.

One problem with constraining the β parameter is that the function essentially becomes a one-parameter function representing the Lorenz curve. One-parameter functions of this type cannot generate Lorenz curves that cross each other (Thistle and Formby 1987). That is to say, given a Lorenz curve, L_1, generated by a one-parameter function, the function cannot generate a Lorenz curve L_2 that intersects L_1 within the interval $\{0,1\}$. Since direct observation reveals that during the period 1984 to 1993, Lorenz curves in the U.S. have crossed, using a one-parameter functional form to represent the Lorenz curve should be a last-ditch solution. Consequently, the Kakwani and Ortega functions warranted further investigation.

An Unsuccessful Alternative to the Rasche Function

The Rasche function was selected because the standard errors of the estimates for this function were, in general, lower than the other functions. However, the standard errors of the β-constrained Rasche function when compared to the unconstrained standard errors of the other three functions were lowest in only 1987 and 1989, though these errors still were lower than the unconstrained Kakwani function in all but 1991. The unconstrained Kakwani function had the lowest standard error in 1991 and the unconstrained Ortega function was lowest in all other years. Given the superiority of

[54]Mexval or the Marginal EXplanatory VALue gives the percent of the variation in the dependent variable that is caused by movements in the independent variable. For additional discussion, see Monaco (1989).

the unconstrained Ortega function over the β-constrained Rasche function, I estimated equations (4.13) and (4.14) for the unconstrained parameters from the Ortega function. The results of these two equations are given in tables 12a and 12b.

TABLE 12a

Estimation Results for α from Ortega Function

SEE = 0.10	RSQ = 0.2379	RHO = -0.15	Obser = 10 from 1984.000			
SEE+1 = 0.09	RBSQ = -0.3718	DW = 2.31	DoFree = 5 to 1993.000			
MAPE = 4.52						
Variable name	Reg-Coef	Mexval	t-value	Elas	NorRes	Mean
0 lao	- - - - -	- - - -	- - - -	- - -	- - - -	-1.83
1 intercept	-1.62626	12.9	-1.171	0.89	1.31	1.00
2 capinc	0.02792	0.8	0.282	-0.26	1.30	17.11
3 un	-0.07435	5.8	-0.775	0.27	1.30	6.56
4 inflate	-0.13162	12.7	-1.161	0.27	1.01	3.70
5 mfctshr	0.01128	0.7	0.270	-0.16	1.00	26.35

TABLE 12b

Estimation Results for β from Ortega Function

SEE = 0.03	RSQ = 0.6361	RHO = 0.06	Obser = 10 from 1984.000			
SEE+1 = 0.03	RBSQ = 0.3449	DW = 1.88	DoFree = 5 to 1993.000			
MAPE = 2.68						
Variable name	Reg-Coef	Mexval	t-value	Elas	NorRes	Mean
0 lbo	- - - - -	- - - -	- - - -	- - -	- - - -	1.08
1 intercept	0.90556	33.5	1.976	0.84	2.75	1.00
2 capinc	-0.03742	12.3	-1.145	-0.59	2.74	17.11
3 un	-0.02853	7.8	-0.901	-0.17	2.61	6.56
4 inflate	-0.00467	0.2	-0.125	-0.02	2.57	3.70
5 mfctshr	0.03866	60.4	2.804	0.94	1.00	26.35

Neither of these equations fit particularly well and some of the signs are perverse. The low R-squared statistic in tables 12a and 12b indicated that the Ortega function be reestimated holding α equal to its mean, making the function one-parameter. The standard errors of the estimated Ortega function holding α constant at its mean exceeded those of the β-constrained Rasche function. Because the α-constrained Ortega functions proved worse than the β-constrained Rasche functions, the β-Rasche function was determined as the superior function, despite being a one-parameter function.

Constructing the Historical Lorenz Curves for the Years 1984 through 1993

All of the work to this point on forecasting the distribution of expenditures has been so that we may construct the total expenditure variables used in the estimation of the system of PCE equations and the forecasting of PCE in the simulation model. The estimated Rasche function is used to "backcast" the Lorenz curve for years prior to 1984 and to forecast the Lorenz curve for years after 1993.

When calculating the size of the five expenditure groups, one needs the ability to calculate the cumulative income held by any person along the x-axis. This is because there is no way of determining, *a-priori*, which person or household is the first household to hold income in each bracket.[55] This means that the Lorenz curve must be known for all values of x between zero and one. Ideally, one has historical data for all or a sufficiently large number of these points. Unfortunately, for the years 1984 through 1993, inclusive, we know only eight to ten of these {x,y} coordinates per year.

Having so few points poses no problem in estimating a function to describe the Lorenz curve since we know that the true Lorenz curve must pass through the known

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[55]The single exception is the year 1972. Because the brackets are defined so that each bracket has twenty percent of the households. Thus, for the year 1972, the first bracket contains the first fifth of the population (x between zero and one-fifth.) The second bracket holds the second fifth of the population (x between one-fifth and two-fifths) and so on.

points. If the estimated functions for the years 1984 to 1993 fit the known points perfectly or slightly imperfectly, then we could use the estimated functions when calculating the expenditure groups. Unfortunately, all of the predicted Lorenz curves using any of the already-presented functional forms fail to fit with an acceptable accuracy; the absolute magnitude of the errors appears small, but the errors, when expressed as a percent of the total income within the bracket, are relatively large.

TABLE 13

Errors in Rasche Function for Year 1984

X-Cell	Percent Error
1	-11.674
2	4.215
3	3.019
4	-0.478
5	1.201
6	-0.012
7	-2.2769
8	0.381

Table 13 shows these errors for the year 1984. The first column gives the x-cell and second column gives the error as a percent of income within the x-cell. As can be seen, the errors are non-trivial. Five of the cells have errors over one percent and the first cell has an error of over ten percent. In all years, the estimated Rasche function, as well as all of the other estimated functions, consistently under-predicts the cumulative income held by the first observation. Because the Rasche function does not perfectly fit the observed data, we are presented with a quandary when calculating the expenditure variables. We can either use the set of x and y coordinates predicted by the Rasche function, which would ignore the observed set of x and y coordinates, or we can use the observed data. The first solution, using the predicted coordinates, was viewed as unacceptable since the known data is discarded. However, the second solution, using the observed set of coordinates, poses its own set of problems.

As mentioned earlier, in order to calculate the expenditure variables, the cumulative income held by any particular person must be calculable. We cannot use the historical data to calculate the cumulative income held at any x-coordinate other than those we observe without making strong assumptions regarding the shape of the Lorenz curve between the observed coordinates. Since the observed data only contains nine or ten points, there will be large segments of the Lorenz curve determined by the assumptions. These assumptions must not only generate a curve that passes through all of the known {x,y} coordinates, but they must also generate a curve that is a valid Lorenz curve.

The simplest assumption is to suppose linear segments along the Lorenz curve between the observed x-coordinates or x-cell. This assumption generates a valid Lorenz curve since the first derivative of the Lorenz curve is always positive and the second derivative of the implied Lorenz curve is always non-negative -- at the known points the second derivative is undefined since the function is not smooth and between the known points it equals zero. Figure 8 shows the case when the known points are connected by line segments. Figure 9 shows the distribution of per-capita income for 1984 if the line-segment assumption is used in calculating the Lorenz curve.

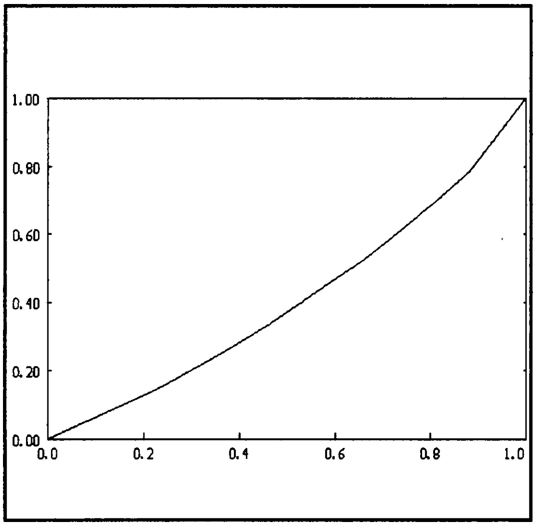

FIGURE 8

Linear Segments Between Points

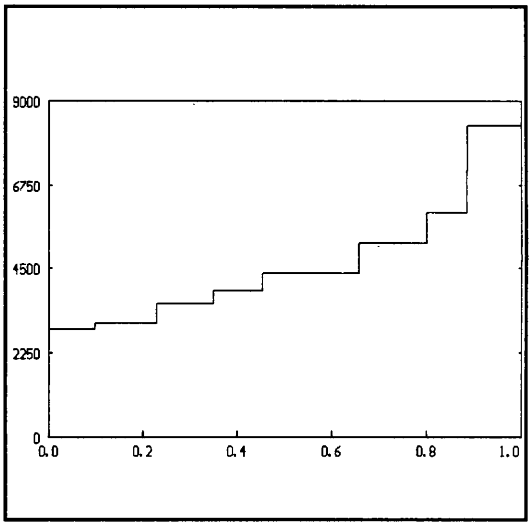

FIGURE 9

Per-Capita Income Distribution

Line Segments

Unfortunately, as can be seen in figure 9, one consequence of the second derivative equalling zero within a given x-cell is that all persons along the segment have the same income. That is to say, for the year 1984, the first 9.78 percent of the population have identical income. The next 12.04 (22.81 less 9.78) percent of the population have identical income, and so-on. Clearly this assumption allocates too much income to the poorest persons within the x-cell. For this reason, a different solution to this problem was required.

A method was needed that assured that the fitted Lorenz curve passed through the observed {x,y} coordinates, but generated a curve that had non-zero second derivatives.

The solution was to predict the slope of the Lorenz curve while keeping the average slope of the points within an x-cell equal to the average slope implied by the end points of the x-cell. For example, if the lower boundary of an x-cell is the coordinates {x=0, y=0} and the upper bound is {x=.2, y=.05}, then the average slope of the x-cell is 0.25. If we assume that the slope is a linear function of the x-coordinate, then we know that at the mid-point of any x-cell, the slope of the Lorenz curve equals the average slope implied by the x-cell boundaries. These two assumptions can be rewritten as:

> Assumption 1: The first derivative of the Lorenz curve is an increasing linear function of cumulative population (x).

> Assumption 2: Between any two known x-coordinates, the Lorenz curve is has a quadratic form:

$$y = y_{i-1} + a_i\gamma_i + b_i\gamma_i^2 \quad (4.15a);$$

$$0 \leq \gamma_i \leq x_i - x_{i-1} \quad (4.15b).$$

The first derivative of this function equals:

$$f_i'(\gamma) = a_i + 2*b_i\gamma_i \quad (4.15c).$$

For the quadratic to generate a valid Lorenz curve, some restrictions must be placed on the coefficients. We know that the first derivative at the right boundary of an x-cell equals:

$$f'_i(\gamma_i = x_i - x_{i-1}) = a_i + 2*b_i(x_i - x_{i-1}) \quad (4.15d);$$

and that the first derivative at the left boundary of the next x-cell equals:

$$f'_{i+1}(\gamma_{i+1} = 0) = a_{i+1} \quad (4.15e).$$

Since the second derivative of the Lorenz curve must be non-negative, we assume:

Assumption 3: The first derivative at the right boundary of the $i^{th}+1$ x-cell is less than or equal to the first derivative at the left boundary of the $i^{th}+1$ x-cell. This is written:

$$0 \leq a_1 \qquad (4.15f);$$

$$a_i + 2*b_i(x_i - x_{i-1}) \leq a_{i+1}, \text{ for all x-cells} \qquad (4.15g);$$

One additional assumption is required since there is no guarantee that b_i will not equal zero -- the case illustrated in figure 9 where all persons within an x-cell have identical income. To avoid this possibility we make the following:

Assumption 4: b_i greater than some small positive number ($b_i \geq 0.005$).

These four assumptions provide us with the basic frame for selecting sets of a_i's and b_i's, but there is an infinite number of a_i and b_i's that satisfy these four assumptions. For this reason, a rule is needed that guides the selection of the a_i's and b_i's. Ideally, one hopes that at the common point, the first derivatives are equal, however, in some instances this solution may not be possible. Consequently, the rule used here is that the set of a_i's and b_i's that minimize the size of the jumps in the first derivatives as we moved between x-cells should be used.

If the gaps equals:

$$Gap_i = a_{i+1} - (a_i + 2*b_i\gamma_i) \geq 0 \qquad (4.16a);$$

then, we select the a_i and b_i to minimize:

$$\sum^{N}_{i=1} (a_{i+1} - (a_{i-1} + 2{*}b_{i-1}\gamma_{i-1})) \qquad (4.16b).$$

The four assumptions can be written as a constrained minimization problem over the coefficients a_i and b_i with equation (4.16b) being the function we wish to minimize:

$$\text{Min } Z = \sum_i (Gap_{i+1} - Gap_i) = \sum_i \{a_{i+1} - (a_i + 2{*}b_i\gamma_i)\} \qquad (4.17a);$$

Subject to the following constraints:

$$Gap_{i+1} - Gap_{i1} = a_{i+1} - (a_i + 2{*}b_i\gamma_i) \geq 0 \qquad (4.18b);$$

(The second derivative must be non-negative)

$$y = y_{i-1} + a_i\gamma_i + b_i\gamma_i^2 \qquad (4.18c);$$

$$0 \leq \gamma_i \leq x_i - x_{i-1} \qquad (4.18d).$$

(The Lorenz Curve must pass through the observed points)

$$b_i \geq 0.005 \qquad (4.18e);$$

The minimization problem is quickly solved using a procedure similar to the one described in Chapter 4 of Almon (1967).

Constructing the Lorenz Curves for Years prior to 1984

The 1984 Lorenz curve constructed by the differential method and the Lorenz curve predicted by the Rasche function differ. While the absolute difference between the two curves is small, the relative differences between the two curves are large for some values of x. Figure 10 plots the percentage difference between the curve predicted by the Rasche function and the curve produced by the above method. If the Rasche function were used in backcasting the Lorenz curve for 1983 (the first year we must backcast), there would be large movements in the predicted values of the total expenditure variables as we moved from 1983 to 1984. These jumps would be an artifact of the switch in the functional form being used in backcasting the Lorenz curve. This set of jumps is known as a joining-on problem.

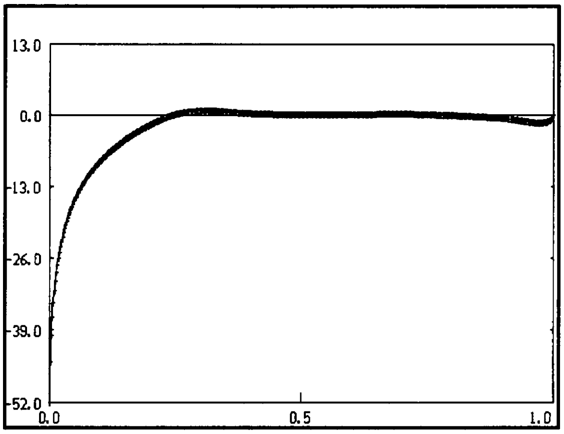

Figure 10

Relative Differences

Between Predicted Lorenz Curves

However, if we use the second property of Lorenz curves, we can alleviate the problem. The second property of Lorenz curves is:

<u>Property 2</u>: If $L_1(x)$, $L_2(x) \in L$, then
$\lambda L_1(x) + (1-\lambda)L_2(x) \in L, \quad 0 \leq \lambda \leq 1.$

Specifically, if we assume:

$$L_t(x) = \rho^{(1984-t)} L_{d,1984}(x) + (1-\rho^{(1984-t)}) L_{R,t}(x) \qquad (4.19);$$

where:

$L_t(x)$ = Predicted Lorenz curve for year t, $1983 \geq t \geq 1959$;
$L_{d,1984}(x)$ = Lorenz curve predicted by differential method, year 1984;
$L_{R,t}(x)$ = Lorenz curve predicted by Rasche function, year t.

The procedure can be thought of as a form of rho-adjustment. Rho-adjustment typically is used to solve a joining-on problem when one is forecasting a variable. Often, an equation possesses excellent simulation properties and has an excellent fit for all years except the last year, year t, of historical data (or last two, etc. years). In year t+1, the first year of the forecast, the equation might predict a value that joins the historical data poorly. One method of avoiding this problem is to use a rho-adjustment. The adjustment process is done in a four-step procedure. First the error in the last year of historical data is calculated:

$$Error = y_t^{true} - y_t^{predicted} \qquad (4.20).$$

Next, the value of ρ must either be estimated or specified. If ρ is estimated, then generally, the estimate is the autocorrelation coefficient of the residual in the regression. The third step is to calculate the adjustment in any year:

$$adjustment_i = error * \rho^{i-t} \qquad 0 \leq \rho \leq 1 \qquad (4.21).$$

The final step is adding the adjustment to the predicted value of the forecasted variable:

$$\hat{y}_i = \hat{y}_i^{predicted} + adjustment_i \qquad (4.22).$$

If ρ equals 1, then the adjustment is constant through out the forecast. If ρ equals 0, then there is no adjustment during the forecast. As ρ decreases, the rate at which the adjustment decays increases. Thus, a ρ of .1 means that in the first year of the forecast the adjustment equals 10 percent of the error in the last historical period and 1 percent of the error in the second year. Larger values of ρ give a slower decay rate for the adjustment term. A ρ of .9 gives an adjustment in the first year equal to 90% of the error and a second year adjustment equal to 81% of the error.

The procedure in (4.19) is slightly different then the standard rho-adjustment. Instead of the adjustment declining as we move forward-in-time, the adjustment is greatest in the latest year (1983) and declines as we move backward through time. In 1984, the function fits the 1984 Lorenz curve exactly. However, in years prior to 1984, the predicted Lorenz curve is a combination of the Rasche Lorenz curve for the specified year and the 1984 differential Lorenz curve. As we move to earlier years, the function slides over to the Rasche curve to a greater degree and the effect of the 1984 differential Lorenz curve decays. The rate of decay depends on the value of ρ.

When constructing the 1959 through 1983 Lorenz curves, a ρ of .8 was used. This value is sufficiently large so as to limit the joining-on problem between the years 1983 and 1984 and is also of a magnitude where the influence of the 1984 differential Lorenz curve quickly decays. For example, in 1979 when t=5, the value of ρ^t equals .32768. This means that the differential Lorenz curve for 1984 influences the predicted Lorenz curve for 1979 by less than one-third and the Rasche predicted Lorenz curve for 1979 accounts for over two-thirds of the predicted curve. By 1959 the influence of the 1984 differential Lorenz curve on the predicted Lorenz curve is less than 0.4 percent.

Forecasting the Lorenz Curve for Years After 1993

As in backcasting the Lorenz curve for the years 1959 through 1983, the second property of Lorenz curves is exploited. The predicted Lorenz curve for 1994 and beyond consists of a linear combination of the differential predicted Lorenz curve for 1993 and the Rasche predicted Lorenz curve for that year. As in the case described above, a ρ of 0.8 is used; ensuring nice joining-on properties as well as ensuring that the rho adjustment decays quickly.

Constructing the Total Expenditure Variables From the Lorenz Curves

At this point, it may be difficult to remember the purpose of this work, to bring the purpose back into focus, let us review. The system of consumption equations uses a Piece-wise Linear Engel (PLEC) curve to forecast consumption. The PLEC allocates expenditures to one of five expenditure categories. The PLEC assumes that the first $2692.87 of per-capita expenditures by anyone are spent in a certain fashion and that the next $396 of expenditures are spent in a second fashion, and so on. In order to calculate the amount of expenditure in each expenditure category, the distribution of expenditures must be known or forecasted. The standard method of forecasting the distribution of expenditures or income for a given year is to calculate a Lorenz curve for that year.[56]

The algorithm described above allows us to generate the Lorenz curve for any given year. The Lorenz curve for any year can be directly converted into the level of cumulative expenditures at any point by multiplying total expenditures and the

[56]The reader is reminded that income and expenditures are analogous in this chapter.

cumulative percent of expenditures (y). The total expenditure variables, however, are not in terms of cumulative income held by any particular person, but are in terms of the actual income held.

Since the Lorenz curve equals:

$$y = L(x) = \frac{1}{\mu} \int_0^x F^{-1}(u)\, du \qquad (4.1);$$

the income held by the person at any percentile equals first derivative of the Lorenz curve multiplied by average per-capita income or:

$$F^{-1}(x) * \frac{M}{Pop} \qquad (4.23);$$

where:
M = Total expenditures;
Pop = total population.

Determining which person sits at the boundary between the total expenditure variables is a simple procedure. Since equation (4.23) is a one variable function, the solution can be found by evaluating the function at two points: one that is to the left of the x that sits at the boundary between the two total expenditure variables, or $x1 \leq x_{boundary\text{-}i}$; and one to the right of the solution, or $x_{boundary\text{-}i} \leq X2$. With the solution bracketed, the function is evaluated at the mid-point between X1 and X2, X3. The technique is known as the bisection method and is illustrated in figure 11.

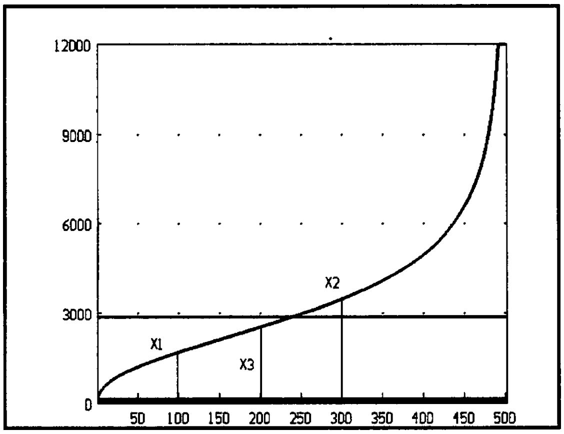

FIGURE 11

Bisection Method of Solution

In figure 11, the horizontal line is the expenditure boundary. (The horizontal scale is in five-hundredths of a percent.) We need to know at what value of X the boundary and the curve plotting per-capita income intersect. X1 is our initial guess at a left bracket and X2 is the initial guess at a right bracket. Since X1 and X2 bracket the solution, we find the mid-point of the line segment X1X2, or X3. If X3 is greater than $x_{boundary-i}$, then X3 becomes the new right bracket, and a new X3 is selected. However, if X3 is less than $x_{boundary-i}$, then X3 becomes the new left bracket and a new X3. The process repeats until the function, when evaluated at one of the selected x points, has converged to B_i.[57]

[57]The process has converged when

$$F^{-1}(x_{boundary-i})*(1 - tolerance) \leq F^{-1}(x_{guess}) \leq F^{-1}(x_{boundary-i}) * (1 + tolerance).$$

With $x_{boundary\text{-}i}$ known, the cumulative amount of expenditures that are in expenditure bracket 1 through i[th] equals:

$$Cumulative\ Expend_i = [capita * \int_0^{x_i} F^{-1}(x)\,dx] + [(1 - x_i) * B_i * Pop]$$

(4.24).

The first term, capita * ∫ $F^{-1}(x)$, equals the product of per-capita expenditures and the Lorenz curve at $x_{boundary\text{-}1}$. The second term, population * $[1-x_{boundary\text{-}1}]*B_i$, equals the expenditures that belong in the i[th] or smaller brackets of persons who have income in brackets above the i[th] bracket. Figure 12 illustrates the technique.

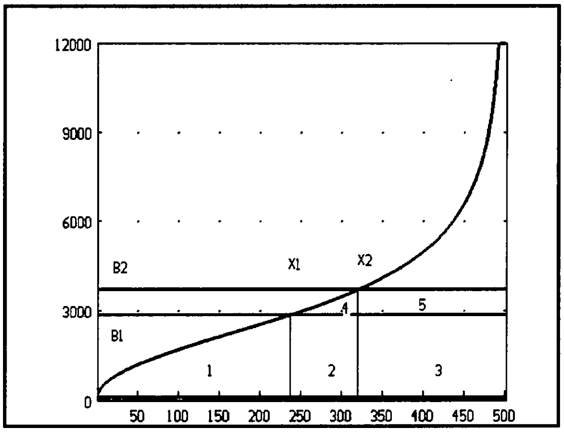

FIGURE 12

Calculating Cumulative

Per-capita Expenditures

Tolerance is the percent the function may deviate from the true solution. The tolerance used for this study was one-tenth percent.

As in figure 11, the horizontal axis is in five-hundredths. Per-capita expenditures intersects the first boundary, B1, at x-coordinate X1. The cumulative expenditures in the first bracket equals the sum of the area under the function and to the left of the first vertical line (X1) -- labeled 1 --- and the area of rectangle 2 and 3. The area under the function is the expenditures of persons who have no expenditures in the higher expenditure brackets. The area of rectangles 2 and 3 is the first $2692.87 of expenditures by persons spending more than $2692.87.

The area of rectangle 2+3 is calculated easily. The height of the rectangle is known and equals the boundary of the bracket, B1. The length of rectangle 2+3 equals the product of total population and (1 - X1). The area under the function, 1, is also easily calculated via integration and, since per-capita income function is the first derivative of the Lorenz curve, the integral is nothing more than the product of per-capita income and the Lorenz curve evaluated at X1.

For the second boundary, B2, the intersection occurs at x-coordinate X2. Cumulative expenditures in the second bracket equals the sum of the area under the function to the left of the vertical line (X2) and the rectangle 5+3. The area under the function at X2 equals the sum of the Area of the regions marked 1,2 and 4. This area equals all of the expenditures of persons who have no expenditures above expenditure bracket 2. The area of rectangle 3+5 equals the first $3088.87 of expenditures of persons who spend more than $3088.87.

As in the case of the first expenditure bracket, the area of rectangle 3+5 is easily calculated because we know the height of the rectangle equals B2 and the width of the rectangle equals the product of total population and (1 - X2). The area under the function is also known since the integral of the function is the Lorenz curve. Thus, the area 1+2+4 equals the product of per-capita expenditures and the value of the Lorenz curve when evaluated at X2.

This procedure continues until the last expenditure bracket. By definition, the cumulative expenditures of the poorest to richest person must equal aggregate total expenditures.

Because the consumption system does not use cumulative expenditures, but instead uses the per-capita expenditures that are in the bracket, the cumulative expenditures in a bracket are converted to the level of expenditures in the bracket by:

$$Expenditures_i = CumulativeExpenditures_i - CumulativeExpenditures_{i-1}$$

(4.25).

In terms of figure 11, the expenditures in the first bracket equal cumulative expenditures in the bracket -- since this is the first bracket -- or the area 1+2+3. The expenditures in the second bracket equal cumulative expenditures in the bracket, area 1+2+3+4+5, less cumulative expenditures in the previous bracket, area 1+2+3. Thus, expenditures in the second bracket equal area 3+5. The per-capita expenditures in any bracket equal:

$$Per\text{-}capita\ Expenditures_i = totex_i = \frac{Expenditures_i}{Pop} \qquad (4.26).$$

(4.25).

Except for the last expenditure bracket, all of the $totex_i$ have an upper-limit equal to their expenditure boundary less the previous expenditure boundary. This is illustrated if we consider the case of the egalitarian distribution where per-capita expenditures equal B2 (see figure 13).

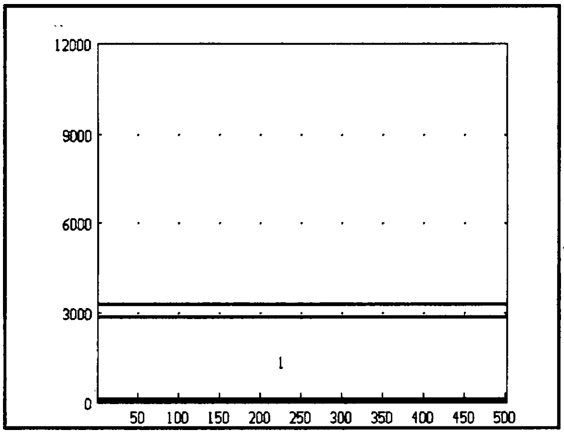

FIGURE 13

Egalitarian Distribution

The lower horizontal line is at the boundary of the first expenditure bracket. Cumulative expenditures in the bracket equal the area of the rectangle A. The per-capita expenditures of the first bracket, $totex_1$, equal the bracket boundary B1. Since the egalitarian distribution is unlikely, $totex_1$ will approach B1 asymptotically.

Figures 14 through 18 show the historical values of the five totex variables. In the first four figures, the horizontal line is the upper-bound of the totex. It should be noted that in figure 14, $totex_1$ appears to equal its upper bound. However, this is not the case and while $totex_1$ gets extremely close to the bound, but does not reach it.

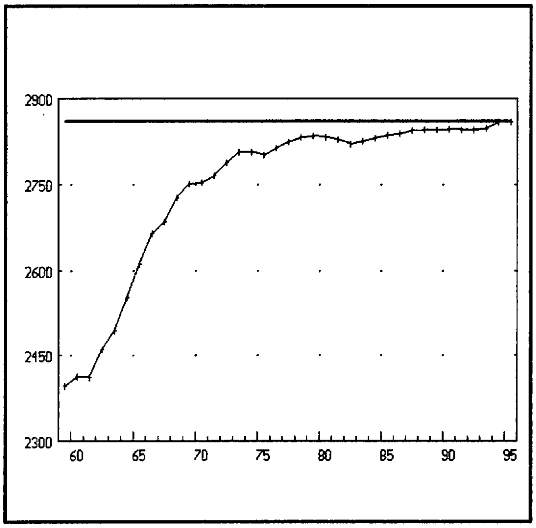

Figure 14

Historical Values: $TOTEX_1$

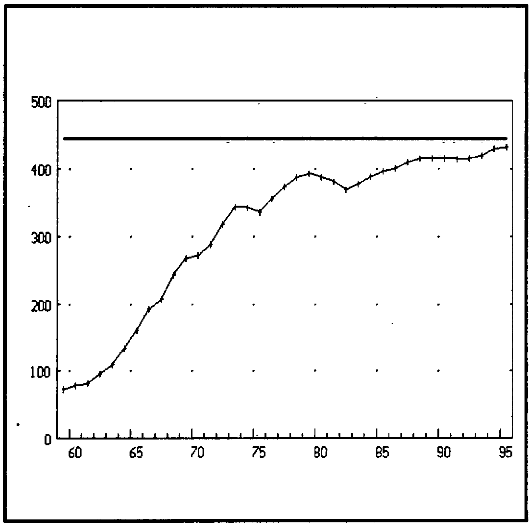

Figure 15

Historical Values: $Totex_2$

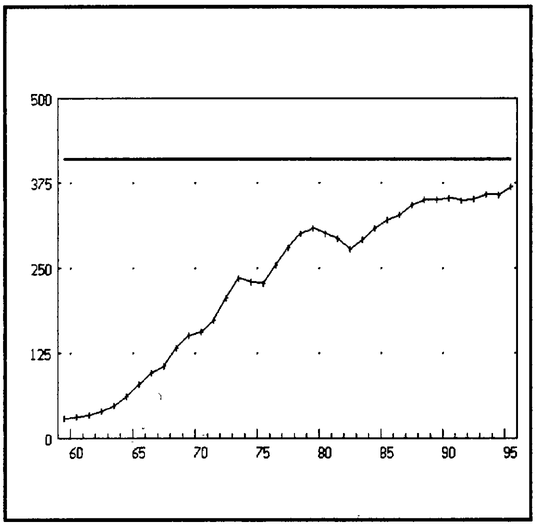

Figure 16

Historical Values: $Totex_3$

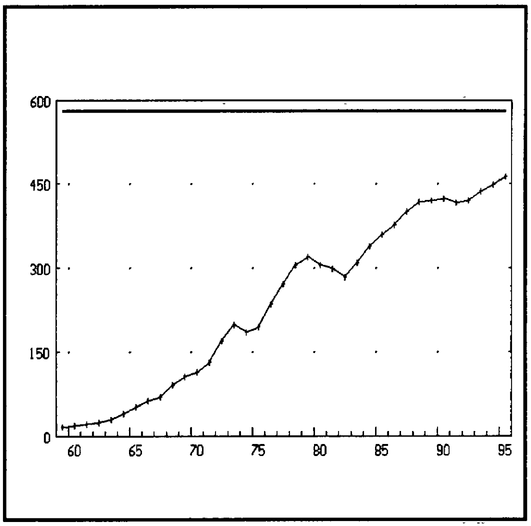

Figure 17

Historical Values: $Totex_4$

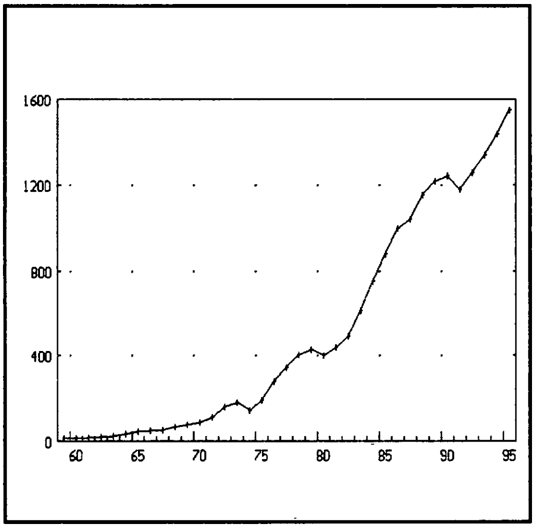

Figure 18

Historical Values: Totex$_5$

Key Differences between the Pollock Method and This Study

Besides the obvious difference in functional form used to represent the Lorenz curve, there are key differences in the method used in calculating the total expenditure variables used in the system of PCE equations. These differences and the advantages of each method are discussed below.

Pollock's main goal was the creation of a tax model with the constraint that the income distribution used in the tax model must generate the total expenditure variables needed for the system of PCE equations. Consequently, his method permits the explicit modeling of alternate tax systems and rates. The goal of the present work was to create an income/expenditure distribution that would generate the total expenditure variables without regards to the tax system.

As already mentioned, Pollock's method forecasted a distribution of Adjusted gross income (AGI). This distribution of AGI was passed through a tax function and then a bridge matrix that converted the post-tax distribution of AGI to a personal distribution of disposable income. The distribution of personal disposable income was converted into a distribution of expenditures and then the total expenditure variables.

One problem with this method is that the AGI-disposable income bridge matrix was constant across time. Thus, if the first ventile received thirty percent of total veterans benefits in 1984, then it would receive thirty percent of total veterans benefits in all years.

Under my method, since it is the distribution of expenditures that is of interest, the expenditure distribution is modeled directly. This distribution automatically accounts for any changes in the way that the reconciliation items are distributed to the ventiles. However, one draw-back of my method is that it is now more difficult to model alternate tax systems.

Conclusion

In this chapter the importance of forecasting the distribution of income was highlighted. The difficulties encountered while modeling the distribution were also highlighted. The method of having one of the two parameters in the Rasche function exogenous (β) and the other parameter endogenous (α) allows the distribution of income to vary automatically from year-to-year and from simulation to simulation.

Since direct observation shows that the distribution varies over time, my innovation will improve the capabilities of the model.

CHAPTER 5

MODELING GOVERNMENT EXPENDITURES ON MEDICAL CONSUMPTION EXPENDITURES[58]

A substantial portion of the gross health care expenditures in the U.S. are reimbursed by either private health insurance, Medicare and/or Medicaid. Such third-party payments financed 80 percent of personal health care expenditures in 1994. In that year, Medicare funded over 17 percent of total health care spending. Clearly, how one models the effects of these third-party payments greatly affects not only forecasts of PCE but also forecasts of output by industry.

Previous versions of the LIFT system of aggregate demand equations (old-LIFT) treated all government transfers as personal income. This treatment is incorrect as some of these programs are in-kind transfers and some are price subsidies. Modeling a price subsidy as an income transfer leads to inaccurate forecasts. For example, if Medicare is a price subsidy, one would expect the effects of an increase in Medicare benefits to be concentrated in health services. In old-LIFT, increased Medicare benefits translated into increased spending in all categories with the greatest changes occurring in commodities with large income elasticities because old-LIFT treated the increase in benefits as an increase in income.

This chapter consists of 4 parts. The first discusses the types of government transfers and identifies the correct transfer type for Medicare. The second section, describes the new treatment of Medicare in the LIFT system. The third section

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[58]I would like to thank and acknowledge the support of the Health Care Financing Administration in financing the majority of the work included in this chapter. This chapter draws heavily on Janoska (1994a; Janoska 1994b, Janoska 1994c).

discusses the estimation procedure and the data used. The fourth presents the new parameters and compare these estimates to the old-LIFT parameters.

Types of Transfers and Their Demand Effects

There are three ways that a government can directly influence individuals' command over goods and services: pure income transfers (or lump-sum payments); in-kind transfers (or commodity gifts); and price subsidies. Most government programs, and in particular, all government transfer programs, belong to one of these three categories, but it is often difficult to determine which definition is appropriate. For example, a government-run low-income housing project is often considered an in-kind transfer since the recipient receives a fixed quantity of housing (one apartment) and the benefit cannot be sold or exchanged. However, since the program is restricted to low-income persons, the program can also be thought of as reducing the price of leisure and thus is a price subsidy. Under strict neoclassical theory, virtually none of the government transfer programs that we traditionally think of as cash transfers are "income" transfers since eligibility is often restricted by employment status, income level or illness.[59]

A pure income or cash transfer, is, by definition, an unconditional (at least as far as economic status is concerned) lump-sum transfer of a fully fungible commodity, like cash. An income or cash transfer is any transfer where eligibility is not contingent

[59] I am concerned with the direct effects transfers have on PCE and not how these programs affect the labor-leisure decision. For the remainder of this study, I ignore any secondary effects of a transfer program and only consider the direct effect on PCE.

upon the recipient using the funds for the purchase of specific goods. In other words, the benefit may be spent in a completely discretionary manner.

An in-kind transfer is one in which the recipient directly receives an amount of a good that can not be legally resold (or equivalently, a voucher with no legal cash value) and where eligibility is not restricted to those who purchase a quantity of the good in excess of the transfer. The Food Stamp program is one such program since it allows the recipient to purchase only one commodity (food) and eligibility is not contingent on the person purchasing any food in excess of the benefit.[60] In contrast, the Medicare program is not an in-kind transfer since recipients do not receive free amounts of medical care but instead bear some portion of the cost of all care they consume.[61]

A consumer price subsidy is a transfer in which the individual determines the amount of the good he will purchase, but the price he pays is subsidized by the government.

Table 14 shows the government transfer programs that the National Income and Product Accounts (NIPA) consider income transfers. Old-LIFT treats all of these programs as income transfers. Using the definitions given above, it is clear that not all of these programs are income transfers. However, this uniform income transfer treatment could still generate the same results as categorizing each transfer

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[60] I assume that the stamps cannot be sold. If the stamps had a legal cash value, then the program would be an income transfer since the benefit would be indistinguishable from a cash grant. In reality, there is a black-market for these stamps, blurring the distinction between an income and an in-kind transfer.

[61] Depending on the type of care purchased, a deductible must be met before Medicare eligibility is established and coinsurance is almost always charged (Petrie 1992).

appropriately. Unfortunately, this is not the case because the three transfer types have dissimilar affects on consumption demand.

TABLE 14

Federal Transfer Payments to Persons (from NIPA Table 3.12)

Billions of Dollars

	1985	1990	1991	1992	1993
Benefits from social insurance funds	320.1	437.5	479.7	526.4	554.9
Old-age, survivors, and disability insurance	183.3	244.1	264.1	281.8	297.9
Hospital and supplementary medical insurance	70.1	107.9	118.2	132.2	146.5
Unemployment insurance	15.8	19.2	26.9	39.6	34.6
State	14.6	18.7	25.6	24.9	21.5
Railroad employees	0.2	0.1	0.1	0.1	0.1
Federal employees	0.3	0.4	0.5	1.2	1.2
Special unemployment benefits	0.8	0.0	0.8	13.5	11.8
Federal employee retirement	41.1	53.9	57.5	59.3	62.1
Civilian	23.5	31.8	33.7	34.2	35.7
Military	17.6	22.1	23.8	25.1	26.3
Railroad retirement	6.2	7.2	7.5	7.7	7.8
Pension benefit guaranty	0.1	0.3	0.2	0.4	0.4
Veterans life insurance	1.6	1.9	1.9	1.9	1.9
Workers' compensation	1.1	1.5	1.6	1.8	1.8
Military medical insurance	0.8	1.5	1.7	1.7	1.8
Veterans benefits	15.0	15.8	16.2	16.7	17.5
Pension and disability	14.0	15.6	15.9	16.2	16.8
Readjustment	0.9	0.3	0.4	0.5	0.6
Other	0.0	0.0	0.0	0.0	0.0
Food stamp benefits	10.7	14.7	18.2	21.2	22.2
Black lung benefits	1.6	1.4	1.4	1.4	1.4
Supplemental security income	8.8	12.9	14.8	18.2	20.7
Direct relief	0.0	0.0	0.0	0.0	0.0
Earned income credit	1.1	4.4	4.9	8.5	9.4
Other	9.6	14.2	14.8	16.4	16.2

A. Effect of Transfers on Demand - The Two Good Case[62]

To illustrate the effects of these various programs a two-good model is constructed with a composite good **C**, and a second good **X**. Figure 19 shows the initial budget constraint faced by a consumer and the indifference curve, $\mathbf{U_0}$, which is

[62]Zellner and Traub (1987) provide an excellent non-mathematical discussion of the effects of these transfers.

tangent to the budget constraint. The consumer purchases the bundle B_0 (C=177.8, X=22.2) --- the point of tangency.[63]

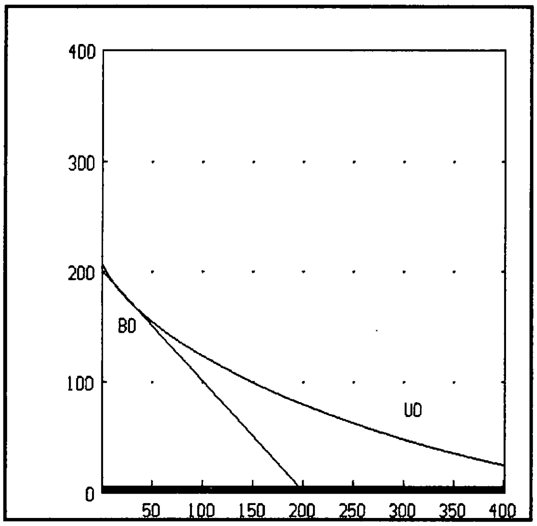

Figure 19

Initial Conditions

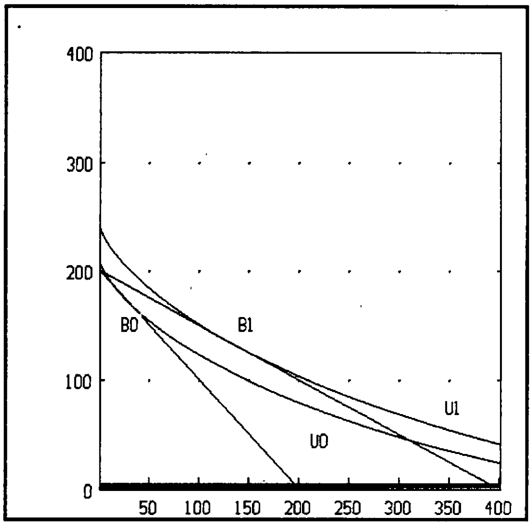

Figure 20

Price Subsidy

A price subsidy of fifty percent on X, costs the government \$67 and increases demand for X by 111.11 units to a total X consumption of 133.33 units (see figure 20). The cost to the government equals one-half times X consumption or \$67.

A pure income transfer of \$67 shifts the budget constraint to the right (see figure 21) -- and allows the consumer complete control over how the transfer is spent. The transfer costs the government \$67. He purchases bundle B_2, X= 29.63 and C = 237.37. Purchases of X increased by 7.50.

[63]Initial prices equal 1.0 and initial income equals \$200.

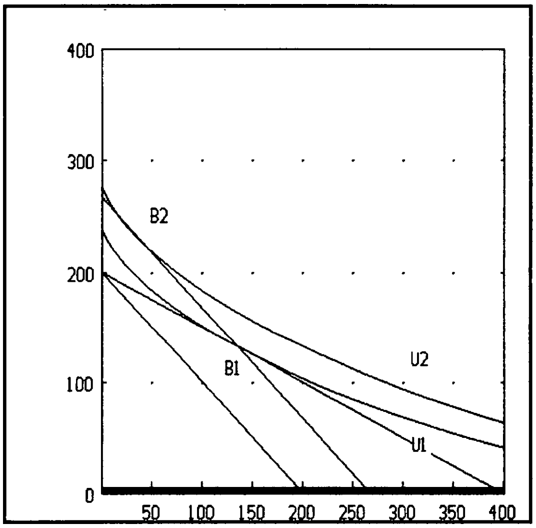

Figure 21

Income Transfer of $67

(Subsidy also shown)

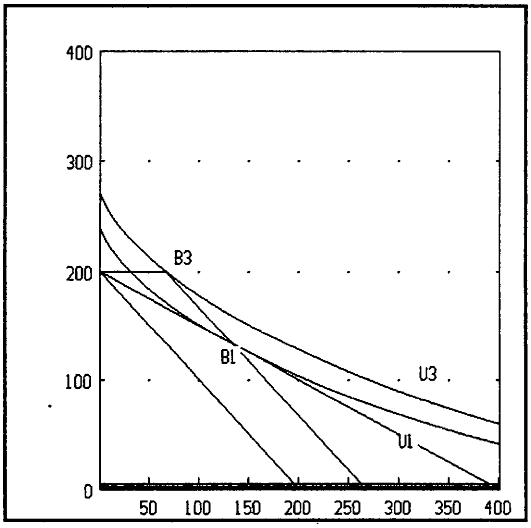

Figure 22

In-Kind Transfer

Figure 22 shows the effect of a $67 in-kind transfer of good X by the government. The government has given the consumer $67 worth of X (or since P_X equals 1, 67 units of X). The consumer is forced to consume B_3 where X equals 67. Had the consumer been able to sell amounts of X, the program would be an income transfer with results equal to figure 20. The government transfer fully displaced the commercially purchased amount of X. Though not shown in a figure, the in-kind transfer generates more demand for X than the income transfer.

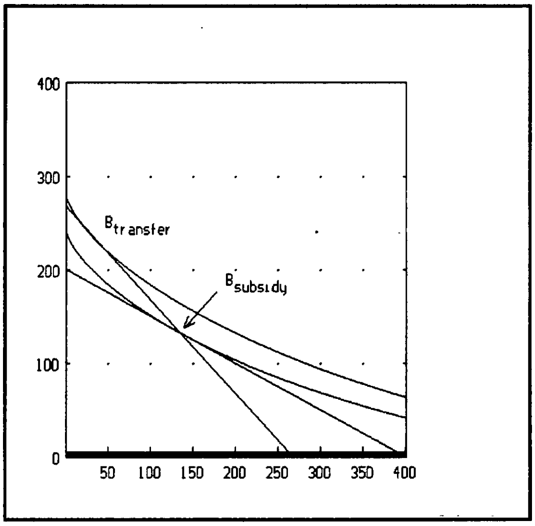

Figure 23

Income Compared to Price Subsidy

In this two-good model, a price subsidy will always generate more demand for the subsidized good than an equally costly income transfer program. Figure 23 shows the demand for X when the price of X is subsidized 50 percent and when the consumer receives a $67 income transfer. The price subsidy also costs the government $67. As can be seen in the diagram, demand for X, the subsidized good, is higher under the subsidy than under the income transfer.

In a multi-good world, it is not clear which will create more demand for the subsidized good: a price subsidy or an income subsidy that costs the government the same amount as the price subsidy. The size and direction of this discrepancy depend on the budget share and the income elasticity of the subsidized good. In general, if the subsidized good has close complements, then there is a greater chance that an income transfer will generate more demand for the subsidized good than the price subsidy.[64] The appendix to this chapter gives the conditions when an income transfer will generate more demand than a price subsidy.

That price subsidies (figure 20) are not equivalent to cash transfers (figure 21) in their effects on welfare and consumer demand is well accepted in economic theory. It is also generally accepted that because some commercial purchases are displaced, in-kind transfers are not equivalent to income transfers (Smeeding 1977; Zellner and Traub 1987; Moffitt 1989). Most of the literature has concentrated on measuring the welfare loss associated with in-kind transfers (Smeeding and Moon 1980; Clemer 1984; Moffitt 1989). Smeeding and Moon (1980) treat the Medicare program as an in-kind transfer rather than a price subsidy. Zellner and Traub (1987) examine the effect on commercial purchases of a commodity when the government provides an in-kind transfer of the good. They found that for certain foods, in-kind transfers can quickly displace all of the commercially purchased quantities.

The examples above indicate that if one models the demand effects of a transfer program incorrectly, then forecasts of the impact of changes in that program will be incorrect. For example, if one treats a program that is a price subsidy as an income transfer, then any increase in the size of the program will affect demand in those commodities with the highest income elasticities since this method of modeling the

[64]Part of the "savings" created by the subsidy must be spent on the unsubsidized complementary good. If the government subsidizes the purchase of Left shoes, but not Right, (shoes being perfect complements), much of the subsidy is wasted since one shoe is virtually useless. If the government had simply given individuals money, more pairs of shoes may have been purchased than under a left shoe subsidy. This of course means that a greater number of left shoes would be purchased under the transfer than under subsidy.

program does not impact price but will affect income. In the LIFT system, the most income elastic goods tend to be consumer durables (Janoska 1994c); thus, any increase in the generosity of a price subsidy has the greatest impact on items such as automobiles and jewelry. Unless the program happens to be a price subsidy on cars and jewelry, this result contradicts our *a-priori* beliefs on the impact of the changes.

A second error in the analysis of changes in a price subsidy occurs because of the manner in which LIFT forecasts PCE by category. The system of demand equations in LIFT uses total expenditures as its income variable. The sum of PCE by category is then scaled to total expenditures to insure additivity. The forecast of total expenditures has four major steps. LIFT first constructs aggregate Personal income (PI) as defined by the National Income and Product Accounts (NIPA) by summing all of the components of PI.[65] The second step consists of calculating Disposable income (DI) as defined by NIPA. DI equals PI less Personal taxes and non-tax payments. LIFT then forecasts the spending rate. The product of the spending rate and DI equals total expenditures. Table 15 shows the relationship between Personal income and total expenditures.

[65]Each of these components is forecasted by a separate set of equations.

TABLE 15

Relationship between Personal Income and Total PCE

NIPA Personal income	=	Wages + Salaries + Other labor income + Proprietor's Income + Rental Income + Dividends + Interest + **Transfer Payments**
NIPA Disposable income	=	NIPA Personal income - Personal income
Total PCE	=	NIPA Disposable income * Spending rate

Since the spending rate is near .95, a dollar of price subsidy, when treated as an income transfer, will generate one dollar of additional disposable income but will generate only 95¢ of additional spending. Thus, the error in forecasting is compounded by allowing only a fraction of the spending to occur. This means that simulations that attempt to evaluate the effects of changes in a price subsidy program will underestimate the effect of the changes on the good that is subsidized.[66] The problem of saving a portion of the price subsidy is not limited to LIFT. Any model that allows either the level or rate of saving to vary will have the problem that the forecasting model allows portions of the subsidy to be saved.

These two items indicate that there are serious simulation problems when one models an income transfer as a price subsidy. For this study, I have focused on the Medicare program. The focus is on whether the program is a price subsidy or income transfer and if the program is a price subsidy, how should one model the effect of the program on PCE.

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[66]This leakage is less serious than the problems caused by misallocating the subsidy to the non-subsidized PCE categories.

B. Analysis of the Medicare Program

Medicare consists of two programs: Hospital Insurance (HI) and Supplementary Medical Insurance (SMI). The first, HI, can be thought of as being a true price subsidy over only certain expenditure ranges because a deductible must be meet before HI coverage activates and after 90 days of hospitalization for a particular "spell of illness", coverage ceases (Petrie 1993).[67] Thus, only charges incurred after the deductible, but before exhaustion of benefits, are subsidized. SMI, however, subsidizes all costs incurred after the deductible is met (Petrie 1993).[68] Since Medicare transfers must be spent on medical goods and services, the program clearly is not an income transfer. Because Medicare recipients determine the amount of care they receive, but are billed at the Medicare coinsurance rate (Petrie 1993), the program cannot be an in-kind transfer (where the transfer is not contingent on the consumer spending his own funds) and is, instead, a price subsidy.

Most authors consider the program a government-run insurance program (Pauly 1986; Hurd 1990; Jacobs 1991; Phelps 1992). The consensus is clear that health insurance increases the demand for medical care (Feldstein 1973; Rosett and Huang 1973; Newhouse and Phelps 1974; Phelps and Newhouse 1974) because it subsidizes the price of care. The effects on medical demand caused by this price subsidy will almost certainly differ from the effects caused by an equal dollar value cash transfer.

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[67]Medicare recipients may also draw on a "lifetime" reserve fund of 60 hospitalization days which can be used if the recipient exhausts the 90 days covered in a benefit period.

[68]Keeler *et al.* (1977) present evidence that Medicare subsidies increase demand even when the deductible is unmet.

This indicates that any attempt to model the effects of changes in Medicare should model the program as a price subsidy.

New Treatment of Medicare

In previous versions of LIFT, Medicare benefits were considered income (Devine 1983; Pollock 1986; Chao 1991). With the completion of this work, LIFT models Medicare benefits as a price subsidy. Estimating the system of demand equations with Medicare modeled as a price subsidy consists of three steps: redefining the income variable in the system; redefining the price deflators; estimating the system of equations using the new income and price variables.

A. Defining Discretionary Income

The first step in estimating the new revised system is the creation of a variable called discretionary income. Discretionary income equals NIPA Disposable income less Medicare benefits and it can be thought of as the income that consumers may spend at their discretion. It differs from NIPA Disposable income since Disposable income includes the value of the Medicare price subsidy that must be spent on medical goods and services. This distinction between NIPA disposable income and discretionary income is very important.

Since NIPA Disposable income includes many in-kind transfers and price subsidies, using NIPA Disposable income to explain the consumption choices of consumers is inappropriate for the reasons given above. Discretionary income, however, excludes the price subsidy and represents the income that the consumer may spend as he sees fit.

B. New Price Deflators

Because Medicare is now modeled as a price subsidy, the price deflators used in the model must be redefined. More specifically, Medicare is treated as an insurance program that subsidizes the price of medical goods and services. The standard treatment for this situation is to define the price that consumers face as equalling the product of the actual price and the co-insurance rate (or fraction that the consumer must pay). The co-insurance rate, C, for the i^{th} good is defined:

$$C_i = 1 - subsidy\ rate_i = \frac{Nominal\ PCE_i - Medicare_i}{Nominal\ PCE_i} \qquad (5.1);$$

Where:
- $Nominal\ PCE_i$ = Nominal PCE spending in category i, as defined by NIPA;
- $Medicare_i$ = Nominal Medicare spending in category i.

The Medicare-adjusted price deflators equal:

$$Medicare\text{-}adjusted\ Price_i = C_i * NIPA\ Price_i \qquad (5.2);$$

This approach assumes that there are no deductibles in the program and that the average coinsurance rate equals the marginal coinsurance rate across all individuals. There are problems with this assumption and these are addressed below.

The new prices differ both in magnitude and movement from the old price deflators. Figures 24 through 33 show the new deflators and the ratio of the old new deflators to the old deflators.

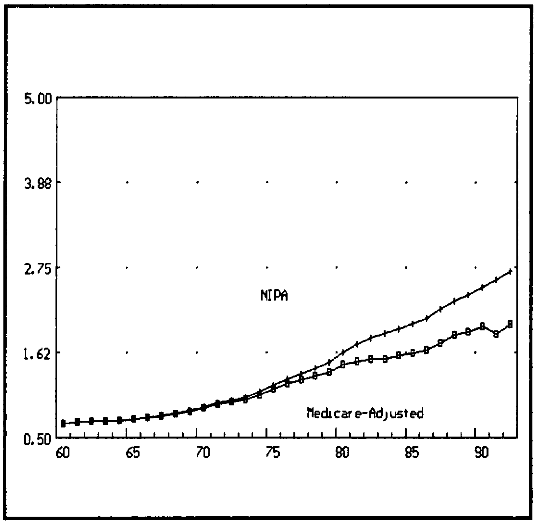

Figure 24

Deflators for Ophthalmic Goods

Note: NIPA Deflators = 1 in 1972

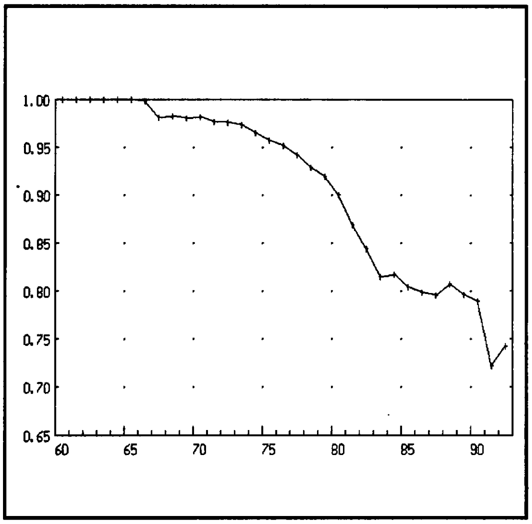

Figure 25

Ratio of Adjusted to NIPA Deflator

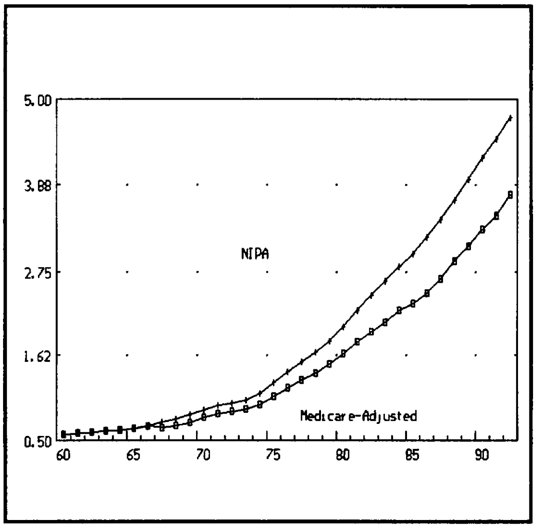

Figure 26

Physicians

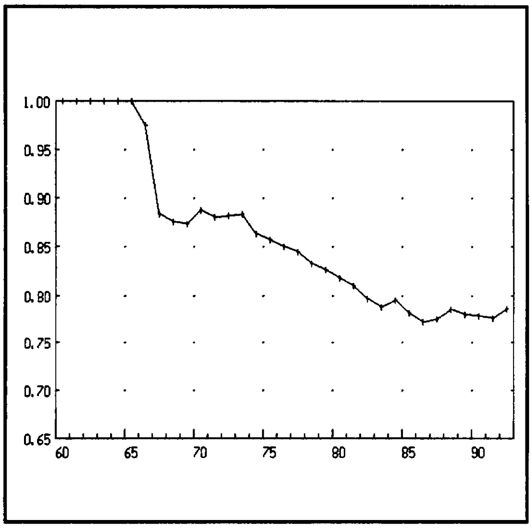

Figure 27

Ratio of Adjusted to NIPA Deflator

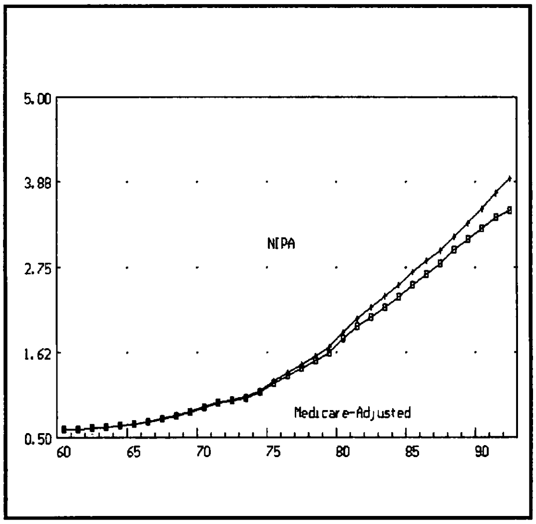

Figure 28

Dentists

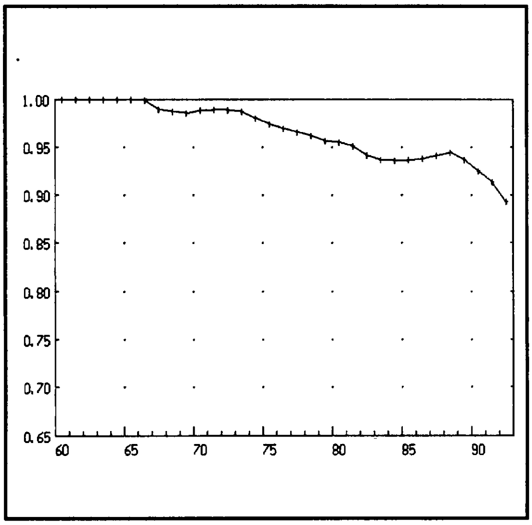

Figure 29

Ratio of Adjusted to NIPA Deflator

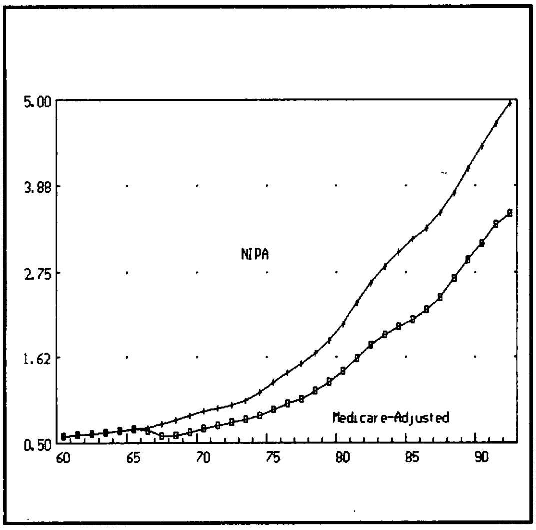

Figure 30

Hospitals

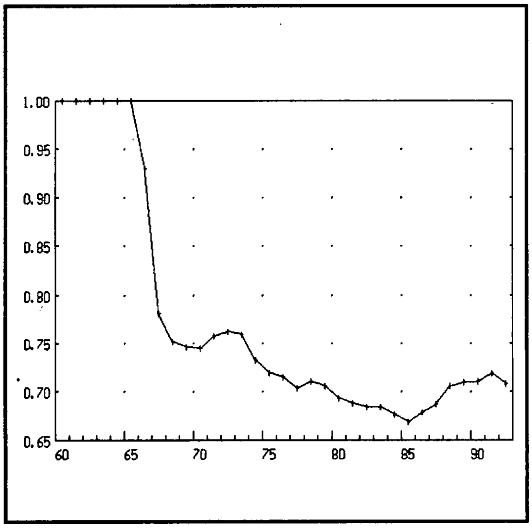

Figure 31

Ratio of Adjusted to NIPA Deflator

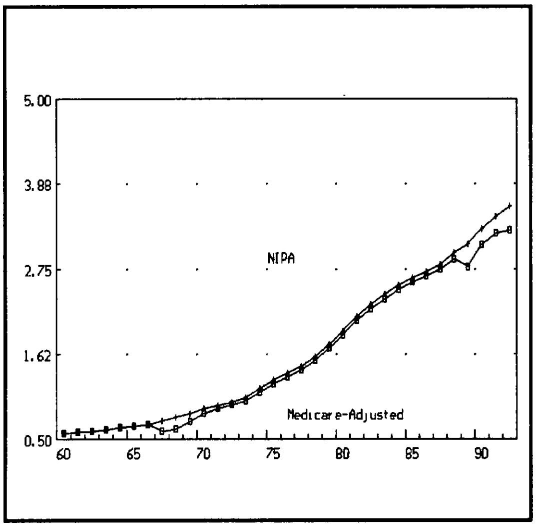

Figure 32

Nursing Homes

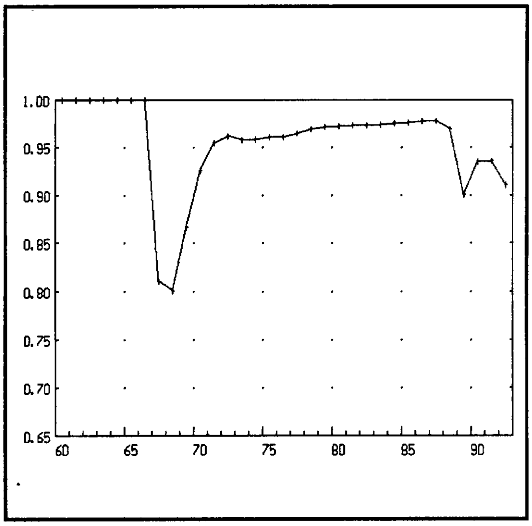

Figure 33

Ratio of Adjusted to NIPA Deflator

C. Problems with this approach

Both these assumptions introduce possible sources of error into the parameter estimates. Under standard consumption theory, marginal price and not average price, determines consumption. Keeler, Newhouse and Phelps (1977) show that estimated price elasticities depend on whether the average or the marginal coinsurance rate is used in the estimation. Thus, the use of an average coinsurance rate may introduce an error in our parameters.

The second error is caused by the treatment of deductibles. Newhouse, Keeler and Marquis (1979) show that treating deductibles as I do will lead to errors in the estimated parameters, but that the direction of the bias cannot be determined *a-priori*. Keeler, Newhouse and Phelps (1977) and Newhouse, Keeler and Marquis (1979) show that there are two solutions to the problem. The first is the removal of all individuals with deductibles from the data set. The second approach is the estimation of demand functions for each level of deductible in the data set.

Neither of these solutions could be implemented. The available data is aggregated to a national total and contains no information on the number of persons who have deductibles or the size of their deductibles. That the estimated price parameters might be in error is acknowledged, but I feel that the size of the error is small relative to the improvement gained through modeling Medicare benefits as a price subsidy.

The data

The data on Medicare benefits by PCE category are from the National Health Accounts. The NIPA are the source for data on total Medicare benefits. Data on PCE by category come from published and unpublished NIPA PCE tables.

Unfortunately, the NIPA value for total Medicare benefits does not equal the sum of the National Health Accounts Medicare benefits by PCE category. This is due to definitional differences between the two sources. The discrepancy is small and consists of items such as the operating surpluses (or deficits) of non-profit organizations[69] and official "statistical discrepancies" that arise through the use of different estimation procedures. Table 16 shows the differences between the NIPA and National Health Accounts data for selected years.

TABLE 16

Differences Between NIPA and National Health Accounts

Total Medicare Benefits

	1968	1970	1977	1980	1984	1990	1992
Medicare Benefits (NIPA)	5628	7075	21704	35582	62644	107937	132139
National Health Accounts	5636	7087	21728	35612	62644	107937	132141
Difference ($Millions)	-8	-12	-24	-30	0	0	-2

To ensure that Medicare benefits by category equals total Medicare benefits as reported by the NIPA, the National Health Accounts data on Medicare benefits by category from the National Health Accounts is scaled to match the reported total NIPA Medicare benefits.

[69]The NIPA exclude this item from personal expenditures for health care, while the National Health Accounts include it.

Estimation Procedure and Results

The LIFT system of aggregate demand equations is restated below:

$$\frac{q_{it}}{WP_{it}} = (a_i + b_i C^*_{it} + c_i \Delta C^*_{it} + d_i TIME + e_i\ OTHER_i) \prod_{L=1}^{M} \left(\frac{P_{it}}{\bar{E}_{Lt}}\right)^{-S_L \lambda_{IL}} \prod_{K=1}^{SG_L} \left(\frac{P_{it}}{\bar{e}^L_{Kt}}\right)^{-SG^I_K}$$

$$q_i \notin Group\ L$$

$$q_i \notin Sub\text{-}group^L_K$$

(3.15);

Where:

$$\bar{E}_L = \left(\prod_{j \in G_L} P_j^{s_j}\right)^{\frac{1}{S_L}}$$

$$\bar{e}_L = \left(\prod_{j \in SG^L_K} P_j^{s_j}\right)^{\frac{1}{S^L_K}}$$

q_{it} = Expenditures on category i during year t;
WP_{it} = Weighted population size, good i, in year t;
C^*_{it} = Cross-section variable, good i, in year t;
P_{it} = Price of good i in year t;
$\bar{E}_{LT}$ = Average price of group L in year t (see above);
$\bar{e}^L_{Kt}$ = Average price of sub-group K, group L in year t (see above);
TIME = Trend variable with 1960 = 1;
$Other_i$ = A non-price, non-income variable affecting good i;
S_L = Share of total consumption, group L, in base year;
$a_i, b_i, c_i, \lambda_{IL}, \gamma^L_K$ = Parameters to be estimated.

The system given by equation (3.15) is difficult to estimate because of the interdependence of the parameters dictated by the Slutsky symmetry and the adding-up constraint. To insure that these two conditions hold, the equations must be estimated as a system. This joint estimation, in turn, must deal with the problem of heteroscedasticity since the variance of the error terms could differ between each equation.[70] The heteroscedasticity is corrected by dividing the data for each category by an estimate of the standard deviation of the error term in the equation for that item prior to estimation (Johnston 1984). These estimates of the standard deviations are obtained by performing separate regressions of a linear version of the consumption function for each of the 80 categories.

In each equation, the coefficient b_i on C^*_{it} is constrained to preserve the cross-section results. The fact that C^*_{it} is by construction a prediction of (q_{iit}/WP_{it}) suggests that the constraint be set to one in the base year. However, to correct for any discrepancies between the definitions used to define the cross-section and time-series items, the value of b_i is chosen so that the elasticity of consumption with respect to C^*_{it} equals one.

A. Estimation Technique

The system represented by equation (3.15) is extremely nonlinear in the price terms. This nonlinearity increases the difficulty of estimating the system. To avoid this problem, the system is estimated iteratively using a linear version of the system.

[70]Since the level of consumption in each category is different by several orders of magnitude, it is assumed that homoscedasticity does not hold.

For purposes of illustrating this technique, suppose we have the following general nonlinear equation:

$$Y_i = F(x_i, B) + U_i \qquad (5.3);$$

Where:
Y_i = The dependent variable in the i^{th} period;
x_i = The independent variable in the i^{th} period;
U_i = The disturbance term in the i^{th} period;
B = The vector of parameters to be estimated.

Estimates of B are selected to minimize the following:

$$\sum_i \{Y_i - F(x_i, B)\}^2 \qquad (5.4).$$

The Gauss-Newton method is employed in estimating iteratively the value of B by performing ordinary least squares regressions.[71]

Then, if we consider the following Taylor expansion of F( ) around B_0, an initial estimate of B. We have

$$F(x_i, B) = F(x_i, B_0) + F'(x_i, B_0)\ \{B-B_0\}$$

$$= F(x_i, B_0) - F'(x_i, B_0)B_0 + F'(x_i, B_0)B \qquad (5.5);$$

where $F'(x_i, B_0)$ is the vector of first derivatives of F( ) with respect to B, evaluated at B_0. If we substitute (5.5) into (5.4) we have:

$$\sum_i \{[Y_i - F(x_i, B_0) + F'(x_i, B_0)B_0] - F'(x_i, B_0)B\}^2 \qquad (5.6).$$

The expression within the brackets contains no unknown parameters. Likewise, $F'(x_i, B_0)$ is a vector that can be calculated for a given value of B_0. It follows that the value of B that minimizes the expression (5.6) is the same as the value that results from

[71] Our description of the Gauss-Newton method is a slight variation of the presentation found in Maddala (1977).

performing an ordinary least squares regression of the expression in brackets on $F^{/}(x, B_0)$. That is:

$$Y_i - F(x_i, B_0) + F^{/}(X_i, B_0) = F^{/}(x_i, B_0)B \qquad (5.7).$$

The estimate of B obtained from this regression is used to re-linearize equation (5.3). Another regression is performed to obtain a second estimate of B. This iterative procedure continues until no further reductions are made in the sum of squared errors. Convergence is usually achieved within five or six iterations and sometimes within three iterations.

B. Equation-Specific Variables

It is a long-established tradition that non-income and non-price variables play a key role in determining household PCE (Heien 1972; Denton and Spencer 1976; Devine 1983; Monaco 1984; Deaton et al. 1989; Chao 1991; Malley and Moutos 1993; Monaco 1994). Most of this work has focused on the effects of demographic and age variables, but some work has examined the effects of "other" variables (Devine 1983; Chao 1991; Malley and Moutos 1993; Monaco 1994). The LIFT PCE system has acknowledged these influences through the use of the cross-section effect variable, C^*, the adult equivalent weights, and equation-specific variables (Devine 1983). Devine included the following equation-specific variables:

Housing Demand Proxy Owner-occupied housing (PCE40) and Tenant-occupied housing(PCE41). A proxy for the speculative demand for housing. Calculated as the ratio of the current price of owner-occupied housing to a three-year moving average of its price.

Natural Gas Price Control Dummy: Natural gas (PCE46), Electricity (PCE45) and Fuel oil (PCE28). A dummy for Natural gas price controls. Equals one for the years 1974, 1975, 1976.

Mortality Rate Funeral expenses and other personal business expenses (PCE72). An attempt to capture the impact of increased longevity on funeral expenses. Expressed in deaths per thousand persons.

Availability of Used Cars Used cars (PCE02). A proxy for the potential stock of cars for the used car market. Equaled a three-year moving average of new car purchases lagged three years.

For my work, I felt that Devine's variables, except for the natural gas price control dummy, were inappropriate. For example, the availability of used cars for market should be reflected in the price term and so this variable was rejected for theoretical reasons.

In my first attempt to estimate the system, the system was estimated without the use of any time trends.[72] Any commodity that appeared trended was examined to determine if a some non-time trend reason might account for the trend. For example, the growth in Nursing home expenditures was thought to be linked to the increased numbers of over-85 years of age persons. Unfortunately, I was forced to estimate the system with time trends included in some equations, and, for three of the commodities, was forced to add a second time trend. The equation-specific variables we used included:

[72]Devine (1983) and Chao (1991) included one or more time trends in their equations. This trend was incorporated in an attempt to capture systematic changes in demand that could not be attributed to price, income, age or demographics.

Two-Year Moving Average of 3-month T-Bill Rate (Interest Rate) Used Cars (PCE02), and New and used trucks (PCE03). Calculated as a two-year moving average of the 3-month treasury bill rate. This variable attempted to capture the sensitivity of automobile financing to changes in the interest rate. During the first estimation attempts, this variable was included in the equation for New cars (PCE01), but was dropped from the equation in subsequent estimations as the sign on the coefficient was perverse and the own-price elasticity of New cars was positive if this variable was included.

Residential Construction Activity (Construction) Furniture and mattresses (PCE06) and Kitchen and other household durables (PCE07). Equalled per-capita spending on Single-family residential construction (STR01) and Additions and alterations (STR04). Purchases of furniture, kitchen appliances and other miscellaneous household items often occur with a new house purchase and/or renovation of an existing structure.

Natural Gas Price Regulation Dummy (Dummy) Fuel oil (PCE28), Electricity (PCE45) and Natural gas (PCE46). Equaled 1 in all years of regulation (1973, 1974, 1975) and 0 in all others. This variable was an attempt to capture the effects of Natural gas price regulation during the early 1970's

Value of Housing Stock (Stock) Owner-occupied housing (PCE41) and Tenant-occupied housing (PCE42). Cumulative housing stock value adjusted for depreciation (2%). Owner-occupied housing (PCE41) is an imputed component of the NIPA. Our formulation is an attempt to bring this sector into a format similar to that used by the NIPA (Carr 1994).

Labor Force Participation Rate (Labor Parti) Net health insurance (PCE67) and Life insurance (PCE70). Equals the labor force participation rate. This variable is an attempt to capture the effect of increased labor force participation among women.

Population 85 years and Older (Elderly) Hospitals (PCE66) and Nursing homes (PCE80). Equals the share of the sixty-five years and older that are eight-five or older. Among the over-65 population, this group tends to use these services more frequently and intensely then those younger than 85 years of age (Harrison 1986; Waldo 1989). Because our equations combine the elderly into a single cohort, our system of weighted-populations cannot capture the "aging-of-the-aged" effect. This variable is an attempt to capture this effect.

Second Time Trend (Second time) Gasoline and oil (PCE27), Intercity railroad (PCE58) and Cleaning and laundering (PCE62). A second time trend beginning in 1982. Some unidentified structural change appears to have occurred in these sectors. This variable is an attempt to account for this change until the reasons for the change can be discovered.

Hurricane Andrew Household Insurance (PCE50) equals premiums less benefits and represents the administrative expenses of the insurance company. IN 1992, a combination of natural disasters caused benefits to exceed premiums and expenditures on household insurance were negative. This variable equals 1 in 1992 and 0 in all other years and attempts to capture the 1992 disasters.

C. Revisions to Commodity Groups and Sub-Groups

The existing group and sub-group structure was examined and some of these groupings were modified -- either through addition/deletion of a PCE category from the group or by changing the sub-groups within the commodity group. For example, under the old system, Food off premise (PCE19) was a member of *Group 1, Food and Alcohol, off premise*. Under the new system, this category is in *Group 1, Food*. Table 17 shows these changes:

TABLE 17

Changes in Group and Sub-Group Structures

Group 1: Food, Alcohol and Tobacco		
PCE Category	Old sub-group	New sub-group
Food, off premise (PCE19)	Food and Alcohol, off premise	Food
Food, on premise (PCE20)	Food and Alcohol, on premise	Food
Alcohol, off premise(PCE21)	Food and Alcohol, off premise	Alcohol
Alcohol, on premise (PCE22)	Food and Alcohol, on premise	Alcohol
Group 6: Medical Services		
Nursing homes (PCE80)	new PCE category, previously included in PCE66- Hospitals	
PCE Category	Old sub-group	New sub-group
Physicians (PCE64)	Physicians and Hospitals	Physicians and Professionals
Dentists (PCE65)	Physicians and Hospitals	Physicians and Professionals
Hospitals (PCE66)	Physicians and Hospitals	Facilities
Nursing homes (PCE80)	Physicians and Hospitals	Facilities
Health insurance (PCE67)	Physicians and Hospitals	Health Insurance
Group 7: Personal Services		
PCE Category	Old sub-group	New sub-group
Brokerage services (PCE68)	Personal Business Services	Financial Services
Life insurance (PCE70)	Personal Business Services	Financial Services
Bank service charges (PCE69)	Personal Business Services	Imputed Service
Legal services (PCE71)	Personal Business Services	Other Business Services
Funerals and other (PCE72)	Personal Business Services	Other Business Services
Group 10: Reading and Education		
PCE Category	Old sub-group	New sub-group
Education (PCE76)	Education and Religious	Education
Education housing (PCE44)	Education and Religious	Education
Religious and Welfare(PCE77)	Education and Religious	Religious

D. Customization of Equations

Prior to my work, the non-price portion of equation (3.15) could take one of two forms:

$$(a_i + b_i C^*_{it} + c_i \Delta C^*_{it} + d_i time_t) \quad (5.8a);$$

or

$$(a_i + b_i C^*_{it} + c_i \Delta C^*_{it} + d_i TIME_t + e_i stock_{it}) \quad (5.8b);$$

where $stock_{it}$ is an equation-specific variable. One could estimate the equations without a time-trend by imposing a soft constraint of zero on the time coefficient, but this was time consuming. Additionally, since the non-linear system is solved in an iterative fashion, the estimated coefficients obtained by imposing a soft-constraint differ from the results obtained by estimating the system without the variable.[73] In the current work and all future updates, a hard-constraint of zero can be placed on all the coefficients in the linear term except the intercept and b_i. There are seven different forms that an equation may take:

TABLE 18

Possible Forms of the Equation

Equation 1	Y = (a + bC* + cΔC* + dtime) * Price effects
Equation 2	Y = (a + bC* + cΔC* + dtime + estock) * Price effects
Equation 3	Y = (a + bC* + cΔC* + estock) * Price effects
Equation 4	Y = (a + bC* + cΔC*) * Price effects
Equation 5	Y = (a + bC* + dtime) * Price effects
Equation 6	Y = (a + bC* + estock) * Price effects
Equation 7	Y = (a + bC* + dtime + estock) * Price effects

Table 19 lists the form of each equation.

[73]Non-linear equations are sensitive to the starting values used in estimating the equation and the path taken in reaching convergence. By imposing a hard-constraint on the system, we change the initial values and the path taken by the system. The differences between the hard-constraint and soft-constraint solutions are, in most cases, trivial and can be attributed to these two properties of non-linear estimation.

TABLE 19

Listing of Equation Forms

NEW CARS	A0 + A1*c + A2*Δc + A3*TIME
NET PURCHASES OF USED CARS	A0 + A1*c + A2*Rate
TRUCKS	A0 + A1*c + A2*Δc + A3*Rate
TIRES AND TUBES	A0 + A1*c + A2*Δc
ACCESSORIES AND PARTS (AUTO)	A0 + A1*c + A2*Δc
FURNITURE, MATTRESSES, AND BEDSPRINGS	A0 + A1*c + A2*Δc + A3*Construction
KITCHEN AND OTHER HOUSEHOLD APPLIANCES	A0 + A1*c + A2*Construction
CHINA, GLASSWARE, TABLEWARE, AND UTENSIL	A0 + A1*c + A2*Δc
RADIO, TV, RECORDS, AND MUSICAL INSTRUME	A0 + A1*c + A2*Δc
FLOOR COVERINGS	A0 + A1*c + A2*Δc
DURABLE HOUSEFURNISHINGS NEC	A0 + A1*c + A2*Δc
WRITING EQUIPMENT	A0 + A1*c + A2*Δc
HAND TOOLS	A0 + A1*c + A2*Δc
JEWELRY	A0 + A1*c + A2*Δc
OPHTHALMIC AND ORTHOPEDIC APPLIANCES	A0 + A1*c + A2*Δc
BOOKS AND MAPS	A0 + A1*c + A2*Δc + A3*TIME
WHEEL GOODS AND DURABLE TOYS	A0 + A1*c + A2*Δc
BOATS, RECREATIONAL VECH., AND AIRCRAFT	A0 + A1*c + A2*Δc
FOOD, OFF PREMISE	A0 + A1*c + A2*Δc
FOOD, ON PREMISE	A0 + A1*c + A2*Δc
ALCOHOL, OFF PREMISE	A0 + A1*c + A2*Δc
ALCOHOL, ON PREMISE	A0 + A1*c + A2*Δc
SHOES AND FOOTWEAR	A0 + A1*c + A2*Δc
WOMENS CLOTHING	A0 + A1*c + A2*Δc
MENS CLOTHING	A0 + A1*c + A2*Δc
LUGGAGE	A0 + A1*c + A2*Δc
GASOLINE AND OIL	A0 + A1*c + A2*Δc
FUEL OIL AND COAL	A0 + A1*c + A2*Δc + A3*Dummy
TOBACCO	A0 + A1*c + A2*Δc
SEMIDURABLE HOUSEFURNISHINGS	A0 + A1*c + A2*Δc
DRUG PREPARATIONS AND SUNDRIES	A0 + A1*c + A2*Δc
TOILET ARTICLES AND PREPARATIONS	A0 + A1*c + A2*Δc
STATIONERY AND WRITING SUPPLIES	A0 + A1*c + A2*Δc
NONDURABLE TOYS AND SPORT SUPPLIES	A0 + A1*c + A2*Δc
FLOWERS, SEEDS, AND POTTED PLANTS	A0 + A1*c + A2*Δc
CLEANING PREPARATIONS	A0 + A1*c + A2*Δc
LIGHTING SUPPLIES	A0 + A1*c + A2*Δc
HOUSEHOLD PAPER PRODUCTS	A0 + A1*c + A2*Δc
MAGAZINES AND NEWSPAPER	A0 + A1*c + A2*Δc
OWNER OCCUPIED SPACE RENT	A0 + A1*c + A2*Stock
TENANT OCCUPIED SPACE RENT	A0 + A1*c + A2*Δc + A3*Stock
HOTELS AND MOTELS	A0 + A1*c + A2*Δc
OTHER HOUSING -- EDUCATIONAL HOUSING	A0 + A1*c + A2*Δc + A3*TIME
ELECTRICITY	A0 + A1*c + A2*TIME + A3*Dummy

TABLE 19

(continued)

NATURAL GAS	A0 + A1*c + A2*Δc + A3*Dummy
WATER AND OTHER SANITARY SERVICES	A0 + A1*c + A2*Δc + A3*TIME
TELEPHONE AND TELEGRAPH	A0 + A1*c + A2*Δc
HOUSEHOLD INSURANCE	A0 + A1*c + A2*Δc + A3*STOCK
OTHER HOUSEHOLD OPERATIONS -- REPAIR	A0 + A1*c + A2*Δc
POSTAGE	A0 + A1*c + A2*Δc
AUTO REPAIR	A0 + A1*c + A2*Δc
BRIDGE, TOLLS, ETC	A0 + A1*c + A2*Δc
AUTO INSURANCE	A0 + A1*c + A2*Δc
TAXICABS	A0 + A1*c + A2*Δc
LOCAL PUBLIC TRANSPORT	A0 + A1*c + A2*Δc
INTERCITY RAILROAD	A0 + A1*c + A2*TIME + A3*Trend
INTERCITY BUSES	A0 + A1*c + A2*Δc
AIRLINES	A0 + A1*c + A2*Δc
TRAVEL AGENTS AND OTHER TRANSPORTATION S	A0 + A1*c + A2*Δc
CLEANING, LAUNDERING AND SHOE REPAIR	A0 + A1*c + A2*Δc + A3*TIME + A4*Trend
BARBERSHOPS AND BEAUTY SHOPS	A0 + A1*c + A2*Δc
PHYSICIANS	A0 + A1*c + A2*Δc
DENTISTS AND OTHER PROFESSIONAL SERVICES	A0 + A1*c + A2*Δc
HOSPITALS	A0 + A1*c + A2*Over-85 Share
HEALTH INSURANCE	A0 + A1*c + A2*Labor Participation
BROKERAGE AND INVESTMENT COUNSELING	A0 + A1*c + A2*TIME
BANK SERVICE CHARGES AND SERVICES W/O PA	A0 + A1*c + A2*Δc
LIFE INSURANCE	A0 + A1*c + A2*Labor Participation
LEGAL SERVICES	A0 + A1*c + A2*Δc
FUNERAL EXPENSES AND OTHER PERSONAL BUSI	A0 + A1*c + A2*Δc
RADIO AND TELEVISION REPAIR	A0 + A1*c + A2*Δc
MOVIES, LEGITIMATE THEATRE, SPECTATOR SP	A0 + A1*c + A2*Δc
OTHER RECREATIONAL SERVICES	A0 + A1*c + A2*Δc
EDUCATION	A0 + A1*c + A2*Δc
RELIGIOUS AND WELFARE SERVICES	A0 + A1*c + A2*Δc
NURSING HOMES	A0 + A1*c + A2*Over-85 Share

<u>E. Estimation Criteria</u>

Because the PCE equations eventually will be used in LIFT, they must be capable of generating reasonable forecasts as well as satisfying economic theory. The were four criteria that each equation had to meet:

<u>1. Non-Positive Own-Price Elasticity</u>: Economic theory suggests that, except in the case of a Giffen good, quantity demanded of a good should be inversely related to its own price. Since, by assumption, none of the PCE categories are Giffen goods, any estimation that results in a category having a positive estimated own-price elasticity would have to be respecified and reestimated.

2. The size and magnitude of the coefficient on the ΔC^* must generate stable long-term forecasts: This coefficient must either be positive or smaller in absolute value than the coefficient on the C^* term. If this did not hold, any long-run increase in income would reduce spending. At the level of disaggregation we use, such a property would lead to unreasonable forecasts. Consequently, if the estimated parameters did not meet this criteria, the system was respecified -- usually by changing the form of the equations.

3. The effect of time must be "small": Time was not allowed to change the absolute value of household consumption by more than 1 percent each year. This was to prevent the time trend from dominating the forecast. Unfortunately, the time trend on New Automobiles did not satisfy this criteria.

4. Equation-specific variables must have the "correct" effect: In other words, the coefficient on these variables had to satisfy my *a-priori* beliefs on the variable's effect.

As already mentioned, the first step was estimating the system without using a time trend in the equations. Those equations that fit poorly or did not satisfy the above four conditions were studied to determine if they required an equation-specific variable. For some of the categories that did not meet (1-4), no equation-specific variable could be found. The system was re-estimated the system using alternate equation forms.

Unfortunately, a set of equations that gave non-positive own-price elasticities for all 76 PCE categories could not be found. Despite many attempts, we were forced to accept results that left Auto repair (PCE53) with a positive own-price elasticity. In contrast, the old-system generated positive own-price elasticities in five categories: Hospitals (PCE66), Physicians (PCE64), Dentists and other health professionals (PCE65), Funeral expenses (PCE72) and Auto repair.

Results

Two sets of estimations were performed -- one set using the new treatment of Medicare and one set using the old treatment of Medicare. This allows a comparison between the fit of the two methods. Traditionally, the regression statistic used in determining "goodness-of-fit" is the R-squared statistic. Under Ordinary-Least Squares (OLS) regression, the R-squared statistic shows the percentage of variation in the dependent variable that is explained by movements in the independent variables. The LIFT system, however, is non-linear, and consequently, the R-squared statistic loses some of its meaning because it is no longer bounded between zero and one.[74] While it is true that larger R-squared values indicate a "better" fit, the values become ordinal -- signifying better or worse, but not indicating the magnitude of improvement. The R-squared statistic is but one of many statistics on goodness-of-fit. The statistic we use is the Average Absolute Percentage Error (AAPE), since it gives information on both the direction and magnitude of changes in fit.[75]

It must also be remembered that because the equations are estimated as a system, the criteria is minimizing the error of the system as a whole. Each category carries the same importance when the software attempts to solve equation (5.6):

$$\sum_i \{[Y_i - F(x_i, B_0) + F'(x_i, B_0)B_0] - F'(x_i, B_0)B\}^2 \qquad (5.6).$$

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[74]The calculation of the R-squared coefficient depends on the relationship between total sum of squares (TSS), residual sum of squares(RSS) and explained sum of squares (ESS). Under OLS, TSS = RSS + ESS. In a non-linear estimation, this relationship no longer holds, TSS $\neq$ ESS + RSS.

[75]The average absolute percentage error (AAPE) is calculated as follows: $\sum$\{Absolute Value[(Predicted Level of PCE - Actual Level of PCE)/Actual Level of PCE\}]\}/(# of observations). For a discussion of alternate measures-of-fit, see Newbold and Bos (1994) or Wilson and Keating (1994).

For this reason, one must look at the AAPE of the system as a whole to determine whether or not one has obtained a "better" fit. Any improvement in the performance of the system, however, could be concentrated in a few categories with the majority of categories performed worse and one should also examine AAPE by PCE category to determine whether the improved overall performance offsets any decline in individual equation performance.

The fit of the system improved under the new treatment of Medicare. This is true regardless of whether one examines the overall AAPE statistic or the number of PCE categories that have improved AAPE. The system-wide AAPE of the new treatment equals 10.05 percent and the system-wide AAPE of the old treatment equals 11.22 percent -- an improvement of slightly more than ten percent. Forty-nine of the eighty PCE categories have improved AAPE while twenty-seven categories have worse AAPE. The four categories that had no change -- Non-durables, not elsewhere classified; Travel by foreigners to the US; Travel by US citizens overseas; and Domestic services -- are not estimated as part of the system.[76] Table 20 lists the sectors with improved AAPE with the health related categories in **bold**. In the table, New AAPE is the AAPE for the new system. Old AAPE is the AAPE for the old system. The values in the difference column equals New AAPE less Old AAPE. The percent change column is the improvement relative to the old system.

[76]Other non-durables is an identity in the model. Domestic servants is forecasted based on an exogenous forecast of domestic servant employment. The two Foreign travel categories are forecasted using foreign prices and foreign income.

TABLE 20

Categories with Improved AAPE

Title	New AAPE	Old AAPE	Difference	%Change
HOUSEHOLD INSURANCE	14.791	50.743	-35.953	-123.278
OTHER RECREATIONAL SERVICES	4.704	13.563	-8.859	-105.888
NURSING HOMES	8.314	22.371	-14.057	-98.979
PHYSICIANS	4.583	8.727	-4.144	-64.409
DENTISTS AND OTHER PROFESSIONAL SERVICES	7.120	12.779	-5.659	-58.485
RELIGIOUS AND WELFARE SERVICES	2.494	3.910	-1.416	-44.957
DURABLE HOUSEFURNISHINGS NEC	4.629	7.122	-2.493	-43.088
NONDURABLE TOYS AND SPORT SUPPLIES	2.830	4.274	-1.444	-41.236
HAND TOOLS	9.966	14.355	-4.388	-36.487
HOSPITALS	6.133	8.621	-2.488	-34.054
HOTELS AND MOTELS	4.303	5.991	-1.688	-33.094
KITCHEN AND OTHER HOUSEHOLD APPLIANCES	6.161	8.474	-2.312	-31.863
FOOD, ON PREMISE	2.866	3.843	-0.977	-29.346
TELEPHONE AND TELEGRAPH	16.146	21.511	-5.366	-28.694
WOMENS CLOTHING	5.240	6.546	-1.306	-22.258
LEGAL SERVICES	5.959	7.440	-1.481	-22.201
ELECTRICITY	1.967	2.438	-0.471	-21.487
LIFE INSURANCE	7.638	9.334	-1.696	-20.052
FOOD, OFF PREMISE	7.099	8.649	-1.550	-19.753
TRUCKS	31.542	37.909	-6.367	-18.386
BROKERAGE AND INVESTMENT COUNSELING	38.056	45.688	-7.632	-18.278
OWNER OCCUPIED SPACE RENT	3.683	4.394	-0.711	-17.655
BARBERSHOPS AND BEAUTY SHOPS	5.468	6.503	-1.034	-17.323
STATIONERY AND WRITING SUPPLIES	5.068	5.870	-0.801	-14.681
FLOWERS, SEEDS, AND POTTED PLANTS	6.102	7.014	-0.913	-13.939
NATURAL GAS	5.819	6.517	-0.698	-11.325
GASOLINE AND OIL	4.725	5.289	-0.564	-11.275
TAXICABS	16.070	17.927	-1.857	-10.936
AIRLINES	19.771	21.911	-2.140	-10.278
SHOES AND FOOTWEAR	2.960	3.277	-0.317	-10.178
JEWELRY	14.587	16.115	-1.528	-9.963
EDUCATION	7.604	8.349	-0.745	-9.351
MENS CLOTHING	2.453	2.657	-0.203	-7.967
POSTAGE	4.050	4.329	-0.279	-6.670
DRUG PREPARATIONS AND SUNDRIES	9.175	9.805	-0.630	-6.642
AUTO REPAIR	3.971	4.153	-0.183	-4.504
RADIO AND TELEVISION REPAIR	15.154	15.668	-0.514	-3.337
TRAVEL AGENTS AND OTHER TRANSPORTATION S	29.210	30.173	-0.964	-3.246
AUTO INSURANCE	2.076	2.144	-0.068	-3.205
ACCESSORIES AND PARTS (AUTO)	9.876	10.192	-0.316	-3.150
TENANT OCCUPIED SPACE RENT	2.997	3.086	-0.090	-2.951
OTHER HOUSING -- EDUCATIONAL HOUSING	5.373	5.518	-0.145	-2.666
BOOKS AND MAPS	10.493	10.751	-0.258	-2.432
BOATS, RECREATIONAL VECH., AND AIRCRAFT	30.381	31.069	-0.687	-2.237
MAGAZINES AND NEWSPAPER	2.918	2.964	-0.046	-1.552
TOILET ARTICLES AND PREPARATIONS	2.188	2.220	-0.032	-1.443
TIRES AND TUBES	8.808	8.903	-0.095	-1.072

The forty-seven categories with improved fit account for over approximately eighty percent total PCE. The five health-care related categories with improved fit -- Drugs and sundries, Dentists and other professionals, Hospitals, and Nursing homes -- account for over ninety percent of health-related PCE. Three of the top five categories are health-related and each improved more than fifty percent relative to the old method of treating Medicare.[77] Table 21 lists the sectors with worse AAPE.

[77] It should be noted that the improvement in the AAPE in Household insurance is somewhat misleading. The category is very small and so any improvement in fit will generate a large percent improvement in fit.

TABLE 21

Categories With Worse AAPE

Title	New AAPE	Old AAPE	Difference	%Change
LIGHTING SUPPLIES	6.371	6.354	0.017	0.265
WRITING EQUIPMENT	45.582	45.418	0.164	0.360
CHINA, GLASSWARE, TABLEWARE, AND UTENSIL	4.472	4.454	0.018	0.406
NET PURCHASES OF USED CARS	7.497	7.453	0.044	0.586
INTERCITY BUSES	18.378	18.225	0.153	0.839
BANK SERVICE CHARGES AND SERVICES W/O PA	7.103	7.041	0.062	0.874
ALCOHOL, OFF PREMISE	11.322	11.156	0.166	1.478
HOUSEHOLD PAPER PRODUCTS	8.663	8.399	0.264	3.096
FUEL OIL AND COAL	28.174	27.180	0.994	3.593
HEALTH INSURANCE	4.364	4.087	0.277	6.559
NEW CARS	9.452	8.837	0.615	6.724
ALCOHOL, ON PREMISE	8.907	8.311	0.595	6.920
BRIDGE, TOLLS, ETC	13.320	12.338	0.982	7.657
INTERCITY RAILROAD	9.253	8.480	0.773	8.722
OTHER HOUSEHOLD OPERATIONS -- REPAIR	6.680	6.063	0.617	9.687
CLEANING, LAUNDERING AND SHOE REPAIR	7.562	6.819	0.743	10.340
LOCAL PUBLIC TRANSPORT	5.200	4.659	0.541	10.994
FLOOR COVERINGS	12.775	11.052	1.723	14.487
TOBACCO	4.464	3.828	0.636	15.376
CLEANING PREPARATIONS	4.801	3.999	0.802	18.279
FUNERAL EXPENSES AND OTHER PERSONAL BUSI	2.924	2.410	0.515	19.362
LUGGAGE	14.140	11.232	2.907	23.020
WATER AND OTHER SANITARY SERVICES	3.015	2.389	0.626	23.293
SEMIDURABLE HOUSEFURNISHINGS	5.874	4.513	1.361	26.353
RADIO, TV, RECORDS, AND MUSICAL INSTRUME	61.806	47.084	14.722	27.208
OPHTHALMIC AND ORTHOPEDIC APPLIANCES	8.748	6.532	2.217	29.221
WHEEL GOODS AND DURABLE TOYS	8.088	5.947	2.141	30.755
MOVIES, LEGITIMATE THEATRE, SPECTATOR SP	7.365	4.830	2.535	42.190
FURNITURE, MATTRESSES, AND BEDSPRINGS	4.350	2.714	1.636	47.170

Table 22 contains the estimated parameters under the new treatment of Medicare. Due to the large number of price parameters, only the non-price parameters are listed in the table. The implied price and income elasticities from the estimated parameters under the new treatment of Medicare are listed in table 23. Table 24 contains the estimated income and price-elasticities from the old method of modeling Medicare benefits.

TABLE 22

Estimated Parameters With New Treatment of Medicare

		1 FOOD, ALCOHOL, AND TOBACCO			ITERATION # 6		
			CONST	INCOME	DEL Y	TIME	OTHER
19	1	FOOD, OFF PREMISE	56.147	0.662	2.249		
21	2	ALCOHOL, OFF PREMISE	20.335	3.602	-2.568		
20	1	FOOD, ON PREMISE	28.639	0.742	0.083		
22	2	ALCOHOL, ON PREMISE	52.784	0.798	0.001		
29	3	TOBACCO	26.793	0.546	2.652		
		2 CLOTHING, ACCESSORIES, & PERSO			ITERATION # 6		
			CONST	INCOME	DEL Y	TIME	OTHER
24	1	WOMENS CLOTHING	-42.995	1.574	-0.396		
25	1	MENS CLOTHING	-2.252	1.234	0.059		
23	2	SHOES AND FOOTWEAR	2.939	0.848	-0.585		
26	2	LUGGAGE	2.702	0.099	0.006		
14	2	JEWELRY	12.820	1.534	1.376		
32	3	TOILET ARTICLES AND PREPARATIO	1.500	0.952	-0.211		
63	3	BARBERSHOPS AND BEAUTY SHOPS	8.676	0.412	0.682		
62	3	CLEANING, LAUNDERING AND SHOE	71.650	0.391	-0.153	-0.783	0.646
		3 HOUSEHOLD DURABLES			ITERATION # 6		
			CONST	INCOME	DEL Y	TIME	OTHER
6	1	FURNITURE, MATTRESSES, AND BED	7.294	0.650	-0.324		0.027
7	1	KITCHEN AND OTHER HOUSEHOLD AP	-27.579	1.217			0.043
9	1	RADIO, TV, RECORDS, AND MUSICA	-188.830	4.837	-2.625		
8	2	CHINA, GLASSWARE, TABLEWARE, A	3.769	4.015	-0.701		
10	2	FLOOR COVERINGS	7.117	0.330	0.001		
11	2	DURABLE HOUSEFURNISHINGS NEC	1.467	0.852	-0.518		
30	2	SEMIDURABLE HOUSEFURNISHINGS	12.953	0.962	-0.905		
		4 HOUSEHOLD OPERATION			ITERATION # 6		
			CONST	INCOME	DEL Y	TIME	OTHER
36	1	CLEANING PREPARATIONS	-0.019	0.112	0.036		
37	1	LIGHTING SUPPLIES	2.756	0.644	-0.219		
38	1	HOUSEHOLD PAPER PRODUCTS	1.002	0.373	-0.073		
50	2	HOUSEHOLD INSURANCE	0.261	0.070	0.077		-4.489
51	2	OTHER HOUSEHOLD OPERATIONS --	2.403	0.389	-0.108		
73	2	RADIO AND TELEVISION REPAIR	1.726	0.419	-0.198		
52	3	POSTAGE	1.692	0.190	-0.085		
48	3	TELEPHONE AND TELEGRAPH	-46.799	1.331	0.001		
		5 HOUSING AND HOUSEHOLD UTILITIES			ITERATION # 6		
			CONST	INCOME	DEL Y	TIME	OTHER
41	1	OWNER OCCUPIED SPACE RENT	-86.746	0.723			9.319
42	1	TENANT OCCUPIED SPACE RENT	-52.205	0.464	0.031		3.396
28	2	FUEL OIL AND COAL	11.727	0.393	18.594		6.754
45	2	ELECTRICITY	-70.873	0.827		0.766	1.686
46	2	NATURAL GAS	7.903	0.487	1.360		-1.107
47	3	WATER AND OTHER SANITARY SERVI	-16.407	0.778	-0.710	0.218	

Table 22

(Continued)

			CONST	INCOME	DEL Y	TIME	OTHER
		6 MEDICAL SERVICES			ITERATION # 6		
			CONST	INCOME	DEL Y	TIME	OTHER
64	1	PHYSICIANS	7.974	1.974	-0.602		
65	1	DENTISTS AND OTHER PROFESSIONA	-16.085	2.450	-1.313		
66	2	HOSPITALS	-94.132	5.851			9.778
80	2	Nursing Homes	-55.011	2.464			5.442
15	3	OPHTHALMIC AND ORTHOPEDIC APPL	0.699	0.329	0.300		
31	3	DRUG PREPARATIONS AND SUNDRIES	12.683	1.179	0.067		
67	4	HEALTH INSURANCE	-70.462	0.345			0.912
		7 PERSONAL BUSINESS SERVICES			ITERATION # 6		
			CONST	INCOME	DEL Y	TIME	OTHER
68	1	BROKERAGE AND INVESTMENT COUNS	-3.921	4.657		-0.193	
69	3	BANK SERVICE CHARGES AND SERVI	3.807	8.763	8.983		
70	1	LIFE INSURANCE	-111.587	0.479			1.601
71	2	LEGAL SERVICES	-0.964	2.978		0.088	
72	2	FUNERAL EXPENSES AND OTHER PER	6.998	2.157	-0.371		
		8 TRANSPORTATION			ITERATION # 6		
			CONST	INCOME	DEL Y	TIME	OTHER
1	1	NEW CARS	373.543	0.512	1.176	-3.801	
2	1	NET PURCHASES OF USED CARS	9.688	0.293			-1.731
3	1	TRUCKS	9.485	0.298	0.145		-1.851
4	2	TIRES AND TUBES	-5.633	0.699	0.403		
5	2	ACCESSORIES AND PARTS (AUTO)	-6.437	0.551	0.093		
53	2	AUTO REPAIR	-2.200	0.894	0.731		
55	2	AUTO INSURANCE	2.315	0.285	-0.156		
54	2	BRIDGE, TOLLS, ETC	0.790	0.025	0.033		
56	3	TAXICABS	1.052	0.214	6.634		
57	3	LOCAL PUBLIC TRANSPORT	3.735	0.388	0.001		
27	4	GASOLINE AND OIL	12.485	0.532	0.176		
		9 RECREATION AND TRAVEL			ITERATION # 6		
			CONST	INCOME	DEL Y	TIME	OTHER
74	1	MOVIES, LEGITIMATE THEATRE, SP	6.159	0.207	0.001		
75	1	OTHER RECREATIONAL SERVICES	8.927	1.716	0.046		
18	2	BOATS, RECREATIONAL VECH., AND	6.677	0.329	0.408		
17	2	WHEEL GOODS AND DURABLE TOYS	-13.227	1.137	-0.369		
34	2	NONDURABLE TOYS AND SPORT SUPP	-18.732	1.456	-0.633		
35	2	FLOWERS, SEEDS, AND POTTED PLA	-3.538	0.391	-0.137		
13	2	HAND TOOLS	-4.468	0.289	0.000		
61	3	TRAVEL AGENTS AND OTHER TRANSP	0.551	0.068	0.130		
43	3	HOTELS AND MOTELS	4.531	0.330	0.059		
58	4	INTERCITY RAILROAD	9.193	0.017		-0.110	0.128
59	4	INTERCITY BUSES	2.701	0.035	0.073		
60	4	AIRLINES	5.524	1.279	0.355		
		10 READING AND EDUCATION			ITERATION # 6		
			CONST	INCOME	DEL Y	TIME	OTHER
16	1	BOOKS AND MAPS	-5.566	0.504	-0.282	0.077	
39	1	MAGAZINES AND NEWSPAPER	4.994	0.689	0.001		
12	1	WRITING EQUIPMENT	1.894	0.024	0.048		
33	1	STATIONERY AND WRITING SUPPLIE	-0.179	0.330	-0.183		
76	2	EDUCATION	2.328	1.519	-0.131		
44	2	OTHER HOUSING -- EDUCATIONAL H	6.758	0.222	0.073	-0.070	
77	3	RELIGIOUS AND WELFARE SERVICES	-10.691	0.969	-0.388		

TABLE 23

Income and Price Elasticities From New Treatment

			SHARE	YELAS	OWN	SG #1	SG #2	SG #3	SG #4
		GROUP 1: FOOD, ALCOHOL, AND TOBACCO							
19	1	FOOD, OFF PREMISE	8.14	0.435	-0.615	0.957	-0.149	0.243	
21	2	ALCOHOL, OFF PREMISE	1.38	0.815	-0.498	-0.025	1.073	0.158	
20	1	FOOD, ON PREMISE	3.79	1.094	-1.127	0.446	-0.069	0.113	
22	2	ALCOHOL, ON PREMISE	0.58	0.723	-1.120	-0.011	0.451	0.066	
29	3	TOBACCO	0.69	0.241	-0.766	0.021	0.079	0.000	
		1 FOOD							
		2 ALCOHOL							
		3 TOBACCO							
		GROUP 2: CLOTHING, ACCESSORIES & PERSON							
24	1	WOMENS CLOTHING	5.27	1.286	-0.745	0.930	0.181	0.127	
25	1	MENS CLOTHING	2.49	1.319	-1.236	0.439	0.086	0.060	
23	2	SHOES AND FOOTWEAR	1.10	0.950	-0.510	0.038	0.056	0.001	
26	2	LUGGAGE	0.06	1.262	-0.563	0.002	0.003	0.000	
14	2	JEWELRY	0.91	2.046	-0.519	0.031	0.047	0.001	
32	3	TOILET ART. & PREPS	0.93	0.966	-0.881	0.022	0.001	0.540	
63	3	BARBERS & BEAUTY SHOPS	0.46	0.611	-1.153	0.011	0.000	0.267	
62	3	CLEANING, LAUNDER., ETC.	0.41	0.708	-1.182	0.010	0.000	0.238	
		1 CLOTHING							
		2 ACCESSORIES							
		3 PERSONAL CARE							
		GROUP 3: HOUSEHOLD DURABLES							
6	1	FURNITURE &MATTRESSES	1.33	1.446	-0.512	0.044	0.029		
7	1	KITCHEN & HHLD APPL.	1.37	1.130	-0.510	0.045	0.029		
9	1	RADIO, TV, RECORDS	6.72	2.289	-0.334	0.221	0.145		
8	2	CHINA, GLASSWARE	0.57	1.230	-0.472	0.012	0.023		
10	2	FLOOR COVERINGS	0.36	1.496	-0.480	0.008	0.014		
11	2	DURABLE HOUSEFURNISHINGS	0.94	1.960	-0.457	0.020	0.037		
30	2	SEMIDURABLE HOUSEFURN.	0.66	1.348	-0.469	0.014	0.026		
		1 MAJOR DURABLES							
		2 MINOR DURABLES							
		GROUP 4: HOUSEHOLD OPERATION							
36	1	CLEANING PREPARATIONS	0.11	1.009	-0.553	0.009	0.048	0.000	
37	1	LIGHTING SUPPLIES	0.63	0.916	-0.509	0.053	0.274	0.001	
38	1	HOUSEHOLD PAPER PRODUCTS	0.36	0.946	-0.532	0.030	0.156	0.001	
50	2	HOUSEHOLD INSURANCE	0.07	1.051	-0.213	0.030	-0.118	0.010	
51	2	OTHER HHLD OPERATION	0.40	0.982	-0.770	0.174	-0.675	0.056	
73	2	RADIO AND TV REPAIR	0.11	0.427	-0.281	0.048	-0.186	0.015	
52	3	POSTAGE	0.20	0.935	-0.470	0.000	0.028	0.013	
48	3	TELEPHONE AND TELEGRAPH	2.70	0.822	-0.305	0.006	0.376	0.179	
		1 CLEANING AND PAPER PRODUCTS							
		2 SERVICES AND INSURANCE							
		3 COMMUNICATION							
		GROUP 5: HOUSING & HOUSEHOLD UTILITIES							
41	1	OWNER OCCUPIED SPACE	9.13	0.788	-0.939	2.104	-0.107	0.029	
42	1	TENANT OCCUPIED SPACE	3.38	0.106	-2.264	0.779	-0.039	0.011	
28	2	FUEL OIL AND COAL	0.19	0.143	-0.814	-0.002	0.077	-0.010	
45	2	ELECTRICITY	1.54	0.440	-0.266	-0.018	0.625	-0.083	
46	2	NATURAL GAS	0.44	0.245	-0.713	-0.005	0.178	-0.024	
47	3	WATER AND OTHER SANITARY	0.47	0.470	-0.106	0.001	-0.025	0.000	
		1 HOUSING OWN							
		2 HOUSing UTILITIES							
		3 SANITATION							

TABLE 23

(Continued)

			SHARE	YELAS	OWN	SG #1	SG #2	SG #3	SG #4
		GROUP 6: MEDICAL SERVICES							
64	1	PHYSICIANS	2.26	1.098	-0.990	0.159	0.351	-0.370	0.401
65	1	DENTISTS & OTHER PROF.	2.18	1.098	-0.996	0.153	0.338	-0.357	0.387
66	2	HOSPITALS	3.98	0.981	-1.060	0.618	-0.791	1.218	-0.494
80	2	NURSING HOMES	0.88	1.030	-0.444	0.137	-0.175	0.269	-0.109
15	4	OPHT. & ORTHOPEDIC	0.29	0.922	-0.252	-0.047	0.089	-0.180	0.007
31	4	DRUG PREPS AND SUNDRIES	1.15	0.824	-0.787	-0.188	0.352	-0.715	0.027
67	3	HEALTH INSURANCE	0.72	0.560	-0.412	0.128	-0.089	0.017	0.000
		1 DENTISTS, DOCTORS AND OTHER PROFESSIONALS							
		2 HOSPITALS							
		3 HEALTH INSURANCE							
		4 DRUGS AND EQUIPMENT							
		GROUP 7: PERSONAL BUSINESS SERVICES							
68	1	BROKERAGE AND INVESTMENT	1.18	1.822	-0.332	0.230	0.121	-0.138	
69	3	BANK SERVICE CHARGES	2.21	1.067	-0.147	-0.258	0.395	0.000	
70	1	LIFE INSURANCE	1.30	1.747	-0.309	0.253	0.134	-0.152	
71	2	LEGAL SERVICES	0.75	0.902	-0.374	0.077	-0.658	0.134	
72	2	FUNERAL EXPENSES	0.54	0.846	-0.190	0.056	-0.473	0.096	
		1 FINANCIAL SERVICES							
		2 REAL SERVICES							
		3 IMPUTED							
		GROUP 8: TRANSPORTATION							
1	1	NEW CARS	2.69	2.531	-0.104	0.262	-0.276	0.729	-0.003
2	1	NET PURCH. OF USED CARS	0.75	1.203	-0.293	0.073	-0.077	0.203	-0.001
3	1	TRUCKS	1.56	2.829	-0.214	0.152	-0.160	0.423	-0.002
4	2	TIRES AND TUBES	0.59	0.850	-0.135	-0.060	0.107	-0.020	-0.022
5	2	ACCESSORIES AND PARTS	0.47	0.933	-0.156	-0.048	0.085	-0.016	-0.017
53	2	AUTO REPAIR	1.91	0.945	0.105	-0.196	0.347	-0.066	-0.071
55	2	AUTO INSURANCE	0.53	0.579	-0.145	-0.054	0.096	-0.018	-0.020
54	2	BRIDGE, TOLLS, ETC	0.05	0.728	-0.233	-0.005	0.009	-0.002	-0.002
56	3	TAXICABS	0.07	0.256	-0.600	0.019	-0.002	-0.291	-0.011
57	3	LOCAL PUBLIC TRANSPORT	0.12	0.187	-0.808	0.033	-0.004	-0.499	-0.019
27	4	GASOLINE AND OIL	2.10	0.532	-0.027	-0.002	-0.078	-0.326	0.000
		1 DURABLE PURCHASES							
		2 MAINTENANCE EXPENSES EXP. GASOLINE							
		3 PUBLIC TRANSPORTATION							
		4 GASOLINE							
		GROUP 9: RECREATION AND TRAVEL							
74	1	MOVIES, THEATER, SPORTS	0.37	1.453	-1.997	0.172	0.037	0.066	-0.028
75	1	OTHER REC. SERVICES	3.11	2.069	-0.720	1.449	0.314	0.558	-0.239
18	2	BOATS, RVS AND ATVS	0.40	2.310	-0.645	0.040	0.007	-0.069	0.046
17	2	WHEEL GOODS & DUR. TOYS	1.02	1.585	-0.634	0.103	0.018	-0.176	0.117
34	2	NONDURURABLE TOYS	1.42	1.333	-0.627	0.143	0.025	-0.245	0.163
35	2	FLOWERS, SEEDS	0.38	1.212	-0.645	0.038	0.007	-0.065	0.044
13	2	HAND TOOLS	0.28	1.447	-0.647	0.028	0.005	-0.048	0.032
61	3	TRAVEL AGENTS	0.04	1.547	-0.947	0.007	-0.007	0.046	0.026
43	3	HOTELS AND MOTELS	0.22	0.956	-0.742	0.039	-0.038	0.251	0.145
58	4	INTERCITY RAILROAD	0.01	0.798	-1.289	-0.001	0.001	0.007	0.011
59	4	INTERCITY BUSES	0.02	0.554	-1.278	-0.002	0.002	0.013	0.022
60	4	AIRLINES	0.69	1.640	-0.541	-0.053	0.079	0.456	0.759
		1 ADMISSIONS							
		2 RECREATIONAL NONDURABLES AND DUR							
		3 HOTELS ETC.							
		4 INTER-CITY TRAVEL							

TABLE 23

(Continued)

		GROUP 10: READING AND EDUCATION SHARE	YELAS	OWN	SG #1	SG #2	SG #3	SG #4
16 1	BOOKS AND MAPS	0.36	0.710	-0.691	-0.036	-0.018	0.087	
39 1	MAGAZINES AND NEWSPAPER	0.49	0.653	-0.704	-0.049	-0.025	0.119	
12 1	WRITING EQUIPMENT	0.02	0.393	-0.657	-0.002	-0.001	0.005	
33 1	WRITING SUPPLIES	0.32	1.002	-0.687	-0.032	-0.016	0.077	
76 2	EDUCATION	1.81	1.316	-0.391	-0.092	0.311	0.144	
44 2	OTHER HOUSING -- EDUC	0.15	1.318	-0.676	-0.008	0.026	0.012	
77 3	RELIG. & WELFARE SVCS	2.76	1.458	-0.650	0.668	0.220	0.000	

1 READING
2 EDUCATION
3 RELIGIOUS

TABLE 24

Income and Price Elasticities From Old Treatment

GROUP 1: FOOD, ALCOHOL, AND TOBACCO

			SHARE	YELAS	OWN	SG #1	SG #2	SG #3	SG #4
19	1	FOOD, OFF PREMISE	8.14	0.442	-0.475	0.944	-0.086	0.240	
21	2	ALCOHOL, OFF PREMISE	1.38	0.874	-0.483	-0.015	1.050	0.263	
20	1	FOOD, ON PREMISE	3.79	1.147	-0.979	0.439	-0.040	0.112	
22	2	ALCOHOL, ON PREMISE	0.58	0.798	-1.092	-0.006	0.441	0.110	
29	3	TOBACCO	0.69	0.246	-0.761	0.020	0.131	0.000	

1 FOOD
2 ALCOHOL
3 TOBACCO

GROUP 2: CLOTHING, ACCESSORIES & PERSON

			SHARE	YELAS	OWN	SG #1	SG #2	SG #3	SG #4
24	1	WOMENS CLOTHING	5.27	1.266	-0.275	0.234	0.253	0.083	
25	1	MENS CLOTHING	2.49	1.375	-0.398	0.110	0.119	0.039	
23	2	SHOES AND FOOTWEAR	1.10	1.037	-0.675	0.053	0.058	0.132	
26	2	LUGGAGE	0.06	1.507	-0.729	0.003	0.003	0.007	
14	2	JEWELRY	0.91	2.332	-0.685	0.044	0.048	0.109	
32	3	TOILET ART. & PREPR	0.93	1.000	-1.089	0.015	0.111	0.737	
63	3	BARBERS & BEAUTY SHOPS	0.46	0.601	-1.462	0.007	0.055	0.365	
62	3	CLEANING, LAUND. ETC,	0.41	0.659	-1.502	0.006	0.049	0.325	

1 CLOTHING
2 ACCESSORIES
3 PERSONAL CARE

GROUP 3: HOUSEHOLD DURABLES

			SHARE	YELAS	OWN	SG #1	SG #2	SG #3	SG #4
6	1	FURNITURE & MATTRESSES	1.33	1.610	-0.181	0.020	0.011		
7	1	KITCHEN & HHLD APPL.	1.37	1.113	-0.181	0.021	0.012		
9	1	RADIO, TV, RECORDS	6.72	2.099	-0.100	0.102	0.058		
8	2	CHINA, GLASSWARE	0.57	1.300	-0.457	0.005	0.099		
10	2	FLOOR COVERINGS	0.36	1.583	-0.493	0.003	0.062		
11	2	DURABLE HOUSEFURNISHINGS	0.94	2.199	-0.393	0.008	0.162		
30	2	SEMIDURABLE HOUSEFURN.	0.66	1.580	-0.441	0.006	0.114		

1 MAJOR DURABLES
2 MINOR DURABLES

GROUP 4: HOUSEHOLD OPERATION

			SHARE	YELAS	OWN	SG #1	SG #2	SG #3	SG #4
36	1	CLEANING PREPARATIONS	0.11	1.057	-0.615	0.025	0.070	-0.001	
37	1	LIGHTING SUPPLIES	0.63	0.963	-0.496	0.144	0.400	-0.006	
38	1	HOUSEHOLD PAPER PRODUCTS	0.36	0.984	-0.558	0.082	0.229	-0.003	
50	2	HOUSEHOLD INSURANCE	0.07	1.138	-0.169	0.044	-0.130	0.010	
51	2	OTHER HHLD OPERATIONS	0.40	1.055	-0.782	0.254	-0.744	0.054	
73	2	RADIO AND TV REPAIR	0.11	0.459	-0.243	0.070	-0.205	0.015	
52	3	POSTAGE	0.20	0.973	-0.317	-0.002	0.027	0.015	
48	3	TELEPHONE AND TELEGRAPH	2.70	0.792	-0.126	-0.024	0.367	0.207	

1 CLEANING AND PAPER PRODUCTS
2 SERVICES AND INSURANCE
3 COMMUNICATION

GROUP 5: HOUSING & HOUSEHOLD UTILITIES

			SHARE	YELAS	OWN	SG #1	SG #2	SG #3	SG #4
41	1	OWNER OCCUPIED SPACE	9.13	0.813	-0.749	1.908	-0.020	0.236	
42	1	TENANT OCCUPIED SPACE	3.38	0.111	-1.951	0.706	-0.007	0.087	
28	2	FUEL OIL AND COAL	0.19	0.162	-0.867	0.000	0.084	-0.005	
45	2	ELECTRICITY	1.54	0.458	-0.271	-0.003	0.679	-0.043	
46	2	NATURAL GAS	0.44	0.251	-0.757	-0.001	0.194	-0.012	
47	3	WATER AND OTHER SANITARY	0.47	0.460	-0.299	0.012	-0.013	0.000	

1 HOUSING OWN
2 HOUSING UTILITIES
3 SANITATION

TABLE 24

(Continued)

GROUP 6: MEDICAL SERVICES

			SHARE	YELAS	OWN	SG #1	SG #2	SG #3	SG #4
64	1	PHYSICIANS	2.26	1.229	-0.004	-0.543	0.039	0.500	0.268
65	1	DENTISTS & PROF.	2.18	1.141	0.015	-0.524	0.038	0.483	0.258
66	2	HOSPITALS	3.98	1.034	-0.042	0.069	0.467	-0.452	-0.063
80	2	NURSING HOMES	0.88	1.224	-0.405	0.015	0.103	-0.100	-0.014
15	3	OPHT. & ORTHOPEDIC	0.29	1.026	-0.274	0.064	-0.033	-0.025	-0.038
31	3	DRUG PREPS & SUNDRIES	1.15	0.956	-0.348	0.255	-0.131	-0.099	-0.149
67	4	HEALTH INSURANCE	0.72	0.498	-0.299	0.085	-0.011	-0.094	0.000

1 DENTISTS, DOCTORS AND OTHER PROFESSIONALS
2 HOSPITALS
3 INSURANCE
4 DRUGS AND EQUIPMENT

GROUP 7: PERSONAL BUSINESS SERVICES

			SHARE	YELAS	OWN	SG #1	SG #2	SG #3	SG #4
68	1	BROKERAGE AND INVESTMENT	1.18	1.770	-0.196	0.130	0.131	-0.070	
69	3	BANK SERVICE CHARGES	2.21	1.093	-0.116	-0.131	0.380	0.000	
70	1	LIFE INSURANCE	1.30	1.422	-0.183	0.143	0.145	-0.077	
71	2	LEGAL SERVICES	0.75	1.011	-0.180	0.084	-0.707	0.129	
72	2	FUNERAL EXPENSES	0.54	0.938	0.018	0.060	-0.509	0.093	

1 FINANCIAL SERVICES
2 REAL SERVICES
3 IMPUTED

GROUP 8: TRANSPORTATION

			SHARE	YELAS	OWN	SG #1	SG #2	SG #3	SG #4
1	1	NEW CARS	2.69	2.676	-0.138	0.290	-0.211	0.824	0.091
2	1	NET PURCH. OF USED CARS	0.75	1.188	-0.348	0.081	-0.059	0.230	0.025
3	1	TRUCKS	1.56	3.360	-0.260	0.168	-0.122	0.478	0.052
4	2	TIRES AND TUBES	0.59	0.843	-0.135	-0.046	0.118	-0.040	-0.026
5	2	ACCESSORIES AND PARTS	0.47	0.910	-0.159	-0.037	0.094	-0.032	-0.020
53	2	AUTO REPAIR	1.91	0.955	0.129	-0.150	0.383	-0.130	-0.083
55	2	AUTO INSURANCE	0.53	0.572	-0.147	-0.041	0.106	-0.036	-0.023
54	2	BRIDGE, TOLLS, ETC	0.05	0.758	-0.244	-0.004	0.010	-0.003	-0.002
56	3	TAXICABS	0.07	0.272	-0.539	0.021	-0.005	-0.305	-0.009
57	3	LOCAL PUBLIC TRANSPORT	0.12	0.188	-0.757	0.037	-0.008	-0.522	-0.015
27	4	GASOLINE AND OIL	2.10	0.531	-0.027	0.071	-0.091	-0.267	0.000

1 DURABLE PURCHASES
2 MAINTENANCE EXPENSES EXP. GASOLINE
3 PUBLIC TRANSPORTATION
4 GASOLINE

GROUP 9: RECREATION AND TRAVEL

			SHARE	YELAS	OWN	SG #1	SG #2	SG #3	SG #4
74	1	MOVIES, THEATER, SPORTSTR	0.37	1.654	-1.645	0.128	0.054	0.141	-0.041
75	1	OTHER RECREATIONAL	3.11	2.378	-0.699	1.074	0.453	1.183	-0.342
18	2	BOATS, REC. SERVICES	0.40	2.598	-0.628	0.058	0.005	-0.121	0.064
17	2	WHEEL GOODS & DUR. TOYS	1.02	1.609	-0.619	0.149	0.014	-0.309	0.164
34	2	NONDURABLE TOYS	1.42	1.339	-0.614	0.207	0.019	-0.430	0.228
35	2	FLOWERS, SEEDS	0.38	1.234	-0.628	0.055	0.005	-0.115	0.061
13	2	HAND TOOLS	0.28	1.437	-0.629	0.041	0.004	-0.085	0.045
61	3	TRAVEL AGENTS	0.04	1.634	-1.418	0.015	-0.012	0.085	0.036
43	3	HOTELS AND MOTELS	0.22	0.971	-1.038	0.084	-0.067	0.465	0.199
58	4	INTERCITY RAILROAD	0.01	0.759	-1.199	-0.001	0.002	0.009	0.010
59	4	INTERCITY BUSES	0.02	0.586	-1.188	-0.002	0.003	0.018	0.021
60	4	AIRLINES	0.69	1.789	-0.485	-0.076	0.111	0.623	0.724

1 ADMISSIONS
2 RECREATIONAL NONDURABLES AND DUR
3 HOTELS ETC.
4 INTER-CITY TRAVEL

TABLE 24

(Continued)

GROUP 10: READING AND EDUCATION

			SHARE	YELAS	OWN	SG #1	SG #2	SG #3	SG #4
16	1	BOOKS AND MAPS	0.36	0.724	-0.725	0.008	-0.037	0.113	
39	1	MAGAZINES AND NEWSPAPER	0.49	0.659	-0.723	0.011	-0.050	0.154	
12	1	WRITING EQUIPMENT	0.02	0.437	-0.733	0.000	-0.002	0.006	
33	1	WRITING EQUIPMENT	0.32	1.047	-0.726	0.007	-0.033	0.101	
76	2	EDUCATION	1.81	1.339	-0.340	-0.184	0.350	0.257	
44	2	OTHER HOUSING -- EDUC.	0.15	1.369	-0.661	-0.015	0.029	0.021	
77	3	RELIG. & WELFARE SVCS	2.76	1.547	-0.693	0.868	0.392	0.000	

1 READING
2 EDUCATION
3 RELIGIOUS

Table 25 is presented as an example of how to read the elasticity tables.

TABLE 25

Sample Elasticity Chart

GROUP 6: MEDICAL SERVICES

			SHARE	YELAS	OWN	SG #1	SG #2	SG #3	SG #4
64	1	PHYSICIANS	2.26	1.098	-0.990	0.159	0.351	-0.370	0.401
65	1	DENTISTS & OTHER PROF.	2.18	1.098	-0.996	0.153	0.338	-0.357	0.387
66	2	HOSPITALS	3.98	0.981	-1.060	0.618	-0.791	1.218	-0.494
80	2	NURSING HOMES	0.88	1.030	-0.444	0.137	-0.175	0.269	-0.109
15	4	OPHT. & ORTHOPEDIC	0.29	0.922	-0.252	-0.047	0.089	-0.180	0.007
31	4	DRUG PREPS AND SUNDRIES	1.15	0.824	-0.787	-0.188	0.352	-0.715	0.027
67	3	HEALTH INSURANCE	0.72	0.560	-0.412	0.128	-0.089	0.017	0.000

1 DENTISTS, DOCTORS AND OTHER PROFESSIONALS
2 HOSPITALS
3 HEALTH INSURANCE
4 DRUGS AND EQUIPMENT

Looking at the first PCE category, we see that *PCE64, Physicians* is a member of *Group 6: Medical Services* and is part of sub-group 1 *Dentists, Doctors and Other Professionals* in Group 6. In 1994, 2.26 percent of total PCE was spent on Physicians. The category has an income elasticity (YELAS) of 1.098.[78] Physicians (PCE64) has

[78]An income elasticity of 1 means that, if income increases by 1 percent, spending increases 1 percent. Similarly, an own-price elasticity of -.5 means that, if the good's own price increased by 1 percent, spending on the good falls by one-half a percent.

an own-price elasticity (OWN) of -0.990. Looking at the values under the headings **SG #**, we see that a 1 percent increase in the cost of Physicians, leads to a 0.159 percent increase in spending on the other categories in sub-group 1. The price increase in PCE64, causes spending in sub-group 2, *Facilities*, to increase 0.351 percent and also causes spending on sub-group 3, *Drugs and Equipment*, to fall -0.370 percent. Finally, the 1 percent increase in the price of PCE64 causes an increase of 0.401 percent on spending on sub-group 4, *Insurance.*

An examination of the tables shows that nearly all of the income and own-price elasticities have changed to some degree. Since the parameters are estimated as a system, this should be no surprise. Most of the income elasticities are roughly the same magnitude as under the old system, but the own-price elasticities have changed dramatically -- particularly Dentists and other professionals. Under the old-system, this category had slightly positive own-price elasticities, but under the new system, the own-price elasticities have the correct sign and the magnitudes are much larger.

Forecast of Medicare Benefits by PCE Category

In old-LIFT, Medicare benefits are forecasted with the use of two equations (Monaco 1994b). From, Monaco (1994b), benefits per person age 65 or older (1977$) are forecasted:

$$\text{Benefit per person age 65+} = -1620.5 + 32.28 * \text{TIME} \qquad (5.9a);$$

Equation (5.9a) states that real spending per person age 65 or older will increase by $32.28 (1977$) per year. This translates into an increase of approximately 2.54 percent per year, based on the 1992 value of benefits per person. Using the forecasted values from equation (5.9a), total nominal Medicare benefits can be easily calculated as:

Nominal Medicare =
(Benefit per person age 65) * PCE Hospital Defl * GPOP8 (5.9b);

Where:

Nominal Medicare	= Medicare Benefits in Millions of Current Dollars;
PCE Hospital Defl	= Deflator LIFT PCE category 66 (1977 = 1.00);
GPOP8	= Millions of persons age 65 and over.

Once calculated, nominal Medicare benefits are then added to nominal personal income, which eventually is converted into real disposable income. Real disposable income in turn is converted into PCE. This method does not forecast Medicare benefits by PCE category nor does it ensure that Medicare benefits are spent on medical PCE. New-LIFT, however, requires forecasts of Medicare benefits by PCE category.

Any forecasting equation or exogenous forecast of Medicare by category of spending must account for the size and age distribution of the population. In particular, the equation or forecast should account for the non-elderly population that receives Medicare benefits. For example, in 1987, over eleven percent of Medicare benefits were received by persons **under** the age of 65 years (Waldo *et al.* 1989). Over 10 percent of Medicare recipients are non-elderly and the number of under-65 beneficiaries is growing at a rate 50 percent higher than the number of over-65 recipients (Petrie and Silverman 1993). Thus, one should recognize that an increase in the under-65 population will increase total Medicare benefits, *ceteris paribus*.

The PCE system provides a ready solution to the problem of incorporating age effects into the forecast of Medicare benefits by category. Waldo *et al.* (1989) provide information on the recipients of Medicare by age and PCE category. These data can be used to create an age-weighted Medicare recipient population. Medicare benefits by category equal the product of the age-weighted Medicare recipient population and Medicare benefits per age-weighted population. By using the AEW concept utilized in the PCE system, the forecast of Medicare benefits by category will account for any changes in the population size and structure.

The equivalency weights used in constructing the age-weighted Medicare recipient population are based on Medicare spending per capita and by age cohort as reported by Waldo *et al.* (1989) Table 26 shows these weights.

TABLE 26

Medicare Adult Equivalency Weights by PCE Category

Age	Physicians	Hospitals	Nursing Homes	Dentists and Professionals	Opth. and orthopedic
Under 20	0.004	$1.03*10^{-4}$	0.000	0.0025	0.0025
20 to 64 Years	0.023	0.0052	0.000	0.0493	0.0493
65 to 69 Years	1.000	1.0000	1.000	1.0000	1.000
70 to 74 Years	1.174	1.2510	2.200	0.9648	0.9648
75 to 79 Years	1.335	1.5610	4.400	0.9366	0.9366
80 to 84 Years	1.414	1.8090	7.400	0.9014	0.9014
Over 85	1.419	1.9630	11.200	0.8521	0.8521

For the Medicare AEWs, the reference group is the 65 to 69 years old cohort. Waldo reports benefits by age for four types of services: Physicians, Hospitals, Nursing homes and Other services. Where the services matched with our PCE categories, the construction of age-weights was simply a one-to-one matching procedure. Unfortunately Other services did not match one-to-one with either Dentists or Ophthalmic goods. For this reason, it was assumed that the age-weights for these last two categories were identical to each other and I used the per-capita benefits data for Other services to derive these weights.

The equations I developed forecast real benefits per age-weighted capita (RBPWP). The procedure is very simple. Real benefits by PCE category are constructed real benefits by PCE category by deflating nominal benefits by:

$$Real\ Benefits_{i,t} = \frac{Nominal\ Benefits_{i,t}}{Deflator_{i,t}} \qquad (5.10);$$

Where:
- $Real\ Benefits_{i,t}$ = Real Medicare benefits in 1977$, category i, year t;
- $Nominal\ Benefits_{i,t}$ = Real Medicare benefits in 1977$, category i, year t;
- $Deflator_{i,t}$ = Price deflator (1977=1.00), category i, year t.

The age-weighted population for each category is constructed:

$$WP_{i,t} = \sum_{j=1}^{7} w_{i,j} * Pop_{j,t} \qquad (5.11);$$

Where:
- $WP_{i,t}$ = Age-Weighted Population, category i, year t;
- $w_{i,j}$ = Weight, category i, age cohort j;
- $Pop_{j,t}$ = Population, age cohort j, year t.

Thus, for any category i, real benefits per age-weighted population (RBPWP) equals:

$$RBPWP_{i,t} = \frac{Real\ Benefits_{i,t}}{WP_{i,t}} \qquad (5.12).$$

Equations (5.10) through (5.12) allow the construction of the historical values of RBPWP.

Unfortunately, it was not possible to develop equations that forecast RBPWP by category. Many different approaches were attempted and no equation simultaneously possessed good simulation and statistical qualities. Equations employing linear time trends exhibited high degrees of autocorrelation and generated implausible forecasts. Equations that employed non-linear time trends (e.g. log-linear or log-log time trends) performed just as poorly. Equations were estimated using the lagged-value of the dependent variable as the explanatory variable, but these resulted in a forecast that either grew exponentially or fell exponentially.

Consequently, real benefits per age-weighted population for each health category are exogenously fixed during a forecast. These values should be considered as "reasonable" forecasts and not the ultimate answer on how real age-weighted Medicare benefits will grow. These forecasts are presented in Chapter 7. In general, simulations indicate that the current rate of growth is unsustainable and a general slow-down must occur (Monaco and Phelps 1994; Carr and Monaco 1995).

Unfortunately, by providing an exogenous forecast of these variables, we have modeled Medicare as a budget item and not as an entitlement program. As a budget item, the government appropriates a given dollar amount for Medicare benefits. In reality, the Medicare program is an entitlement program. In an entitlement program, the government establishes eligibility requirements and the benefits to which the recipient is **entitled**. For example, eligibility requirements for Medicare exclude the non-elderly, non-disabled population. Eligible persons must be over-65, disabled, or suffering from end-stage renal disease.

Any person eligible for Medicare is guaranteed that the government will partially fund certain types of medical treatment. Thus, government obligations depend on both the **price** of health care and the **demand** for health care. If one modeled the program as an entitlement, real Medicare benefits would be endogenous to the model, unless one assumes that the government changes the entitlement structure. One implication of this is that any increase in demand will increase the real benefit. For example, if the income of Medicare eligible persons were to increase, the government obligation would also increase as lower income eligible persons could afford to consume more health services and goods. Modeling real benefits as a budget item ignores this simultaneity between health PCE and Medicare benefits.

I chose to model the program as a budget item because it is my belief that the government will take steps to limit its obligations, if faced with skyrocketing Medicare benefits. The 1984 switch to the prospective payment system (PPS) illustrates this point. The government was faced with rapidly accelerating Medicare hospital benefits and, in an attempt to control this increase, acted to control its obligations.

However, one interesting question that specifying real benefits cannot answer is: What is the government's Medicare obligation given the current state-of-the-law? Only when real benefits are endogenous can this question be answered. For this reason, the default real benefits equations, used only in the absence of exogenous assumptions, are designed to keep the Medicare subsidies constant at their last historical values. The general form of the equation is:

Where:

$$RBPWP_{i,t} = rate_{i,YR} * \frac{\frac{PCE_{i,t}}{WP_{i,t}}}{Pop_t} \qquad (5.13);$$

$RBPWP_{i,t}$= Real Benefits per-weighted population, year t, category i;
$PCE_{i,t}$ = Real PCE in year t, category i;
$WP_{i,t}$ = Age-Weighted recipient population, year t, category i;
Pop_t = Total population, year t;
$rate_{i,YR}$ = Subsidy rate, category i in year YR - the last year of historical data.

Using equation (5.13) forces real benefits per-weighted population to grow with per-capita PCE and endogenizes Medicare spending. Use of the entitlement real Medicare benefit equations, rather than exogenous assumptions, models the program as an entitlement, **given the current status of the law**. The potential Medicare obligation caused by the use of equation (5.13) is limited only by the level of GDP in a given year. Use of equation (5.13) assumes that the Federal government will take no action to reduce its potential obligations.

Given the current debate regarding the impending Medicare crisis, it is my belief that the government, if faced with upwardly-spiraling Medicare obligations, will act so as to limit the rate of growth in Medicare benefits. The exogenous forecasts of real Medicare benefits used in the simulation chapter incorporate this belief and consequently, I make no use of the Medicare as an entitlement equations.

It should be noted that the modifications to LIFT implemented as part of this work allows the modeler to specify control values for either total nominal or total real Medicare benefits. Similarly, the modeler may specify control values for both parts of Medicare (Part A and Part B). Table 27 shows the relationship of the categories to the two parts.

TABLE 27

Relationship of Medicare Categories to Total Medicare

Total Medicare = Hospital Insurance [HI] (Part A) + Supplemental Medical Insurance [SMI] (Part B)	
Part A (HI) = Hospitals	Part B (SMI) = Physicians + Dentists + Nursing Homes + Opth. and Orthopedic Appliances

If either Part A or Part B is specified exogenously, the component pieces are scaled so that their sum equals the control. If total Medicare is specified exogenously, both Parts are scaled to total Medicare and then the components of each Part are scaled to the new Part total. This insures consistency at an aggregate level. Technically, one could specify all components, both Parts and total Medicare benefits, though the controls for the less-aggregated items are ignored in favor of the control at a higher level of aggregation.

Conclusion

The treatment of Medicare as a price subsidy is not only correct on theoretical grounds, but it has improved the fit of the system of equations. In Chapter 7, the new equations are tested in simulation runs that show their true worth.

CHAPTER 6

THE DEMOGRAPHICS PROJECTIONS MODEL

Until the present work, the long-run forecasting model lacked the ability to simulate the full effects of changes in birth, death and immigration rates.[79] While the effects of changes in population and its age distribution were easily simulated, the model did not account for changes in the indirect age demographic variables under alternate scenarios of the age structure or total population. The model relied on a exogenous and separately projected forecast of indirect-age demographics such as:

Households: The number of households in the U.S.

Age of Household Head: The system of consumption equations uses three categories of household heads. These are the percent of households whose head of household is: under thirty-five; between thirty-five and fifty-five; and over fifty-five. A fourth category, the percent of households headed by persons aged twenty-five to thirty-five, is forecasted as well. This last category is used in the equations that forecast residential construction.

Region: The percent of households living in the North East, North Central, South and West.

Two or More Earners: The percent of households with two or more earners.

Household Size: The percent of households that have: one person; two persons; three or four persons; five or more persons.

Education: The percent of households with a head of household with four or more years of college education.

Prior to this study, no attempt was made to insure that the forecasts of the indirect-age demographic variables were consistent with the forecasts of total population and its age structure. This chapter discusses the new model that forecasts

[79] See chapters 3 and 5 for a discussion on how these variables affect consumption.

these indirect-age variables. The new model insures that there is consistency between the age and total population forecasts and the forecasts of the various indirect-age demographic variables. Thus, when examining the effects of alternate population projections, the simulations will automatically account for changes in the indirect-age demographic variables.

Chapters 3 and 5 describe how the indirect-age and age demographic variables affect consumption patterns. Historical values for these age and indirect-age demographic variables are readily available, but forecasts are not obtained so easily. The Census Bureau publishes several forecasts of population and its age distribution (US Census Bureau, Current Population Reports Series P-25). Using these projections, one can easily obtain a forecast of the age variables used by the consumption equations. Obtaining projections of the indirect-age demographic variables is more difficult. Historical data are available from the Census Bureau (US Census Bureau, Current Population Reports Series P-20, P-23 and P-25), but forecasts are not so easily obtained.

Monaco *et al* (1996) and Dowd (1996a) describe the method used to project the size and composition of the population. Starting from basic demographic assumptions -- fertility rates, survival rates and immigration -- the model creates population projections by age and sex. The model uses the cohort component approach, and mimics the method used by the Census Bureau when it creates its projections. The model allows changes in some of these basic assumptions. Using these age projections and assumptions on labor force participation rates, the Demographics Projections Model, or DPM, forecasts the total labor force. DPM also forecasts, based on the size and age distribution of the population, the indirect-age demographic variables. It is this indirect-age demographic portion of DPM that was undertaken as part of this study.

These variables are termed indirect-age variables because they are indirectly determined by the age composition of the population. For example, the age of household head variables are determined by the age composition of the population.

This chapter is broken into two sections. The first discusses the historical trends since 1960 of the age variables. This section also describes the forecast of these historical trends. The second section describes the equations that were developed to forecast the indirect-age demographic variables. The second section also presents and discusses the forecasts of the indirect-age demographic variables.

Trends in the Age Structure of the Population

When examining the historical and forecasted values of the age variables, there are several features that immediately appear. Perhaps the most obvious is the aging is the baby-boom generation. Starting in 1946 and ending in 1964, the fertility rate in the US increased dramatically. The aging of this generation is seen very clearly in the indirect-age demographic variables. For example, in 1971, when the leading edge of the baby-boom was twenty-five, the number of households with heads aged twenty-five to thirty-four began a dramatic climb that began to slow in 1981, when the same leading edge was thirty-five years old. Until the last of this generation passes away, any variable using age as a defining factor will see the aging of this generation.

A second important factor is the increased number of births in the *circa*-1982 to 1995 period. This group is known as the baby-boomlet generation. The increase in births was a natural outgrowth of the aging of the baby-boom generation. However, unlike the baby-boom generation, this surge in births was not the result of an increase in fertility rates, but was caused by an increase in the number of women of child-bearing age.[80] This generation, like the baby-boom generation, is larger than the generations that proceed and follow, and, like the baby-boom generation, its aging can be tracked in the forecast.

A third factor is the increase in life expectancy. The over sixty-five population will rise dramatically as the baby-boom generation reaches this age, but the increase will persist for a long period because these persons are living longer lives. The surge begins in 2011 and finally levels off near the end of the forecast horizon. By 2050, the leading edge of the baby-boom will be over 100 years old and the entire generation will be eighty-six years or older. However, the leading edge of the baby-boomlet will be sixty-eight years old and in this age category. Thus, the decline in the share of the population aged sixty-five or older that one would expect to see does not occur.

These three features of the historical and projected population data are mentioned because the forecasts of both the age and indirect-age demographic variables may look odd to the reader if he is unaware of these three factors. The sharp increase in fertility during the baby-boom years coupled with the decline in fertility and births in the years between the end of the boom and the beginning of the boomlet gives the age structure

[80]The baby-boom generation occurred through an increase in the number of births per woman of child-bearing age. The baby-boomlet generation occurred through an increase in the number of women of child-bearing age.

of the population two peaks that march through **all** of the demographic variables. Over time, the echoes of the baby-boom will dampen, but during the period through 2050, these waves are quite visible.

Equations and Forecasts

There are fifteen indirect-age demographic variables that must be forecasted. Ten of these -- all four age of household head variables, all four region of residence variables, the percentage of households with a college-educated head, and the percentage of households with two or more earners -- are forecasted via regression equations. The total number of households is calculated as an identity and equals the summation of the four age of household head variables. The four household size variables -- single-person households, two-person households, three-or-four-person households, and households with five or more persons -- are forecasted via equations where the coefficients are not estimated by a statistical method but are deduced instead from Census publications.

For each indirect-age demographic variable, the equation and estimation results are presented and discussed. Following this discussion, there are several graphs showing the fit of the equation and the forecast of the variable. The forecast of each variable is discussed following the graphs.

A. Number of Households by Age of Head

There are four households-by-age variables. These are households where the head is: under twenty-five; twenty-five to thirty-four; thirty-five to fifty-four; and fifty-five or older. For all four categories of households, the equations forecast the number of households in the category. The sum of these four categories equals the total number of households.

The number of households in each category must undergo a two-step calculation before the consumption system can use the forecasts. The first calculation consists of summing the youngest two age categories -- the number of households with: heads under twenty-five and heads between twenty-five and thirty-four.[81] The second step is the conversion of the household head variables into shares. However, before summing any of the categories or converting the levels into shares, the number of households of each type must be forecasted.

Each household by age of head variable is a function of an intercept and the age cohorts that could head a household of that type. For example, the number of persons who are twenty to twenty-four years old is used in determining the number of households headed by persons under the age of twenty-five, but the number of persons in this age cohort is not used in determining the number of households headed by persons twenty-five years or older.

Unfortunately, the age variables are collinear and the estimated coefficients from an unconstrained estimation were unsatisfactory. For example, the unconstrained coefficients implied that persons aged thirty to thirty-four was only about twice as likely to head a household as persons aged fifteen to nineteen. This was deemed

[81]DPM forecasts the two variables because the construction equations used in the model use the twenty-five to thirty-four category (Monaco 1993).

TABLE 28

The Number of Households Headed by Persons Twenty-four Years Old or Younger

```
:
SEE    =      423.24 RSQ    = 0.8030 RHO =    0.76 Obser  =    36 from 1960.000
SEE+1  =      277.62 RBSQ   = 0.7911 DW  =    0.49 DoFree =    33 to   1995.000
MAPE   =        6.87
  Variable name              Reg-Coef  Mexval t-value  Elas   NorRes     Mean
0 headlt25                  - - - - - - - - - - - - - - - - - - - - -  4929.22
1 intercept               -100.58149     0.1  -0.229  -0.02     5.08      1.00
2 pop15to19                 19.03989   122.2  11.402   0.07     5.08     18.62
3 pop20to24                257.51843   125.3  11.597   0.95     1.00     18.15
```

Table 28 shows the estimation results from the equation that forecasts the number of households headed by persons twenty-four years or younger. The number of households headed by persons twenty-four years old or younger (headlt25) is a function of the number of people aged fifteen to nineteen (pop15to19) and the number of people aged twenty to twenty-four (pop20to24). Technically, while it is possible for a person under the age of fifteen to head a household, the number of households with these juvenile heads is negligible. For this reason, it is assumed that these persons do not head households.

Figure 34 shows the historical versus predicted values for the number of households headed by persons under twenty-five years old. The predicted values are denoted by a "+" and the actual number of households is plotted with "□". Figure 35 shows the forecast of the number of households headed by persons under the twenty-five years old.

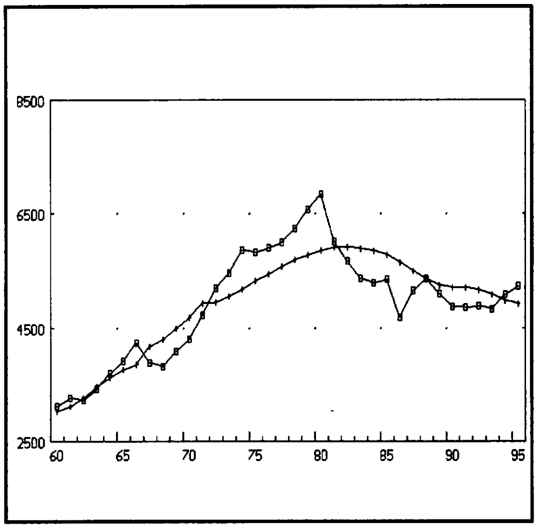


Figure 34

Number of Households:

Head under 25 (1000's)

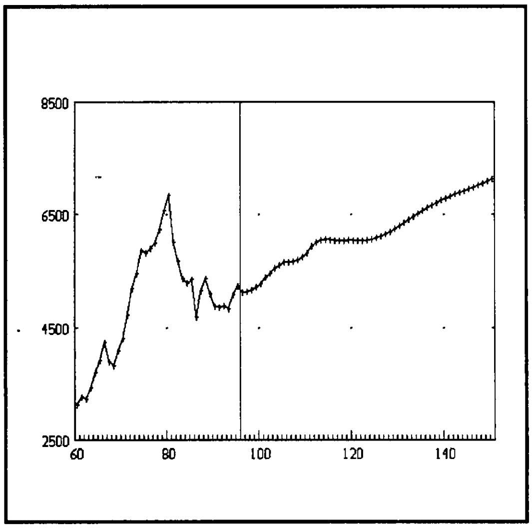


Figure 35

Forecast of Households:

Head Under 25 (1000s)

The number of households headed by persons under the age of twenty-five peaked in 1980 and continued to fall until the early 1990's. This decline should have been expected since the under-twenty population peaked in 1978 and the number of persons between twenty and twenty-four years old peaked in the early 1980's. This category of household will continue to fall until the late 1990's when the leading-edge of baby-boomlets reach the age of fifteen.

Starting in the late 1990's, this category of household will grow until the late 2010's, when the last of the baby-boomlet generation hits twenty-five. There will be little or no growth for the next ten years. Unlike the 1980's, there will not be a severe drop in these households. After the early 2020's the number of households in this category resumes growing and continues through the forecast horizon of 2050.

Table 29 shows the results for the equation that forecasts the number of households headed by persons twenty-five to thirty-four.

TABLE 29

The Number of Households Headed by Persons Twenty-five to Thirty-Four Old

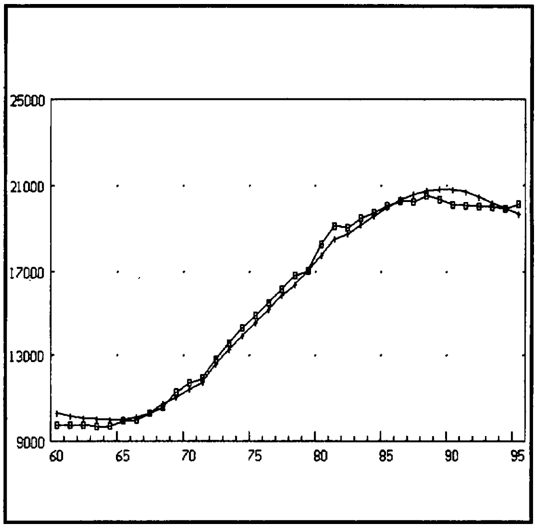
:

SEE = 355.06 RSQ = 0.9930 RHO = 0.83 Obser = 36 from 1960.000
SEE+1 = 216.47 RBSQ = 0.9925 DW = 0.33 DoFree = 33 to 1995.000
MAPE = 2.08

Variable name	Reg-Coef	Mexval	t-value	Elas	NorRes	Mean
0 head25to34	- - - - - -	- - - -	- - - -	- - -	- - - -	15623.96
1 intercept	-1534.14111	43.6	-5.924	-0.10	142.05	1.00
2 pop25to29	481.43071	1091.8	68.222	0.53	142.04	17.13
3 pop30to34	547.07943	1091.8	68.223	0.57	1.00	16.29

The number of households headed by persons twenty-five to thirty-four years old is determined by the number of persons twenty-five to twenty-nine and the number of persons thirty to thirty-four. Unlike the younger household category, the two age cohorts have approximately the same effect on the number of households.

Figure 36 shows the historical versus predicted values for the number of households headed by persons aged twenty-five to thirty-four. Figure 37 shows the forecast of the number of households in this category. The forecast of the number of households in this category is very similar to the forecast of the households headed by persons under twenty-five -- except the peaks and troughs of the twenty-five to thirty-four category occur approximately ten years after the under twenty-five category's peaks and troughs.



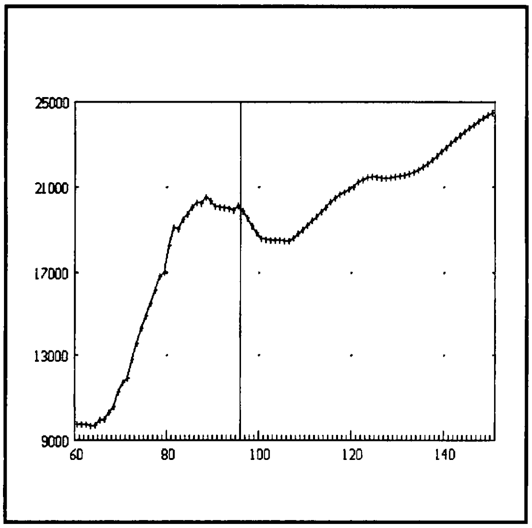


Figure 36

Number of Households:

Head Aged 25 to 34 (1000's)

Figure 37

Forecast of Households:

Head Aged 25 to 34 (1000s)

Households headed by persons twenty-five to thirty-four began to grow in the late 1960's and peaked in the late 1980's. The start of the increase coincides with the years that the leading edge of the baby-boom reached twenty-five. The late 1980's drop occurs approximately when the majority of the baby-boom generation has already entered the twenty-five to thirty-four bracket and the leading edge of the baby-boom has reached thirty-five. The number of households does not drop as sharply as the under twenty-five category because, at first, the in-flow of twenty-four year olds is approximately the same as the out-flow of thirty-four year olds. Figure 37 shows the forecast of households in this category. The number of households begins a steep decline in the late 1990's until reaching a trough just after 2000. This trough coincides with the years that the last of the baby-boom reach thirty-four. The growth in the number of households in this category is near zero until the mid-2000's, when the leading-edge of the baby-boomlet reaches twenty-five. As is expected, there is a second, smaller trough that occurs about twenty years after the first trough. This second trough occurs in the years just after the baby-boomlet has turned thirty-five, or exited the category.

Table 30 shows the results for the equation that forecasts the number of households headed by persons thirty-five to fifty-four.

TABLE 30

The Number of Households Headed by Persons Thirty-five to Fifty-four Years Old

```
:
SEE    =    1035.75 RSQ   = 0.9642 RHO =    0.94 Obser  =    36 from 1960.000
SEE+1  =     407.56 RBSQ  = 0.9621 DW  =    0.11 DoFree =    33 to   1995.000
MAPE   =       3.25
  Variable name            Reg-Coef  Mexval t-value  Elas   NorRes      Mean
0 head35to54             - - - - - - - - - - - - - - - - - - - - -  27988.41
1 intercept             -5728.92913    32.6  -5.005  -0.20   27.96      1.00
2 pop35to44               637.74885   428.8  29.829   0.65   27.96     28.59
3 pop45to54               655.62831   428.8  29.827   0.55    1.00     23.62
```

The number of households headed by persons aged thirty-five to fifty-four is determined by the number of persons aged thirty-five to forty-four and the number of persons aged forty-five to fifty-four.

Figure 38 shows the historical versus predicted values for the number of households headed by persons aged thirty-five to fifty-four. Figure 39 shows the forecast of the number of households in this category.

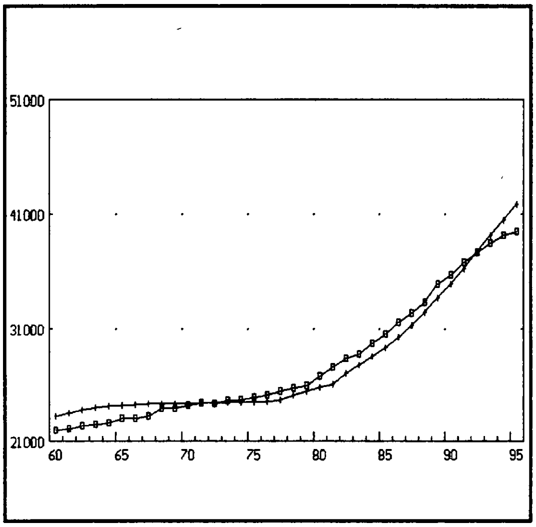


Figure 38

Number of Households:

Head Aged 35 to 54 (1000s)

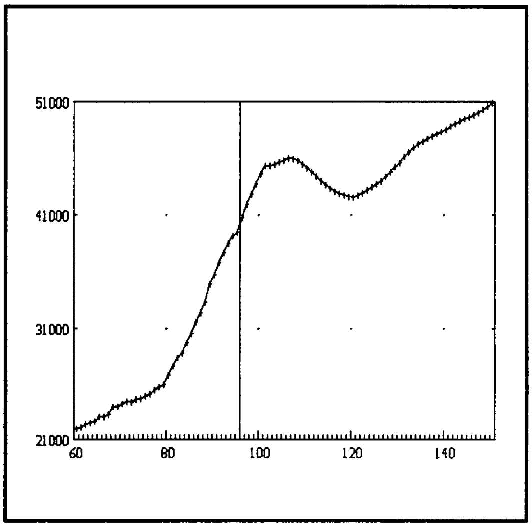


Figure 39

Forecast of Households:

Head Aged 35 to 54 (1000s)

The number of households with heads thirty-five to fifty-four has increased since 1960, but the rate of growth increased in 1980 as the leading-edge of the baby-boom reached thirty-five. The rate of growth has decline slightly in the post-1989 period but households in this category will continue growing until just after 2000. The decline will slight until around 2010, when the majority of the baby-boom has exited this age bracket. The decline will continue until reaching a trough around 2020 when the leading-edge of the baby-boomlet generation reaches this age bracket.

Like the previous household by age of head forecasts, the passage of the baby-boomlet generation is apparent as the growth rate levels off around 2030. The growth rate is still positive through the forecast horizon of 2050 but there is no third wave in this forecast. The absence of a third wave is because the children of the baby-boomlet do not reach the age of thirty-five before we reach the forecast horizon.

Table 31 shows the results for the equation that forecasts the number of households headed by persons fifty-five and older

TABLE 31

The Number of Households Headed by Persons Fifty-five and Older

:

SEE = 294.43 RSQ = 0.9966 RHO = 0.46 Obser = 36 from 1960.000
SEE+1 = 264.66 RBSQ = 0.9961 DW = 1.08 DoFree = 31 to 1995.000
MAPE = 0.90

Variable name	Reg-Coef	Mexval	t-value	Elas	NorRes	Mean
0 headover55	- -					26835.91
1 intercept	-4937.54682	180.0	-14.560	-0.18	291.24	1.00
2 pop55to64	708.10176	1606.5	94.853	0.52	291.24	19.83
3 pop65to69	772.25862	1606.5	94.850	0.24	291.23	8.29
4 pop70to74	796.02039	1606.5	94.850	0.19	291.21	6.55
5 pop75over	629.68775	1606.5	94.850	0.23	1.00	9.71

The number of households headed by persons fifty-five or older is determined by four age groupings: fifty-five to sixty-four, sixty-five to sixty-nine, seventy to seventy-four and seventy-five and older.

Figure 40 shows the historical versus predicted values for the number of households headed by persons fifty-five or older. Figure 41 shows the forecast of the number of households in this category.

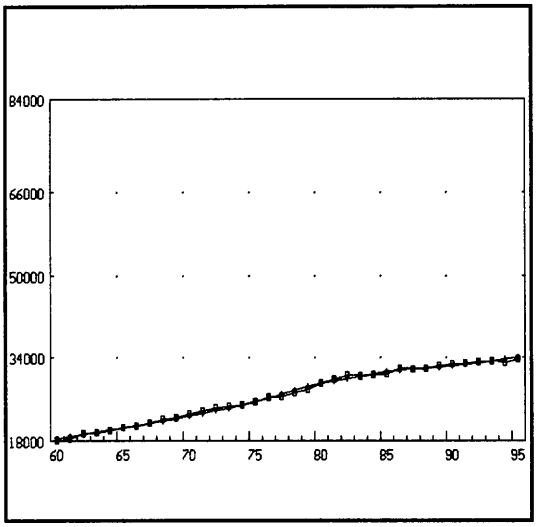


Figure 40

Number of Households:

Head 55 or Older (1000s)

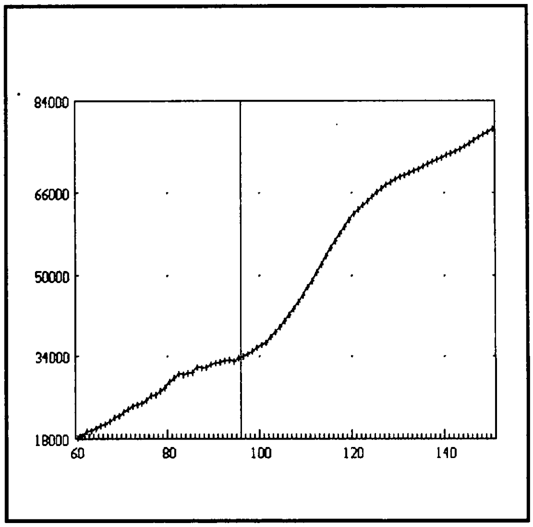


Figure 41

Forecast of Households:

Head 55 or Older (1000s)

The number of households with heads fifty-five or older has gradually increased since 1960. The baby-boom wave is not present in the historical data because the leading edge of the baby-boom generation will not reach the age of fifty-five until 2001. However, one can see the aging of the baby-boom in the forecast.

Beginning in 2001, the growth rate of households in this category dramatically increases and does not slow until 2020. The period 2001 to 2020 corresponds with the years that the majority of the baby-boom generation reaches fifty-five. The baby-boom generation continues to enter this category until around 2229. However, unlike the other household head categories, there is no trough following the slowing growth.

There are two reasons why there is no trough. The first is that these persons do not abruptly exit to the next older category but die away at only a few percent per year. The second is that this category is much longer than the previous categories.

For example, in 2009, all of the persons born in 1955 are in the household head category thirty-five to fifty-four, but in 2010, all of these persons are no longer in that category but are now in the over fifty-five household head category. Over time, the number of persons born in 1955 who remain alive will decline, but there is no abrupt exit and instead their number gradual decays.

The second reason there is no trough in the forecast is that this category contains households over a forty year age range.[83] The other categories hold ten or twenty years. Thus, even if persons in this category were to age into an older category of households, they remain in this household category for a very long time. In fact, when the leading-edge of the baby-boom is eighty-five the leading edge of the baby-boomlet is starting to near fifty-five. Thus, during the years in which the death rate of the baby-boom generation increases, a large influx of baby-boomlets begins.

[83]Technically, the range of this group is infinite since there is no upper bound. However, saying that it has a range of forty years covers all householders under ninety-five years old.

Figure 42 shows the forecast of the total number of households.

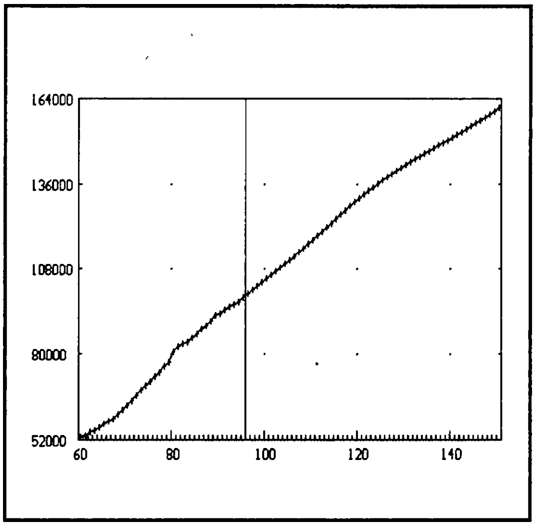


Figure 42

Number of Households:

Total (1000s)

The forecast for the total number of households is made by summing all four household head categories. The forecast is somewhat unremarkable and appears as if the total was forecasted by simply continuing the recent trend for the total. In 1980, there was a slight increase in the growth rate in the total number of households that can be attributed to the jump in the number of households headed by persons aged twenty-five to thirty-four.

The forecasted growth rate slows around 2020. Prior to 2020, the annual growth rate in the number of households is above one percent for every year except 2000. The average annual growth rate for the period 1995 to 2020 is 1.16 percent per year. After 2020, all of the annual growth rates are below one percent per year and tend to fall each year. The average annual growth rate for the period 2020 to 2030 is 0.80 percent per year and the average annual growth rate for the period 2040 to 2050 is 0.61 percent per year.

The decline in the growth rates is caused by the slowed growth in households headed by persons twenty-four and younger and a very slow decline in the growth rate of households headed by persons fifty-five or older.

Figures 43 through 45 show the forecasts of the household variables used in the consumption system. The consumption system does not use the number of households in a given category, but instead uses the share of total households in the category. In addition, the consumption system uses the share of households headed by persons thirty-four years or younger and so the forecast of this variable is constructed by summing under twenty-four and the twenty-five to thirty-four household categories.

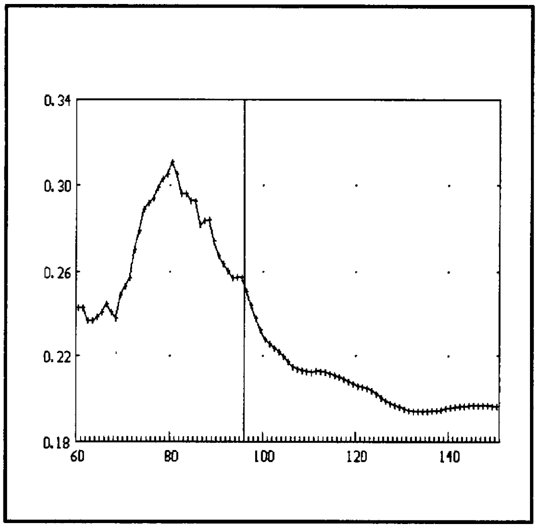


Figure 43

Share of Heads Under 34

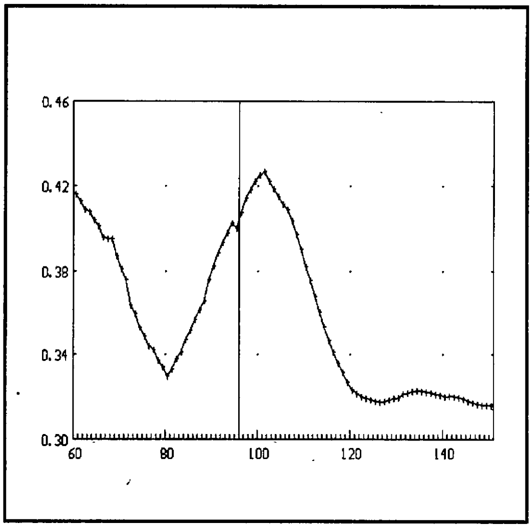


Figure 44

Share of Heads 35 to 55

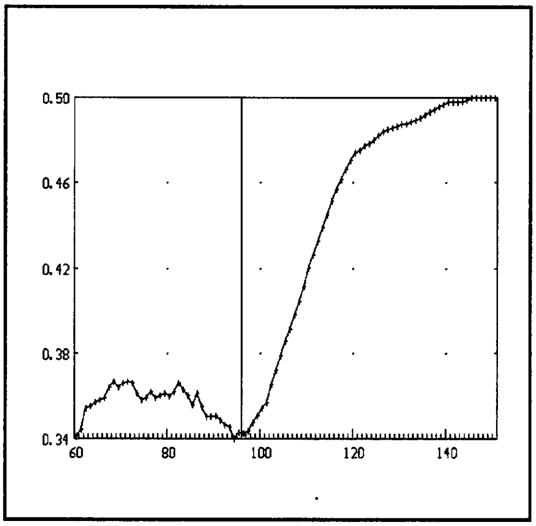


Figure 45

Share of Heads 55 and Older

When expressed as a share of total households, one can easily see the aging of the baby-boom in the household head variables. The youngest category, heads thirty-four or younger, peaked as a share in 1980 and will fall through 2030. The decline slows in the post-2000 period and the share is approximately constant in after 2030.

The middle category, heads thirty-five to fifty-four, fell over the period 1960 to 1981. From 1981 through 2000, the share of households in this category will increase -- peaking in 2002. After 2002, this share will fall through 2020 when the decline is arrested. From 2020 through the early 2030's, the share remains constant remains constant, but in the mid- and late 2030's, the share falls until 2040 when the fall is stopped.

However, the share of households headed by persons fifty-five or older increases dramatically during the 2000 to 2020 period. After 2020, the growth declines each year until growth is negligible in the late-2040's. The growth in this share is caused by one major factor -- as a share of total population, the baby-boom generation is larger than any generation forecasted to occur through 2050.

When this generation reaches fifty-five, the over fifty-five population and households headed by persons over fifty-five will dramatically increase. The over fifty-five household category will increase as a share because there is no younger generation creating households as rapidly as the baby-boom generation. This means that the two younger categories must decline and the oldest category will increase. This will have a dramatic effect on consumption since these older households typically do not purchase households durables at the same rate that younger households purchase durables.

B. Two-Earner Households

The number of two or more earner households is forecasted as a function of three age groups: the number of dependents -- defined as persons under twenty or over seventy-five; the number of persons aged twenty to thirty-nine; and the number of persons aged forty to seventy-four.

It was thought that an increase in the under twenty population should decrease the number of two or more earner households because of a tendency for one parent to exit the work force with the birth of a child. The effect of an increase in the over seventy-four population was thought to decrease the number of households with two or more earners. One reason for this is that these persons typically are not in the workforce, so an increase in this population translates into an increase in the number of households with no earners.

My *a-priori* belief was that an increase in either of the other two population groups should increase the number of households with two or more earners. There was no *a-priori* belief regarding the relative effects of the two age groups on the number of two or more earner households.

Table 32 shows the estimation results.

TABLE 32

The Number of Households with Two or More Earners

```
:
SEE    =      554.68 RSQ    = 0.9975 RHO =     0.47 Obser  =    36 from 1960.000
SEE+1  =      492.54 RBSQ   = 0.9972 DW  =     1.06 DoFree =    32 to   1995.000
MAPE   =        1.33
  Variable name            Reg-Coef  Mexval t-value  Elas   NorRes      Mean
0 ntwoy                    - - - - - - - - - - - - - - - - - - - - -  30092.79
1 intercept            -44314.16850   136.7 -12.136  -1.47  397.44       1.00
2 ndependent              -70.22133     2.1  -1.155  -0.19  256.88      83.49
3 pop20to39               317.57456   182.7  14.959   0.70   12.03      66.50
4 pop40to74               822.08020   246.8  18.787   1.97    1.00      71.95
```

The coefficients on all three independent variables have the correct sign. The coefficient on the dependent population (ndependent) -- the under twenty and seventy-five and older population -- is not significant, but the other coefficients are significant. An increase in the population aged forty to seventy-four has a larger effect on the number of households with two or more earners than does an increase in the population aged twenty to thirty-nine.

There are several reason for this result. The younger grouping tends to live in single person households more frequently than the older grouping and by definition a single person household cannot contain two or more earners. The younger grouping also contains persons who are more likely to not participate in the labor force for a variety of reasons including the pursuit of collegiate and graduate degrees and a greater probability of having a young child in the household.

Figure 46 shows the historical versus predicted values for the number of households with two or more earners. Figure 47 shows the forecast of the number of households in this category. Figure 48 shows the forecast of the share of households with two or more earners.

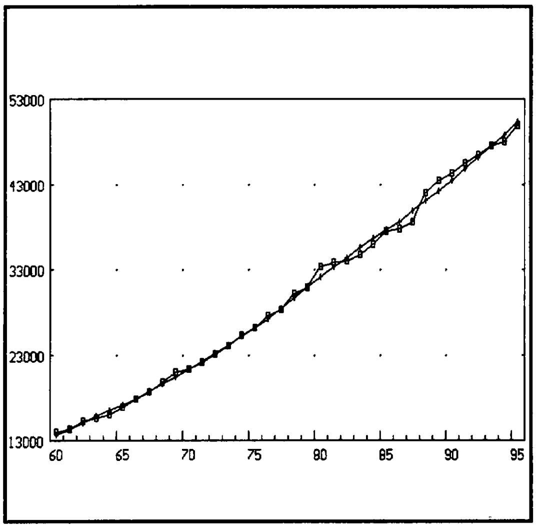


Figure 46

Households with 2 or More Earners

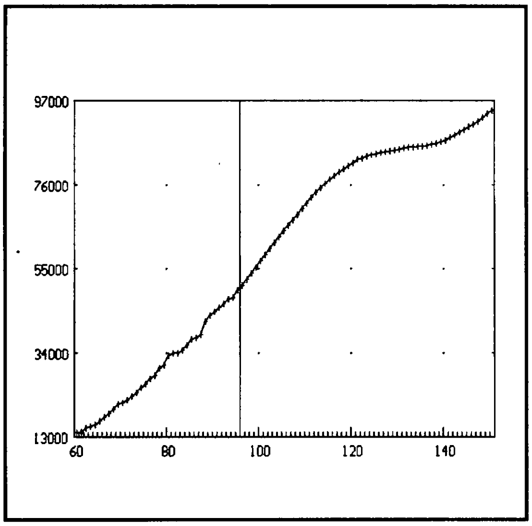


Figure 47

Forecast of 2+ Earner Households

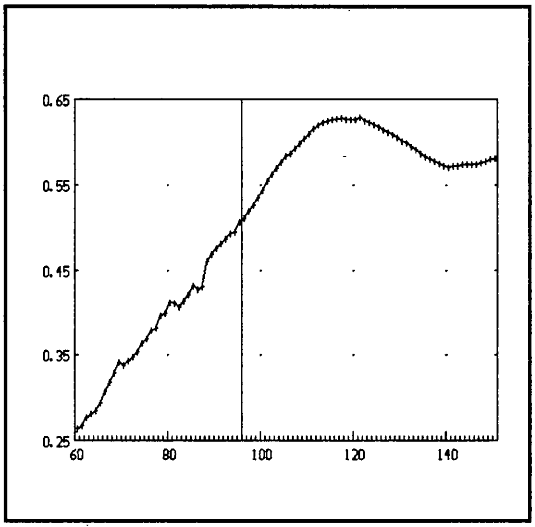


Figure 48

Forecast of Share

The number of two or more earner households has risen steadily during the 1960 to 1995 period. This trend is forecasted to continue until 2020 when there will be some leveling off caused by the increased number of elderly persons. Around 2040 the growth rate again increases as the share of the non-dependent population increases slightly.

The share of households with two or more earners will peak in the 2010 to 2020 period and will fall as the baby-boom increasingly leaves the work force. The share will fall until 2040 when there will be a slight increase in this share. This is probably due to the slowing in the growth of the elderly population and an increase in the growth rate of persons in the working population.

C. College-Educated Heads

The number of households with heads with four or more years of college education is forecasted as a function of the number of persons twenty-five years or older with four or more years of college. Historical data on the over-twenty-five population with four or more years of college is from the Current Population Reports Series P-20. The data is reported for five-year age groups.[84] The historical data is readily available, but since this will be used in the forecasting model, the population college-educated population twenty-five years or older must now forecasted.

The total number of persons with a college-education (allcoll) equals the number of persons in each of the five-year age brackets that has a college-education. The number of persons within the five-year bracket that have a college-education equals the product of the population in that five-year bracket and the percentage of persons in that bracket with a college-education. Since the population numbers are forecasted by DPM, only the percentage of persons in an age bracket that are college-educated must be forecasted.

This forecast is fairly easy to construct since the percent of persons aged fifty to fifty-four in 1998 with a college-education is determined by the percent of persons aged forty-five to forty-nine in 1993 with a college-education plus an adjustment to

[84]Persons twenty to twenty-four, twenty-five to twenty-nine, and so-on.

account for new persons receiving degrees. Thus, in any year the percent of persons in the cohort with a college-education equals:

$$\text{Percent}_{g,t} = \text{Percent}_{g-1,\,t-5} + \text{Adjust}_g \qquad (6.1);$$

where:
$\text{Percent}_{g,t}$ = Percent of g^{th} cohort with college-education, year t.
Adjust_g = Adjustment factor for new college graduates.

The adjust factor accounts for any persons in the cohort being recent college graduates. Only three of the five-year age groups use an adjustment factor -- persons aged twenty-five to twenty-nine, persons aged thirty to thirty-four and persons aged thirty-five to thirty-nine. In each case, the adjustment factor equals 1.0.

Using equation (6.1), the forecast of college-educated persons can be constructed, provided that the percent of persons aged twenty-five to twenty-nine is forecasted through 2050. The growth in this ratio, college-educated to population, virtually has ceased during the last several years and it appears that an upper-bound of some type has been reached with the ratio hovering near .23 and .24. No equation is used to forecast this ratio, instead an exogenous assumption that this ratio will increased linearly to .25 by the year 2050. Figure 49 shows the forecast of the college-educated population over twenty-five.

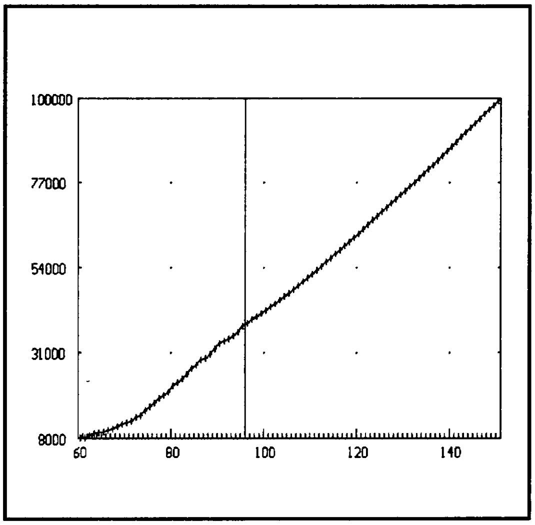


Figure 49

College-Educated Population

Table 33 shows the estimation results for the number of households with a college-educated head.

TABLE 33

The Number of Households with a College-Educated Head

```
:
SEE   =      509.28 RSQ   = 0.9922 RHO =   0.82 Obser  =   36 from 1960.000
SEE+1 =      313.87 RBSQ  = 0.9919 DW  =   0.36 DoFree =   34 to   1995.000
MAPE  =        3.03
  Variable name          Reg-Coef  Mexval t-value  Elas   NorRes      Mean
0 ncollege             - - - - - - - - - - - - - - - - - - - - -  13437.96
1 intercept            1176.59306    39.9   5.707  0.09  127.80       1.00
2 allcoll                 0.59594  1030.5  65.659  0.91    1.00   20574.74
```

The coefficient on collegepop says that about six hundred of every thousand persons in the twenty-five and older college-educated population (collegepop) head a household. Figure 50 shows the fit of the equation. Figure 51 shows the forecast of the number of households with a college-educated head.

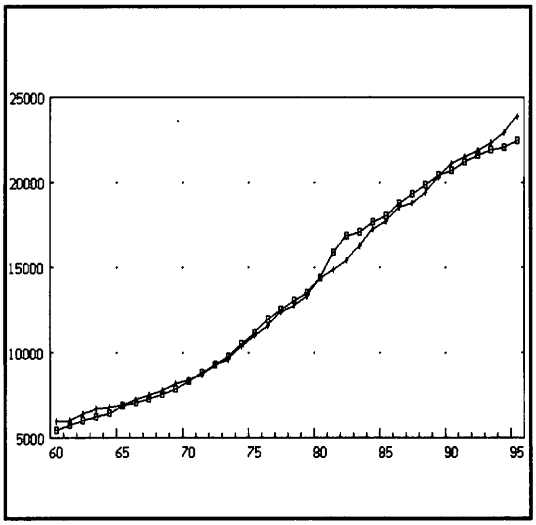


Figure 50

Historical vs. Predicted:

College-Educated Heads

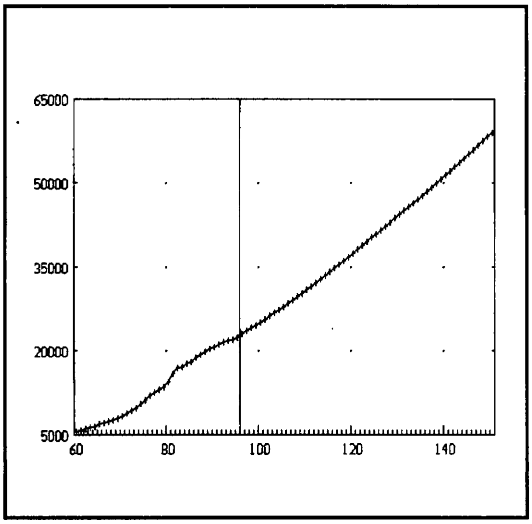


Figure 51

Forecast:

College-Educated Heads

D. Region of Residence

There are four possible regions in which a household may live: North East, North Central, South and West. There are many factors that determine whether a household relocates. Among these are the age of the household members and economic factors. The long-term model is a model of the aggregate U.S. economy and does not forecast any regional variables. Developing a model by region is not within the scope of this study. Consequently, no regional economic variables could be used in forecasting the region of residence variables.

Experimentation showed that attempting to forecast the region of residence variables using the age distribution of the population generated implausible coefficients and ridiculous forecasts. In addition, while it is clear that elderly persons tend to relocate to warmer climates, the relocation decision of the other age groups is probably determined by economic variables which are unavailable in the forecast. For this reason, time trends were used to forecast the four region- of-residence variables.

The variables are shares and must sum to one. This imposes cross-equation constraints on the parameters. Namely, the sum of the intercept terms must equal zero and the sum of the time coefficients must equal one. In addition, the cross-equation constraints suggest that a seemingly-unrelated regression (SUR) be performed to account for any correlation across equations in the error terms. However, because the same variables are used in each equation, the ordinary-least squares estimators are identical to the SUR estimators and the constraints on the coefficients are automatically satisfied.

Tables 34 through 37 show the estimation results for the four regional variables.

TABLE 34

The Share of Households in the North East

```
:
SEE    =        0.00 RSQ    = 0.9726 RHO =    0.93 Obser  =    36 from 1960.000
SEE+1  =        0.00 RBSQ   = 0.9718 DW  =    0.14 DoFree =    34 to   1995.000
MAPE   =        1.12
  Variable name            Reg-Coef  Mexval t-value  Elas    NorRes      Mean
0 neast                  - - - - - - - - - - - - - - - - - - - - -      0.22
1 intercept                 0.26369  3577.3 214.344   1.17    36.51      1.00
2 tiempo                   -0.00166   504.2 -34.748  -0.17     1.00     23.50
```

TABLE 35

The Share of Households in the South

```
:
SEE    =        0.00 RSQ    = 0.9482 RHO =    0.93 Obser  =    36 from 1960.000
SEE+1  =        0.00 RBSQ   = 0.9466 DW  =    0.15 DoFree =    34 to   1995.000
MAPE   =        1.01
  Variable name            Reg-Coef  Mexval t-value  Elas    NorRes      Mean
0 south                  - - - - - - - - - - - - - - - - - - - - -      0.33
1 intercept                 0.29057  3059.4 184.134   0.89    19.29      1.00
2 tiempo                    0.00153   339.3  24.940   0.11     1.00     23.50
```

TABLE 36

The Share of Households in the West

```
:
SEE    =        0.00 RSQ    = 0.9879 RHO =    0.92 Obser  =    36 from 1960.000
SEE+1  =        0.00 RBSQ   = 0.9875 DW  =    0.16 DoFree =    34 to   1995.000
MAPE   =        0.94
  Variable name            Reg-Coef  Mexval t-value  Elas    NorRes      Mean
0 west                   - - - - - - - - - - - - - - - - - - - - -      0.19
1 intercept                 0.14496  2808.3 169.484   0.78    82.40      1.00
2 tiempo                    0.00175   807.7  52.608   0.22     1.00     23.50
```

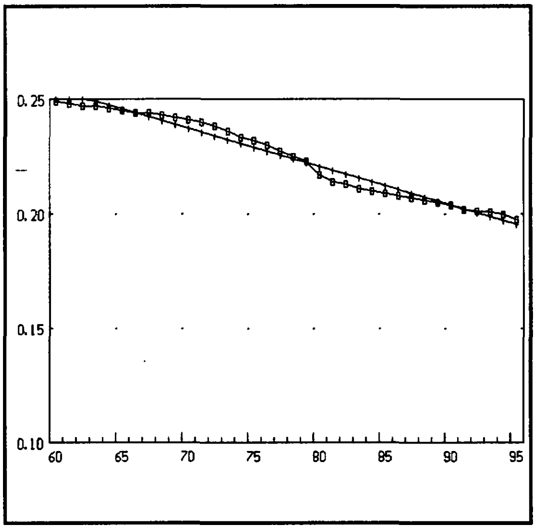
TABLE 37

The Share of Households in the North Central

```
:
SEE   =        0.00 RSQ   = 0.9728 RHO =   0.94 Obser  =   36 from 1960.000
SEE+1 =        0.00 RBSQ  = 0.9720 DW  =   0.11 DoFree =   34 to   1995.000
MAPE  =        0.91
  Variable name         Reg-Coef  Mexval t-value  Elas   NorRes      Mean
0 ncent                 - - - - - - - - - - - - - - - - - - - -      0.26
1 intercept              0.30078  4228.5 252.323   1.14   36.83      1.00
2 tiempo                -0.00162   506.9 -34.902  -0.14    1.00     23.50
```

The four region of residence variables are forecasted as a share of households living in a given region. Each is a function of an intercept and a time trend. The forecast is for an increasing share of households to live in the west and the south and for a decreasing share of households to live in the north east and north central.

Figures 52 through 59 show the fit of the equation and the forecast for each regional variable.



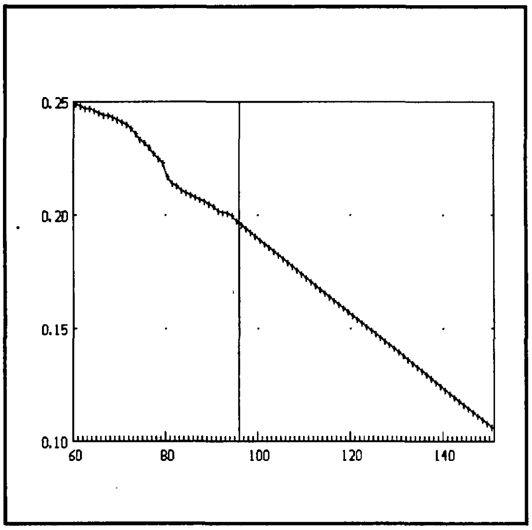


Figure 52

Region of Residence: North East

Figure 53

Forecast: North East

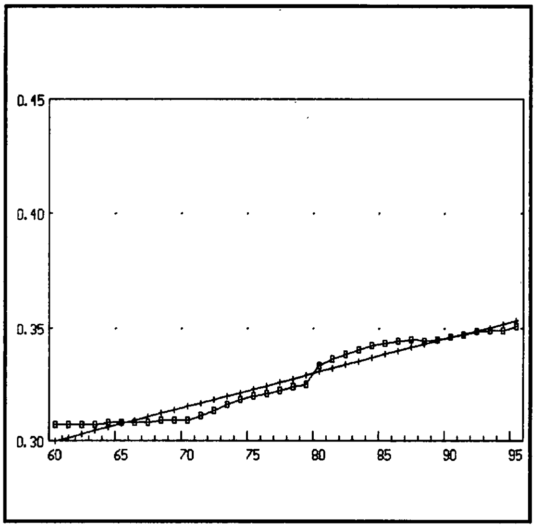


Figure 54

Region of Residence: South

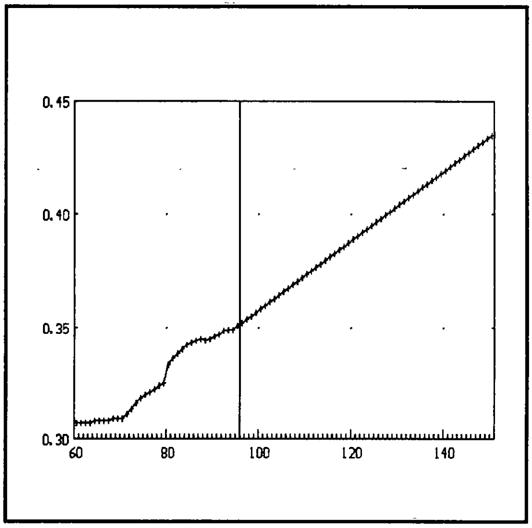


Figure 55

Forecast: South

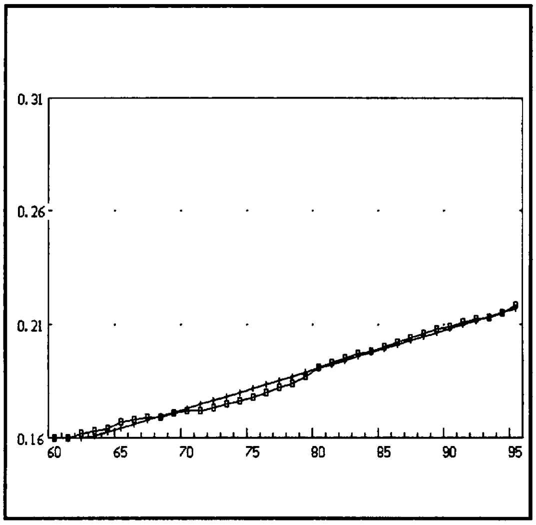


Figure 56

Region of Residence: West

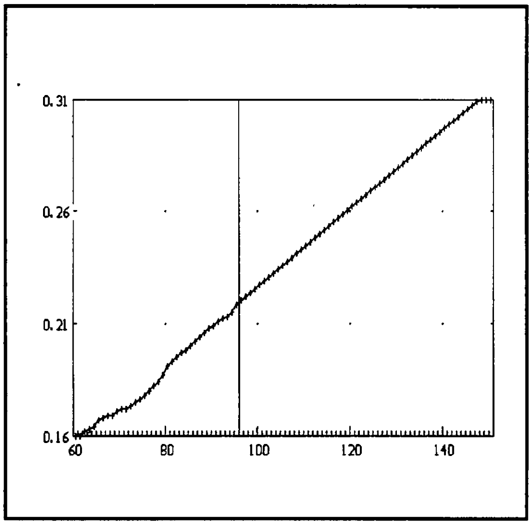


Figure 57

Forecast: West

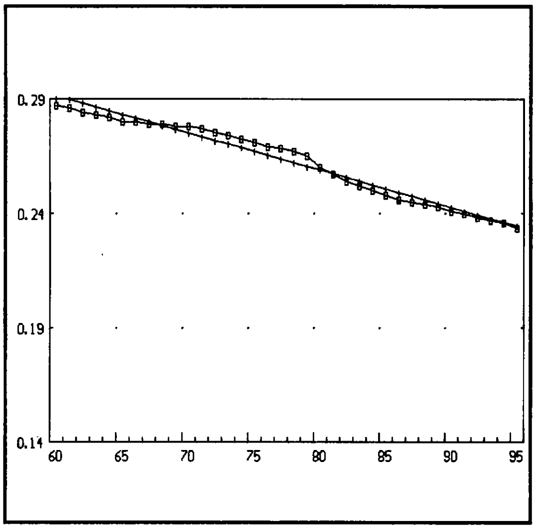


Figure 58

Region of Residence: North Central

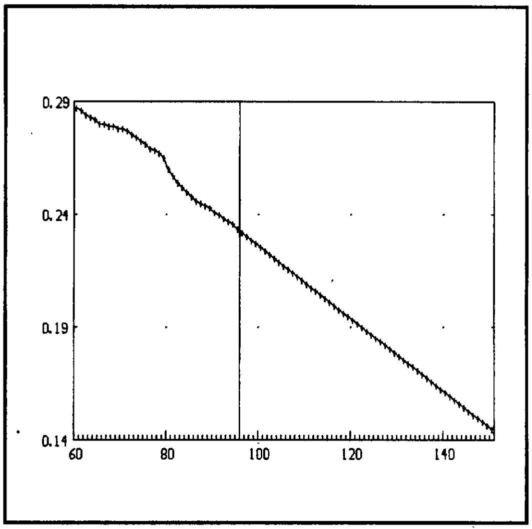


Figure 59

Forecast: North Central

E. Household Size

The four household size variables -- one person, two person, three or four person and five or more person -- are forecasted using coefficients based on Current Population Reports Series P-20 data. The number of persons living in each household size category is a function of the three populations variables: the number of persons fifteen or younger, the number of persons sixteen to sixty-four, and the number of persons sixty-five or older.

Attempts to estimate equations to forecasts the household size variables were unsuccessful. Several specifications were investigated and in each case, the estimated coefficients were implausible -- e.g. implying that a person of a certain age lived in several households or that none of the age group lived in a household -- and generated forecasts that were fanciful at best. One reason for the "bad" estimated coefficients is that most of the age variables, even when joined in different combinations, are so collinear that the estimated coefficients are not reliable.

Because the attempts to estimate parameters by regression methods were unsuccessful, an informal estimation procedure was used. Data in the Current Population Reports Series P-20 is reported in a form that allows the construction of a table that shows the age distribution of seven family size variables -- one, two, three, four, five, six, and seven or more person households.

The "raw" data is reported by age of household head and the number of households of each size headed by persons in that age bracket. The data also reports the number of persons under the age of eighteen, the number of persons eighteen to sixty-four, and the number of persons sixty-five or older that live in households headed by persons in the age bracket. Thus, we have a table that shows the distribution of household size by age of household head and a table that shows the age distribution of the population by age of household head.[85] Unfortunately, what we require is the union of these two tables or, in other words, the age distribution of the population by household size.

By making several assumptions regarding the two distributions, we can use the two available tables to construct the unavailable table that is needed. The first assumption is that none of the households headed by persons under twenty years old

[85]The age of household head Series P-20 data is reported by households with heads: under twenty, twenty to twenty-four, twenty-five to twenty-nine, thirty to thirty-four, thirty-five to thirty-nine, forty to forty-four, forty-five to fifty-four, fifty-five to sixty-four, sixty-five to seventy-four, seventy-five to eighty-four, and eighty-five and older.

is headed by a person under the age of eighteen. The second assumption is that the average number of persons living in households with three or four people equals 3.5. The third assumption is that the average number of persons living in households with five or more persons equals six. The final assumption is that the number of persons in the k^{th} age category (under eighteen, eighteen to sixty-four and over sixty-five) living in the i^{th} household size and with the j^{th} head equals the product of the share of people living in the i^{th} household with the j^{th} head and the total number of persons in the k^{th} age category. This is written as:

$$\text{Persons}_{k,i,j} = (\textstyle\sum_k \sum_j \text{Persons}_{k,i,j}) * (\text{Persons}_{i,j})/(\sum_i \text{Persons}_{i,j}) \qquad (6.3).$$

The first assumption, that no persons under the age of eighteen head a household, is needed because of the way that the Series P-20 data is tabulated. The Series P-20 data has data on the number of persons under the age of eighteen and if we allowed some of these persons to head a household, then any change in the under eighteen population would change the number of households in the economy. Since an increase in the population of minors usually occurs because of changes in the number of births, not making this assumption would imply that newborns head a household. This implication is absurd and the assumption that no persons under the age of eighteen head a household prevents it.

One problem with making this assumption is that almost certainly there are households with heads aged fifteen to seventeen. By not allowing these persons to head a household, the effect of a change in the number of persons fifteen to seventeen is not included.

The second assumption, that the average number of persons living in a household with three or four persons equals 3.5, is based on the time-series of this average. Since 1960, this average has ranged between 3.4 and 3.6 and has tended to remain fairly close to 3.5.

The third assumption, that the average number of persons living in a household with five or more persons equals six is based on the time-series of this average. Since 1960, this average has fallen from a peak of slightly above 6.6. The average has tended to decline since the peak and it appears that it is falling to the neighborhood of an average of six persons. Figure 60 shows the historical values of the average number of persons in households with five or more persons:

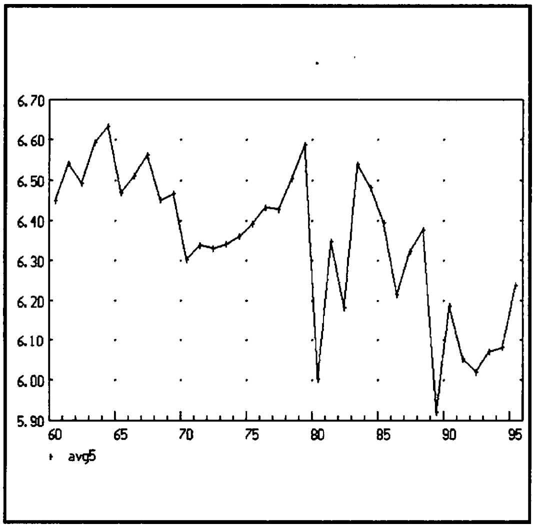


Figure 60

Average # of Persons

Size 5+ Households

As can be seen in the graph, there were two years that the average was below six, but the general trend has been for the average to fall and remain somewhere near six persons.

The solution to equation (6.3) gives us an approximation of the age distribution by household size. If the solution to equation (6.3) for the k^{th} age grouping is divided by the total number of persons in the k^{th} age group, the result is the proportion of persons in the k^{th} age group that live in the i^{th} household size and this proportion can be used as the coefficient that determines the number of persons in the k^{th} age group that live in the i^{th} household.

Several years worth of tables were created and based on the coefficients from these tabulations, the DPM coefficients are specified exogenously. The coefficient for any particular age group and household size varies from year to year, but they tend to remain in the neighborhood of the values given in table 38.

TABLE 38

Household Size Membership Coefficients by Age

Effect of 1000 Additional Persons in Age Group

	Size = 1	Size = 2	Size = 3 or 4	Size = 5+
Age under 15	0	185	525	290
Age 15 to 64	90	230	440	240
Age 65 and over	310	445	165	80

From this table, we see that no person under the age of fifteen lives alone and that about half of these minors live in a household with three or four persons. Among the non-elderly adult population, persons aged fifteen to sixty-four, about one-tenth live alone and about half live in a household with three or four persons. The elderly, unlike the other age groupings, tend to live in households with one or two people. Approximately one-third of the elderly live alone and one-half live in a two person household.

The values reported in table 38 are used as the coefficients in DPM. With the total number and average number of persons in each household size known, the number of households in each household size category is known. The share of households in each category equals the number of households in the size category divided by the sum of the number of households in the four size categories.

Figures 61 through 64 show the forecast of these shares.

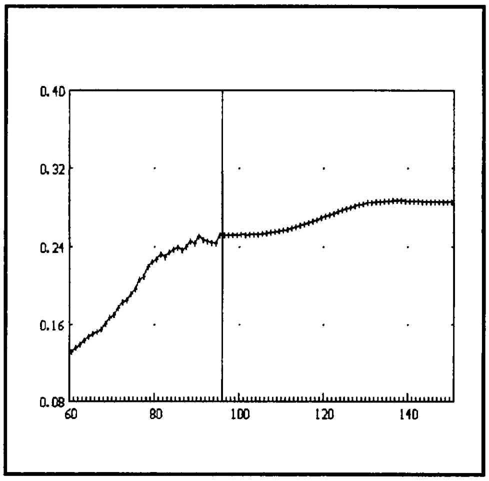


Figure 61

Share of Size = 1 Households

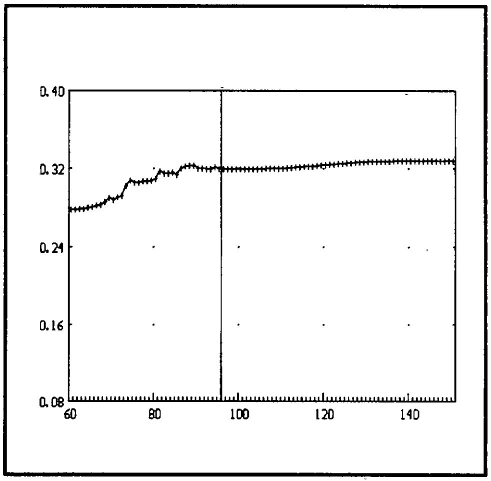


Figure 62

Share of Size = 2 Households

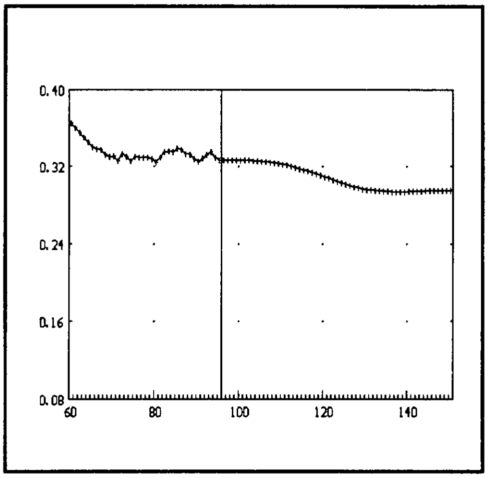


Figure 63

Share Size = 3 or 4 Households

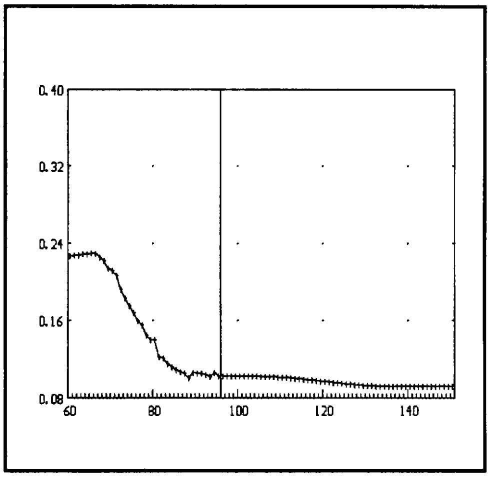


Figure 64

Share Size = 5+ Households

The forecast is for all four of the household sizes to remain approximately constant. The large movements seen in the 1960 to 1995 period are not present in the forecast for one major reason -- the forecast does not include a second baby-boom. One can easily detect the aging of the baby-boom generation in the household size variables.

The share of households with five or more persons peaked in the early 1960's, just as the leading edge of the baby-boom reached adulthood. As the baby-boom aged, they left the large households where they spent their childhood and formed single-person or two-person households. The decline in the share of households with three or four persons also shows the aging of the baby-boom, but the drop in this share is not as steep. One reason this share does not decline as rapidly as the share of households with five or more persons is because most of the households that left the five or more category immediately entered the three or four person category.

During the next fifty or so years, the share of single person households will increase as the baby-boom ages. Since approximately three-fourths of the elderly live in either a single person or two person household, the increased number of elderly will cause the other two household size shares to decline as well. The greater impact occurring in households with three or four persons. The movement in these shares is a natural consequence of the aging of the baby-boom generation.

Concluding Remarks

Using DPM, the implications of various population assumptions can finally be modeled in a consistent manner. Prior to this work, simulations modeling various population projections could not account for the effects of changes in the indirect-age demographic variables and thus could not correctly model the effects on consumption expenditures of the alternate population projections.

With the description of the indirect-age demographic model complete, we can now turn our attention to the simulations that illustrate the importance of the new work.

CHAPTER 7

SIMULATIONS HIGHLIGHTING THIS WORK

As previously mentioned, the work done in this study will be used in LIFT, a long-term macroeconometric forecasting model. One purpose of this study is improving the simulation properties of LIFT. This study improves the simulation properties in three major areas. The first area, described in chapter 5, is the new treatment of Medicare as a price subsidy. The second area, described in chapter 6, is the non-age demographics model that allows LIFT to project the effects of changes in the size and age structure of the population. The improvement in the modeling of the distribution of income is the third area.

Prior to this study, LIFT treated Medicare as an income transfer and changes in Medicare affected PCE by increasing disposable income. By treating Medicare as an income transfer, the initial effect of a change in the size of the program occurs through an income effect. Since the initial effect comes through the income effect, the PCE categories that are influenced most are those with the largest income elasticities and not necessarily the medical goods and services categories. The work done in this study treats the Medicare program as a price subsidy to the health care PCE categories. Under the Medicare-as-a-price-subsidy approach, the initial effect of a change in the size of the Medicare program occurs through a change in the perceived price of most of the medical goods and services categories. The change in the price of these sectors means that if Medicare decreased, the relative price of medical goods and services increased and spending in these categories will be affected directly.

The immediate effects of a change in the Medicare program on PCE is called the primary or first round effect. However, in both versions of the model, there will be secondary effects from the change in the Medicare program. These secondary effects occur because a change in the composition of PCE affects many of the other variables in the model. For example, the primary effect of a decrease in the size of the Medicare program under the new treatment is an increase in the relative price of medical goods and services that causes PCE to fall in these categories. However, a decrease in the size of Medicare decreases the size of the Federal deficit, causing a decline in government interest payments to persons. Since interest payments are a component of personal income, the decline in the deficit causes personal income to decline. The link continues through disposable income until total PCE falls because of the decline in interest payments from the government. This secondary effect is akin to a Keynesian multiplier.

An additional secondary effect occurs because of the "goods mix" effect. In LIFT, as well as in any model in which total income is generated by aggregating income-by-industry, the level of income depends upon the mix of goods purchased. Increased purchases of goods produced in high labor productivity sectors will generate greater income than will increased purchases of "low-productivity" goods.[86] This result has been shown in a number of cases (Meade 1995; Monaco 1994c; Monaco and Phelps 1995).

[86]This is true regardless of whether-or-not the purchases are due to increased PCE, government purchases, investment, etc.

In addition to the new treatment of Medicare, an indirect-age demographics model was constructed as part of this study (see chapter 6). Prior to the construction of this model, LIFT could not forecast the full effects of a change in the size or age structure of the population. As described in chapters 3 and 5, the system of PCE equations accounts for changes in various non-age demographic variables. Until the work described in chapter 6, the forecast of these variables was not linked in a consistent manner to the age demographics that drive the movements in these variables. With the work described in chapter 6, the non-age demographic variables are forecasted in a consistent manner that allows LIFT to forecast the full effects of a change in the underlying demographic assumptions.

To compare and contrast the simulation properties of the two models, three simulations were run with each model -- a Base scenario, a Medicare scenario and a Fertility scenario -- for a total of six simulations. In the Medicare scenario, real Medicare benefits were 10 percent lower than the Base scenario in all years 1996 through 2050, inclusive. To put this in perspective, total Medicare benefits were $160.8 billion (1994$) in 1994. Total personal income in that year equalled $5,750.2 billion (1994$) (Survey of Current Business 1996). Thus, Medicare accounted for only 2.8 percent of total income, as measured by NIPA. In terms of PCE, in 1994, Medicare spending accounts for only 3.4 percent of total PCE, but more than 19 percent of medical PCE. A 10 percent decrease in real Medicare benefits represents an initial shock of less than .5 percent of GDP. Thus, relative to the entire economy, the shock introduced is small but in terms of medical PCE it is a large shock.

In the Fertility scenario, total fertility is increased from the Base scenario. In both scenarios, total fertility is 2.079 births per woman in 1994. While the Base scenario has this increasing linearly to 2.245 births per woman in 2050, the Fertility scenario increases total fertility linearly to 2.4 birth per woman by 2010 and then it is constant through 2050. Figure 65 shows the total fertility assumptions and figure 66 shows the effect on total population.

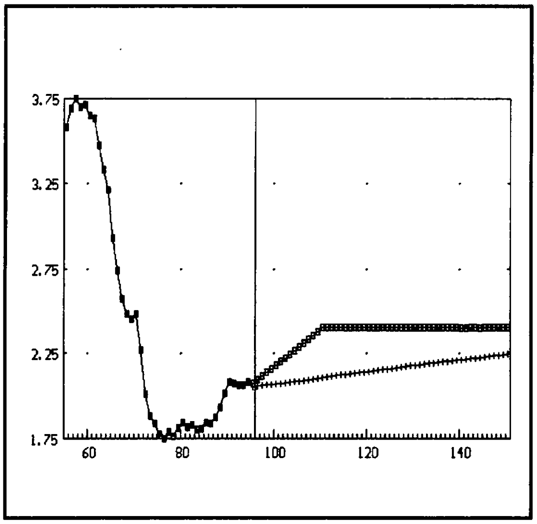


Figure 65

Total Fertility Assumptions

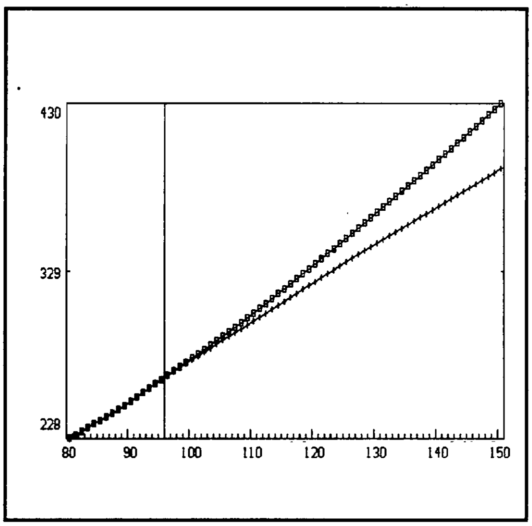


Figure 66

Total Population

Under the Fertility scenario, total population is higher in all forecast years and by 2050, total population equals 423 million compared to a total population of 390.5 million under the Base scenario. The largest deviations begin occurring around 2020, or twenty-five years after the start of the Fertility scenario. The deviations get larger in 2020 because the children born at the beginning of the Fertility scenario have reached childbearing age and start having children at the new increased fertility rate.

In this chapter, the macroeconomic properties of LIFT are not discussed except in regards to their reaction to the change in Medicare and the total fertility rate. This study takes these properties as given and does not intended to explain why LIFT possesses certain macroeconomic properties, but instead attempts to improve LIFT's treatment of Medicare, how Medicare impacts the rest of the model and the impact of demographic changes. Comparisons are base-to-alternate or difference-from-base.

This chapter consists of three sections. The first section is a brief discussion of the Base Scenario. This discussion is brief since the Base scenario for both versions of LIFT are used only in measuring LIFT's response to different Medicare and population assumptions. The second section discusses the Medicare scenario. The third presents the Fertility scenario. Concluding remarks are in chapter 8.

The Base Scenario

The Base forecasts are very similar with the deviations in the macroeconometric aggregates generally less than one percent. In both Base scenarios, real Medicare benefits are identical.

In both Bases, total nominal Medicare expenditures equal the sum of nominal Medicare by category. Nominal Medicare in each category equals the product of real Medicare and the appropriate PCE deflator. As described in chapter 5, real Medicare benefits by category equals real benefits per weighted recipient and the weighted recipient population. The weights reflect how heavily a particular population cohort utilizes Medicare transfers.

Figures 67 through 71 show the exogenous assumptions for the five PCE categories that are subsidized by Medicare. Figure 72 shows the growth rate of real Medicare benefits. Three of the categories, Medical durables, Physicians, and Dentists and other professionals, grow at 1.5 percent per year through the forecast horizon. Hospitals and Nursing Homes are forecasted to grow at an annual rate of 1 percent through the forecast horizon.

Total real benefits will grow at approximately 2.5 percent per year through 2010. In 2010, the leading edge of the baby-boom reaches sixty-five and Medicare eligibility and the growth rate increases until the last of the baby-boom has reached sixty-five.

The growth rate starts to decline in the late 2020's and continues to fall through the forecast horizon.

With the discussion of the Base completed, we can turn our attention to the alternate scenarios.

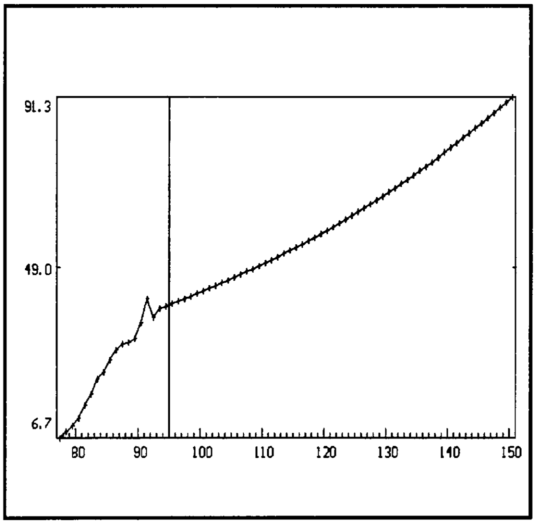


Figure 67

Benefits Per Weighted Population:

Medical Durables

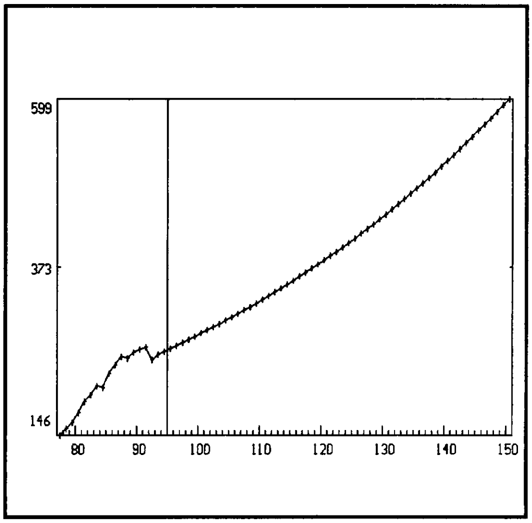


Figure 68

Benefits Per Weighted Population:

Physicians

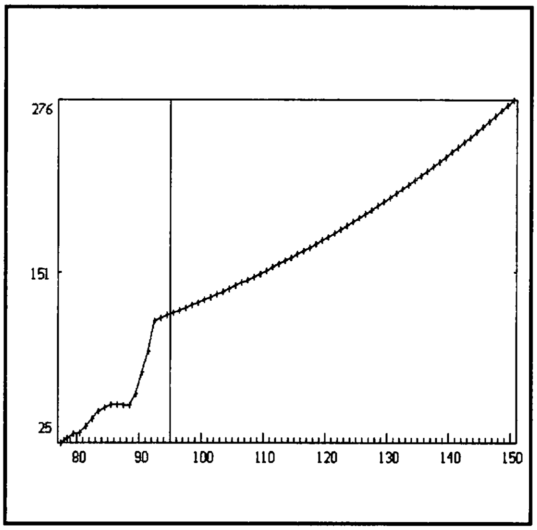


Figure 69

Benefits Per Weighted Population:

Dentists and Other Professionals

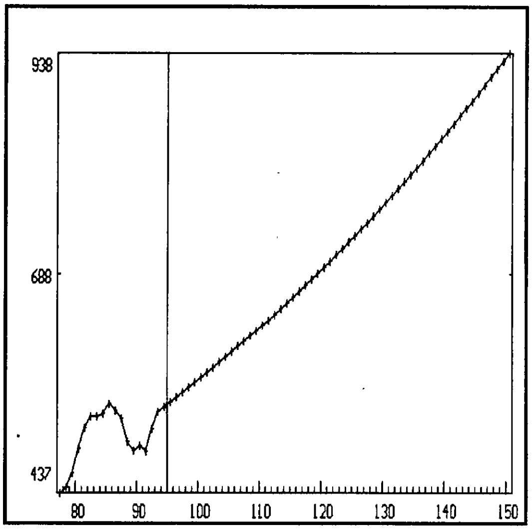


Figure 70

Benefits Per Weighted Population:

Hospitals

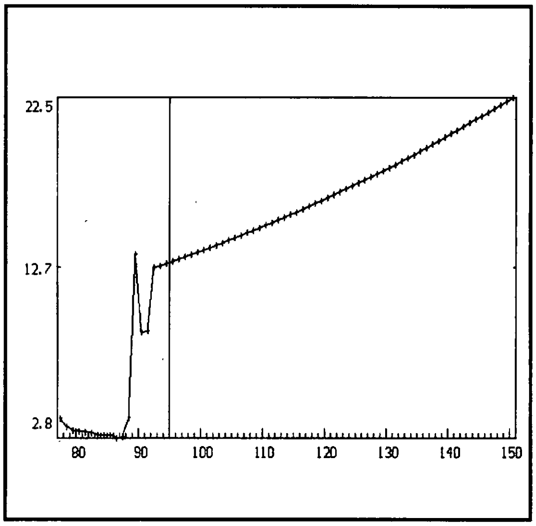


Figure 71

Benefits Per Weighted Population:

Nursing Homes

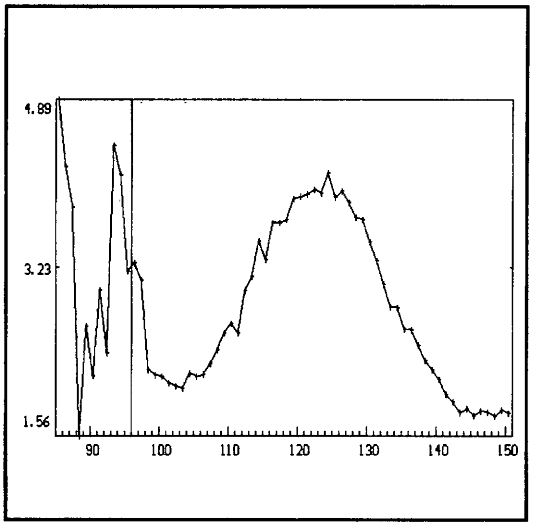


Figure 72

Four-year Average Growth Rate:

Total Real Medicare Benefits

Medicare Scenario

In the Medicare Scenario, real Medicare benefits were cut ten percent from the Base scenario real benefits. This represents a cut of approximately three-tenths percent in personal income as measured by NIPA. The cut in Medicare is kept small to mitigate the secondary effects from the cut in benefits. While a larger cut would induce a larger first round effect, the secondary effects would also be larger and we would have difficulty determining whether the changes under this scenario were a product of the different method of modeling Medicare or the macroeconomic properties of the model.

Table 39 is a summary of the macroeconomic effects under the two scenarios. The first line is the difference-from-base using old-LIFT. The second line is the difference-from-base when the scenario is run using new-LIFT.

TABLE 39

Medicare Scenario: Macro Summary

Line 1: Medicare Scenario OLD LIFT
Line 2: Medicare Scenario NEW LIFT

	2000	2010	2020	2030	2040	2050
Gross Domestic Product, bil $	-31.0	-54.4	-147.9	-769.9	-2977.1	-8687.4
	-32.9	-54.3	-149.8	-792.6	-2931.8	-8293.2
Gross Domestic Product, bil 77$	-11.5	-15.5	-27.1	-31.1	-41.0	-59.4
	-12.4	-15.5	-27.0	-31.0	-39.5	-55.5
GDP Components, bil 77$						
Personal consumption	-14.4	-19.0	-33.2	-49.1	-73.5	-107.7
	-14.8	-18.8	-33.0	-48.1	-69.5	-98.6
Fixed investment	0.1	-0.5	-0.5	4.2	6.6	10.4
	-0.1	-0.6	-0.8	3.5	5.8	7.9
Inventory change	0.1	0.0	-0.0	0.3	0.3	0.4
	0.1	-0.0	-0.0	0.2	0.3	0.2
Exports	-0.7	-0.8	-1.5	-0.9	3.5	6.0
	-0.8	-0.7	-1.2	-0.6	3.5	6.2
Imports	-4.0	-5.9	-10.5	-20.3	-30.7	-42.0
	-3.8	-5.8	-10.2	-19.5	-28.3	-38.0
PCE deflator (77=100)	0.1	0.5	0.1	-13.4	-55.9	-149.0
	0.1	0.6	0.2	-13.7	-54.9	-142.4
Total jobs, mil	-0.4	-0.5	-0.9	-1.1	-1.3	-1.8
	-0.5	-0.5	-0.9	-1.0	-1.3	-1.7
Unemployment rate, %	0.3	0.3	0.5	0.6	0.7	0.8
	0.3	0.3	0.5	0.5	0.6	0.8
Financial Indicators						
Three month T-bills, %	-0.2	-0.2	-0.3	-0.8	-1.1	-1.4
	-0.1	-0.2	-0.3	-0.8	-1.0	-1.2
Federal deficit, bil $	-30.6	-70.8	-170.9	-480.8	-1148.1	-2539.8
	-29.2	-68.8	-161.4	-447.2	-1045.1	-2204.4
relative to GNP	0.3	0.5	0.7	1.2	1.6	2.0
	0.3	0.5	0.7	1.1	1.5	1.8

The macroeconomic effects of a ten percent cut in Medicare benefits appear similar under both versions of LIFT. However, from the table it is clear that under old-LIFT the cut in Medicare has a greater negative effect on the economy than under new-LIFT. Real Gross Domestic Product (GDP) measured in 77$ falls $59.4BN in 2050 under old-LIFT, but only $55.5BN in 2050 under new-LIFT. This is slightly less than a one percent difference. PCE also falls more under the old treatment of Medicare than under the new treatment, primarily because the decline in GDP causes a drop in disposable income.

Table 40 shows the effects on employment by industry under the two versions of LIFT. The table gives the percent of the change in total employment caused by changes in industry employment. A positive number indicates that employment fell in the industry and negative indicates that employment increased. If we look the Medical services lines, we see that in 2000 and under old-LIFT, 9.65 percent of the decline in jobs is in the Medical services sectors. Using new-LIFT, 13.61 percent of the decline was in the Medical services sectors. For Nursing homes, under old-LIFT, 2.02 percent of the decline in jobs occurred in the Nursing home sector. However, under new-LIFT, the number of jobs in this sector actually increased by a number equal to 4.94 percent of the change in the total number of jobs.

TABLE 40

Medicare Scenario: Employment by Industry

Percent of Decline in Jobs

Line 1: Medicare Scenario OLD LIFT
Line 2: Medicare Scenario NEW LIFT

	2000	2010	2020	2030	2040	2050
Agriculture, forestry, fisher	1.21	0.56	0.84	0.69	0.44	0.50
	1.15	0.49	0.78	0.65	0.40	0.46
Mining	0.32	0.22	0.29	0.16	0.09	0.12
	0.32	0.20	0.28	0.16	0.08	0.11
Construction	-0.93	-1.32	-0.62	-4.39	-5.56	-5.02
	-0.26	-1:43	-0.70	-4.41	-5.73	-4.74
Nondurables manufacturing	4.95	3.45	3.56	2.33	1.09	0.89
	4.60	3.12	3.26	2.02	0.81	0.61
Durables manufacturing	2.02	0.50	1.97	-2.63	-4.99	-3.74
	2.78	0.45	2.06	-2.41	-4.86	-3.73
Durables, exc. Medical	1.95	0.45	1.90	-2.67	-5.00	-3.75
	2.61	0.36	1.99	-2.46	-4.87	-3.75
Medical Inst. and Opth.	0.07	0.05	0.07	0.05	0.01	0.01
	0.17	0.09	0.06	0.05	0.01	0.02
Transportation	4.14	3.71	3.58	3.32	3.16	3.27
	3.69	3.68	3.56	3.28	3.13	3.05
Utilities	1.52	1.08	0.98	0.80	0.63	0.60
	1.43	1.01	0.95	0.77	0.61	0.58
Trade	41.58	38.52	35.10	37.91	41.52	42.42
	36.07	36.45	33.83	36.42	39.64	40.81
Finance, insurance, real estate	8.51	9.54	8.77	9.70	9.81	9.43
	7.60	8.56	7.70	8.39	8.64	8.42
Services, nonmedical	25.67	27.24	27.24	28.86	28.61	27.48
	26.85	27.74	27.29	28.72	28.32	27.61
Medical services	9.65	14.16	15.32	19.48	21.33	20.52
	13.61	17.08	18.23	22.86	25.42	23.23
Private hospitals	4.16	7.23	8.42	10.73	11.26	10.51
	6.41	8.28	7.96	10.16	10.36	10.82
Physicians	1.93	2.48	2.17	2.85	3.62	3.70
	4.58	5.40	5.22	5.11	6.03	5.00
Other medical services	1.53	1.06	0.44	0.71	1.19	1.24
	7.56	7.88	7.12	9.58	10.07	6.83
Nursing homes	2.02	3.39	4.30	5.20	5.26	5.06
	-4.94	-4.48	-2.07	-2.00	-1.04	0.58

Under both versions of LIFT the industries with the greatest effect on total employment are Medical services, Nonmedical services and Trade. However, in New-LIFT, the effect of a fall in Medical services is greater than in old-LIFT. Since a cut in Medicare increases the relative price of Medical goods, we expect this result. There is one odd, or at least odd for now, result under new-LIFT -- employment in Nursing homes increases relative to the new-LIFT Base scenario. This apparent contradiction disappears when we consider table 41, PCE spending by category.

TABLE 41

Medicare Scenario: Personal Consumption Expenditures

Percent of Change in PCE

Line 1: Medicare Scenario OLD LIFT
Line 2: Medicare Scenario NEW LIFT

	2000	2010	2020	2030	2040	2050
Durable Goods	27.2	23.3	23.4	22.0	21.6	22.1
	25.2	22.7	23.4	22.1	21.6	22.1
Motor Vehicles and Parts	11.7	7.8	9.6	8.0	7.3	8.0
	10.4	7.2	9.4	8.0	7.3	7.9
Other Non-Durables	15.5	15.5	13.8	14.0	14.3	14.1
	14.8	15.5	14.0	14.1	14.4	14.2
Non-Durable Goods	28.8	26.1	26.6	26.1	25.3	25.5
	25.6	23.0	23.8	23.1	22.4	23.2
Food and Alcohol	11.2	9.3	10.1	9.7	9.4	9.6
	10.0	8.7	9.7	9.5	9.2	9.5
Clothing	8.9	8.9	8.3	8.3	8.3	8.2
	8.5	8.8	8.4	8.3	8.3	8.2
Other Non-Durable	8.6	7.8	8.2	8.0	7.6	7.6
	7.1	5.5	5.7	5.4	4.9	5.5
Services	44.0	50.6	50.0	52.0	53.1	52.4
	49.2	54.3	52.8	54.7	56.0	54.7
Housing & Household operations	11.1	10.3	9.2	9.3	9.4	9.2
	9.2	9.5	8.9	9.1	9.3	9.1
Transportation	3.7	3.3	3.5	3.5	3.4	3.7
	3.5	3.2	3.5	3.4	3.4	3.6
Medical Services	8.4	11.4	12.3	13.1	13.5	13.2
	13.1	14.7	15.1	16.1	16.5	15.8
Physicians	2.7	3.0	2.7	2.8	3.2	3.3
	5.0	5.4	5.3	4.6	5.0	4.4
Dentists & other prof svcs	1.6	1.3	0.6	0.7	0.8	0.9
	5.8	5.6	5.1	5.9	5.8	4.1
Hospitals	3.1	5.5	7.1	7.7	7.6	7.3
	5.1	6.3	6.6	7.3	7.0	7.7
Nursing homes	0.8	1.3	1.7	1.7	1.7	1.7
	-1.7	-1.3	-0.6	-0.5	-0.1	0.3
Other Services	20.8	25.6	25.0	26.1	26.7	26.3
	23.3	26.9	25.3	26.1	26.7	26.2

Table 41 shows the percent of the total change in PCE by several broad PCE categories. As in table 40, a positive value indicates that PCE in the category fell.

Examining the table, we see that in new-LIFT, a greater percent of the fall in PCE occurs in Medical services than in old-LIFT. In fact, for Physicians and Dentists and other professionals, the relative effect is twice as great under new-LIFT than under old-LIFT. The apparent oddity of increased employment in Nursing homes under new-LIFT is explained by the increase in Nursing home PCE.

Unfortunately, the increased Nursing home result seems odd until we consider what happens to the price of Nursing homes relative to the price of the other Medical services PCE categories. Compared to the Medicare subsidy in the other PCE categories that Medicare finances, the Nursing home subsidy is very small. An across-the-board ten percent cut in the subsidy rate causes the price of low subsidy categories to fall relative to high subsidy categories. For example, consider two goods with equal unsubsidized prices ($P_x = P_y = 1.0$). Good x receives a ten percent subsidy and its effective price equals 0.9. Good y's subsidy is fifty percent and its effective price equals 0.5. The price of x relative to the price of y equals 1.8. Now, with the subsidy rate cut ten percent, P_x-effective equals 0.91 and P_y-effective equals 0.55. With the cut in the subsidy rate, the price of x relative to the price of y equals 1.65. X is now less expensive than y.

Since the Nursing home subsidy is so small compared to the other subsidized categories, an equiproportional cut in benefits causes the price of Nursing homes to fall relative to the other subsidized categories. The increase in Nursing home PCE in new-LIFT is caused by a substitution away from the more "expensive" goods and services and into the less expensive Nursing homes.

Table 42 is a detailed listing of the eighty PCE categories. The table gives the percent of the change in total PCE caused by changes in the category. The categories are aggregated into broad groupings that are in **bold**. These broad groupings are then aggregated into Durable goods, Nondurable goods an Services.

TABLE 42

Medicare Scenario: Detailed Personal Consumption Expenditures

Percent of Change in Total PCE

	2000	2010	2020	2030	2040	2050
Durable Goods	27.17	23.34	23.38	21.96	21.62	22.11
	25.22	22.71	23.40	22.14	21.63	22.07
Motor Vehicles and Parts	11.70	7.84	9.58	7.98	7.32	8.04
	10.41	7.20	9.42	8.02	7.27	7.91
1 New cars	7.58	4.76	6.48	5.99	5.39	5.92
	7.03	4.46	6.35	5.95	5.32	5.77
2 Used cars	0.10	-0.01	-0.18	-0.74	-0.69	-0.65
	-0.14	-0.13	-0.19	-0.70	-0.67	-0.61
3 New & used trucks	3.31	2.46	2.66	2.12	2.04	2.18
	2.90	2.28	2.63	2.16	2.04	2.17
4 Tires & tubes	0.38	0.32	0.33	0.32	0.30	0.30
	0.34	0.30	0.33	0.32	0.29	0.30
5 Auto accessories & parts	0.33	0.31	0.29	0.29	0.29	0.28
	0.29	0.29	0.29	0.29	0.29	0.28
Furniture & Household Equip.	10.21	10.51	8.50	8.48	8.78	8.43
	9.02	10.31	8.70	8.63	8.96	8.50
6 Furniture,mattresses	1.86	1.99	1.91	1.87	1.89	1.87
	1.97	2.06	1.93	1.87	1.89	1.87
7 Kitchen, household app.	0.78	0.70	0.55	0.38	0.39	0.40
	0.59	0.64	0.56	0.39	0.41	0.40
8 China,glass&tableware	0.75	0.75	0.67	0.65	0.65	0.64
	0.69	0.72	0.67	0.64	0.64	0.63
9 Radio,tv,records,musical	4.21	4.23	2.80	2.90	3.05	2.78
	3.24	4.03	2.95	3.05	3.21	2.86
10 Floor coverings	0.54	0.64	0.52	0.53	0.54	0.50
	0.53	0.64	0.53	0.52	0.54	0.50
11 Durable housefurnishings nec	1.71	1.88	1.70	1.79	1.87	1.84
	1.68	1.89	1.72	1.79	1.88	1.84
12 Writing equipment	0.04	0.02	0.04	0.04	0.03	0.04
	0.04	0.02	0.04	0.03	0.03	0.04
13 Hand tools	0.32	0.31	0.31	0.33	0.35	0.36
	0.28	0.30	0.31	0.33	0.35	0.36

TABLE 42 (Continued)

	2000	2010	2020	2030	2040	2050
Other Durables	5.26 5.79	4.98 5.21	5.30 5.28	5.50 5.50	5.51 5.39	5.64 5.66
14 Jewelry	1.78 2.22	1.83 1.98	2.01 2.06	2.02 2.05	1.99 2.01	2.04 2.03
15 Ophthalmic & orthopedic	0.26 0.43	0.25 0.35	0.20 0.09	0.19 0.16	0.14 0.01	0.10 0.23
16 Books & maps	0.39 0.32	0.29 0.27	0.31 0.30	0.33 0.32	0.35 0.34	0.39 0.37
17 Wheel goods & durable toys	1.20 1.01	1.11 1.04	0.98 0.97	1.03 1.02	1.08 1.07	1.06 1.03
18 Boats, rec vech., & aircraft	1.63 1.81	1.51 1.57	1.81 1.85	1.93 1.95	1.95 1.96	2.05 2.00
Non-Durable Goods	28.80 25.62	26.07 23.01	26.64 23.77	26.06 23.15	25.28 22.41	25.46 23.20
Food and Alcohol	11.23 10.00	9.30 8.74	10.07 9.74	9.67 9.46	9.35 9.19	9.63 9.47
19 Food, off premise	4.22 3.18	1.85 1.39	3.06 2.82	2.74 2.59	2.17 2.07	2.45 2.24
20 Food on premise	5.10 4.78	5.23 5.04	4.89 4.79	4.74 4.68	4.86 4.81	4.86 4.92
21 Alcohol, off premise	1.22 1.28	1.38 1.39	1.26 1.24	1.13 1.10	1.00 0.99	0.86 0.85
22 Alcohol, on premise	0.69 0.76	0.85 0.91	0.87 0.89	1.08 1.08	1.32 1.33	1.46 1.46
Clothing	8.92 8.49	8.93 8.79	8.34 8.35	8.35 8.33	8.34 8.35	8.21 8.18
23 Shoes & footwear	0.98 1.12	1.24 1.33	1.23 1.25	1.30 1.31	1.35 1.36	1.32 1.33
24 Women's clothing	5.05 4.56	4.76 4.56	4.38 4.36	4.32 4.31	4.27 4.27	4.21 4.19
25 Men's clothing	2.78 2.66	2.79 2.75	2.58 2.58	2.56 2.55	2.56 2.56	2.52 2.50
26 Luggage	0.12 0.15	0.14 0.15	0.15 0.16	0.16 0.16	0.17 0.17	0.17 0.17
27 Gasoline & oil	1.51 1.20	1.31 1.15	1.16 1.10	1.07 1.03	1.04 1.01	1.03 1.00
28 Fuel oil & coal	0.23 0.39	-0.35 -0.46	0.37 0.35	0.16 0.16	-0.03 -0.06	0.21 0.18
29 Tobacco	0.19 0.19	0.07 0.05	0.18 0.17	0.13 0.12	0.08 0.07	0.10 0.09
31 Drug preparations & sundries	3.31 2.90	2.46 2.28	2.66 2.63	2.12 2.16	2.04 2.04	2.18 2.17

TABLE 42 (Continued)

	2000	2010	2020	2030	2040	2050
Other Non-Durables	5.83 5.47	5.66 5.53	5.48 5.48	5.62 5.57	5.59 5.57	5.54 5.41
30 Semidurable housefurnishing	0.97 1.13	1.14 1.21	1.14 1.16	1.17 1.17	1.18 1.18	1.17 1.16
32 Toilet articles & preps	0.86 0.81	0.79 0.75	0.83 0.81	0.76 0.73	0.60 0.58	0.56 0.53
33 Stationery & writing supp.	0.52 0.48	0.44 0.43	0.46 0.47	0.51 0.50	0.54 0.54	0.61 0.58
34 Nondurable toys, sport supp.	1.29 1.11	1.30 1.25	1.13 1.14	1.24 1.25	1.33 1.34	1.28 1.26
35 Flowers, seeds, plants	0.33 0.29	0.31 0.30	0.28 0.28	0.27 0.28	0.27 0.28	0.26 0.25
37 Cleaning preparations	0.65 0.60	0.65 0.62	0.61 0.60	0.60 0.59	0.57 0.57	0.53 0.52
36 Lighting supplies	0.12 0.11	0.11 0.10	0.11 0.11	0.11 0.11	0.11 0.11	0.11 0.11
38 Household paper products	3.31 2.90	2.46 2.28	2.66 2.63	2.12 2.16	2.04 2.04	2.18 2.17
39 Magazines & newspaper	0.56 0.47	0.41 0.38	0.44 0.43	0.47 0.46	0.49 0.49	0.55 0.51
40 Other nondurables -- identity	0.04 0.04	0.04 0.04	0.04 0.04	0.04 0.04	0.04 0.04	0.04 0.04
Services	44.03 49.16	50.60 54.28	49.99 52.83	51.98 54.71	53.10 55.97	52.43 54.73
Housing	7.76 6.39	7.21 6.58	6.36 6.18	6.45 6.32	6.54 6.48	6.39 6.27
41 Owner occupied space	6.76 5.08	6.15 5.51	4.91 4.75	5.02 4.97	5.14 5.15	4.97 4.89
42 Tenant occupied space	0.48 0.87	0.62 0.66	1.02 0.99	0.94 0.87	0.86 0.79	0.88 0.87
43 Hotels, motels	0.26 0.22	0.23 0.22	0.23 0.23	0.27 0.27	0.30 0.30	0.27 0.26
44 Other housing	0.26 0.22	0.21 0.19	0.20 0.20	0.22 0.22	0.24 0.24	0.26 0.25

TABLE 42 (Continued)

		2000	2010	2020	2030	2040	2050
	Household Operation	3.35	3.14	2.83	2.88	2.90	2.85
		2.83	2.91	2.74	2.81	2.86	2.79
45	Electricity	0.91	0.84	0.79	0.83	0.82	0.79
		0.78	0.77	0.75	0.80	0.80	0.76
46	Natural gas	0.17	0.06	0.10	0.07	0.06	0.08
		0.11	0.05	0.09	0.07	0.06	0.08
47	Water & oth sanitary svc	0.32	0.31	0.27	0.28	0.27	0.25
		0.25	0.26	0.25	0.26	0.25	0.24
48	Telephone & telegraph	1.25	1.15	0.98	0.99	0.97	0.91
		1.01	1.06	0.96	0.99	0.98	0.90
50	Household insurance	0.09	0.09	0.09	0.08	0.09	0.10
		0.09	0.09	0.09	0.08	0.09	0.10
51	Oth hhld operations:repair	0.40	0.45	0.38	0.38	0.42	0.46
		0.39	0.45	0.38	0.38	0.42	0.46
52	Postage	0.21	0.25	0.22	0.23	0.26	0.26
		0.20	0.25	0.22	0.23	0.26	0.26
	Transportation	3.68	3.29	3.52	3.46	3.45	3.66
		3.52	3.19	3.47	3.43	3.43	3.60
53	Auto repair	1.81	1.61	1.72	1.72	1.72	1.82
		1.73	1.55	1.71	1.71	1.72	1.79
54	Bridge, tolls, etc	0.05	0.04	0.05	0.05	0.05	0.05
		0.05	0.04	0.05	0.05	0.05	0.05
55	Auto insurance	0.21	0.21	0.14	0.14	0.14	0.11
		0.14	0.17	0.13	0.13	0.13	0.11
56	Taxicabs	0.08	-0.02	0.06	0.03	0.00	0.03
		0.08	-0.02	0.06	0.03	0.00	0.02
57	Local public transport	0.05	0.04	0.04	0.02	0.01	0.02
		0.04	0.03	0.04	0.02	0.01	0.02
58	Intercity railroad	0.01	0.01	0.01	0.01	0.00	0.00
		0.01	0.01	0.01	0.01	0.00	-0.00
59	Intercity buses	0.02	0.02	0.03	0.02	0.01	0.02
		0.03	0.02	0.03	0.02	0.01	0.02
60	Airlines	1.38	1.35	1.40	1.41	1.47	1.55
		1.39	1.34	1.39	1.40	1.46	1.53
61	Travel agents,oth trans svc	0.06	0.04	0.06	0.06	0.05	0.06
		0.06	0.04	0.06	0.06	0.05	0.06

TABLE 42 (Continued)

		2000	2010	2020	2030	2040	2050
	Medical Services	8.40	11.39	12.26	13.12	13.50	13.25
		13.07	14.70	15.15	16.10	16.54	15.83
64	Physicians	2.68	3.03	2.67	2.83	3.22	3.35
		5.04	5.39	5.28	4.56	4.97	4.41
65	Dentists & other prof svcs	1.63	1.29	0.64	0.66	0.85	0.86
		5.78	5.57	5.12	5.92	5.76	4.12
66	Hospitals	3.07	5.52	7.09	7.72	7.63	7.34
		5.06	6.32	6.63	7.26	6.97	7.65
67	Health insurance	0.19	0.22	0.18	0.17	0.08	0.00
		-1.07	-1.24	-1.24	-1.14	-1.02	-0.69
80	Nursing homes	0.82	1.34	1.68	1.74	1.73	1.70
		-1.74	-1.34	-0.64	-0.50	-0.13	0.34
76	Education	2.10	1.87	1.53	1.49	1.49	1.45
		1.68	1.69	1.50	1.48	1.49	1.44
	Other Services	18.69	23.60	23.39	24.54	25.16	24.76
		21.61	25.11	23.68	24.53	25.11	24.75
62	Laundries & shoe repair	0.39	0.38	0.34	0.36	0.44	0.45
		0.36	0.37	0.33	0.36	0.43	0.45
63	Barbershops & beauty shops	0.46	0.37	0.32	0.33	0.40	0.41
		0.36	0.34	0.30	0.33	0.39	0.40
68	Brokerage,invest. counseling	1.11	1.39	1.42	1.44	1.44	1.41
		1.36	1.54	1.48	1.47	1.47	1.41
69	Bank service charges	2.47	3.18	3.30	3.36	3.36	3.28
		3.07	3.48	3.37	3.37	3.36	3.23
70	Life insurance	1.47	1.77	1.34	1.35	1.33	1.29
		1.49	1.79	1.35	1.35	1.33	1.28
71	Legal services	0.68	0.99	0.95	0.93	1.03	1.04
		0.85	1.11	0.97	0.94	1.05	1.07
72	Funerals, oth pers business	0.42	0.56	0.59	0.56	0.61	0.64
		0.55	0.62	0.61	0.57	0.62	0.66
73	Radio & tv repair	0.05	0.05	0.04	0.04	0.05	0.05
		0.04	0.04	0.04	0.04	0.04	0.05
74	Movies, theater,spec sports	0.29	0.51	0.44	0.39	0.29	0.23
		0.43	0.58	0.44	0.38	0.28	0.25
75	Other recreational services	5.41	6.47	7.33	7.59	7.35	7.27
		7.18	7.21	7.56	7.72	7.43	7.34
77	Religious & welfare service	3.17	4.33	3.65	3.53	3.38	2.73
		3.36	4.45	3.64	3.49	3.35	2.86
78	Foreign travel by U.S.	2.78	3.60	3.66	4.66	5.46	5.96
		2.55	3.60	3.60	4.54	5.36	5.74
79	Travel in U.S. by foreigner	0.05	0.09	0.11	0.05	0.06	0.06
		0.05	0.09	0.11	0.04	0.06	0.05

For most of the categories listed in table 42, the effect of modeling Medicare as an income transfer is very similar to the effect of modeling Medicare as a price subsidy. However, for goods and services that are health care related -- Ophthalmic and orthopedic goods (PCE31), Drug preparations and sundries (PCE31), Physicians (PCE64), Dentists and other professionals (PCE65), Hospitals (PCE66), Health insurance (PCE67) and Nursing Homes (PCE80) -- it does matter whether Medicare is modeled as an income transfer or a price subsidy. The difference from the Base scenario PCE in these categories is larger than the other categories.

In particular, if one looks at the pre-2020 columns, the difference in the results is more apparent. Under new-LIFT, in 2000, four years after the cut in the program, 13.1 percent of the fall in PCE occurs in Medical services. Under old-LIFT, only 8.4 percent of the change in PCE occurs in Medical services. The effect of the ten percent cut in Medicare on Medical services in new-LIFT is about double the effect in old-LIFT for the early years of the forecast.

To help put this in perspective, in 2000 and using old-LIFT, a ten percent reduction in Medicare has about the same effect on New car purchases as it does on **total** Medical services expenditures. In old-LIFT, the change in New car purchases accounts for about 7.6 percent in the change in total PCE and the change in Medical services PCE is around 8.4 percent of the change in total PCE. Spending on Motor vehicles and parts accounted for more of the change in total PCE than did the change in Medical services. In new-LIFT, however, the effect of the cut in Medicare is felt more heavily in Medical services. The percent of the change in total PCE caused by

Medical services is nearly twice that of New cars and the effect on Medical services is about thirty percent greater than the effect on Motor vehicles and parts.

The new-LIFT results are more intuitively appealing than the old-LIFT results. We expect that a decrease in Medicare benefits should have a greater effect on medical goods and services than on the other PCE categories. In old-LIFT, the failure to link Medicare to medical PCE insures that any changes in the size of the program will have the greatest impact on those PCE categories with the largest income elasticities and not necessarily medical goods and services.

Fertility Scenario

In the Fertility scenario, total fertility was increased to 2.4 births per women in the year 2010. The fertility rate was held constant at 2.4 births from 2010 to 2050. Under the Base scenario, total fertility increases linearly to 2.245 births per woman in 2050 from 2.079 in 1994. This increase in fertility causes a second baby-boom. This second baby-boom is different from the boomlet of the 1980's and early 1990's because the boomlet was caused by an increase in the number of births and not an increase in the fertility rate.[87]

The birth rate is the number of births in a particular year. The fertility rate is the expected number of children that a women just entering childbearing age will have. The birth rate may increase even if the fertility rate is falling. For example, if the number of childbearing-age women increases, births may increase even if the fertility

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[87]Boom is a good description for this increase in fertility since the new fertility rate causes an explosive population growth rate.

rate is declining. Similarly, if the number of women in their childbearing years is falling, the birth rate may fall despite an increase in the fertility rate.

In old-LIFT, the indirect-age demographic variables do not change when the population projection changes. Thus, in old-LIFT, the results of the Fertility scenario represent changes in the age structure and size of the population and there are no changes to the indirect-age demographic variables. However, in new-LIFT, the indirect-age demographic variables will change because of the work described in chapter 6. Thus, the results of the new-LIFT Fertility scenario reflect the changes caused by the age variables and the indirect-age demographic variables.

Table 43 shows the effects on the size and age structure of the population of this second baby-boom (second-boom). The first line shows the Base forecast of the variables. The second line shows the change under the Fertility scenario. The initial effect is to increase the number of persons under the age of fifteen. By 2010, however, the leading edge of the second-boom is just reaching sixteen years old and will soon enter both the labor market and their childbearing years. During the period 2000 to 2010 (the first full decade of the second-boom), there are 4.7 million more people in the total population than in the Base scenario. During the 2010 to 2020 period, total population is 10.6 million above the Base scenario. In each subsequent decade, the rate of increase continues to grow. This is because, in increasing numbers, the children of the second-boom begin to have children around 2010. The forecast horizon extends through 2050 which is sufficient time for a third generation of second-boom children to be born (the grandchildren of the leading edge of the second-boom).

There is no change in the sixty-five and older population because the leading edge of the second-boom is only fifty-five years old in 2050.

TABLE 43

Population Variables: Fertility Scenario

Difference from Base

	2000	2010	2020	2030	2040	2050
Total Population (Millions)	275.2	298.4	322.5	345.8	367.8	390.5
	0.7	4.7	10.6	18.1	28.0	39.5
Under 15 Population	59.1	60.9	65.3	69.7	74.1	79.8
	0.7	4.7	8.4	10.6	14.1	16.9
15 to 19 Population	19.3	20.7	20.8	22.6	24.0	25.4
	0.0	0.0	1.5	2.9	3.4	4.7
20 to 24 Population	18.3	20.3	21.1	22.0	23.8	25.0
	0.0	0.0	0.7	2.5	3.0	4.1
25 to 34 Population	38.2	39.5	42.9	43.9	46.5	49.7
	0.0	0.0	0.0	2.2	5.3	6.4
35 to 44 Population	44.2	38.8	40.1	43.5	44.5	47.1
	0.0	0.0	0.0	0.0	2.1	5.2
45 to 54 Population	37.4	43.4	38.3	39.7	43.1	44.2
	0.0	0.0	0.0	-0.0	0.0	2.1
55 to 64 Population	24.0	35.6	41.4	36.8	38.4	41.9
	0.0	0.0	-0.0	0.0	0.0	0.0
65 to 74 Population	18.1	20.9	31.2	36.6	32.9	34.8
	0.0	0.0	0.0	0.0	0.0	0.0
75 to 84 Population	12.3	12.7	15.1	23.1	27.6	25.5
	0.0	0.0	0.0	0.0	0.0	0.0
Over 85 Population	4.2	5.6	6.3	8.1	13.0	17.1
	0.0	0.0	0.0	0.0	0.0	-0.0

Table 44 shows the effect on the indirect-age variables. Since old-LIFT lacks an indirect-age model, there is no change in these variables in that version of the model. The first line of the table shows the values in the Base scenario and the second line shows the difference from the Base scenario in new-LIFT. For the share variables, the first line shows the Base scenario value and the second line shows the Fertility scenario value.

TABLE 44

Indirect-age Variables: Fertility Scenario versus Base

Difference from Base (new-LIFT only)

	2000	2010	2020	2030	2040	2050
Total Number of Households	104873.2	117942.7	131548.0	142552.5	151434.0	161040.9
(1000's)	0.0	0.7	217.6	1798.6	4931.7	9126.0
Heads under 35	23880.9	25041.9	27047.8	27825.4	29641.8	31626.0
	0.0	0.6	217.6	1777.4	3547.8	4420.7
Heads 35 to 54	44581.8	45067.4	42516.4	45597.8	48477.2	50869.3
	0.0	0.0	0.0	21.2	1383.9	4683.0
Heads over 55	36410.5	47833.5	61983.8	69129.4	73314.9	78545.5
	0.0	0.0	-0.0	-0.0	0.0	22.4
Share of Households						
Heads under 35	22.8	21.2	20.6	19.5	19.6	19.6
	22.8	21.2	20.7	20.5	21.2	21.2
Heads 35 to 54	42.5	38.2	32.3	32.0	32.0	31.6
	42.5	38.2	32.3	31.6	31.9	32.6
Heads 55 and over	34.7	40.6	47.1	48.5	48.4	48.8
	34.7	40.6	47.0	47.9	46.9	46.2
Household Sizes						
One person	26354.5	30200.8	35538.3	40494.7	43299.9	45899.6
	-52.4	-359.2	-644.5	-494.3	-2.1	815.2
Two persons	33388.0	37653.6	42441.5	46463.8	49451.9	52608.0
	-0.1	-1.6	48.1	516.9	1489.9	2798.2
Three or four persons	34080.0	37841.3	40495.0	42051.2	44399.9	47323.5
	39.8	273.9	617.0	1346.0	2609.5	4177.2
Five or more persons	11050.7	12247.1	13073.1	13542.8	14282.2	15209.8
	12.7	87.6	197.0	430.0	834.4	1335.4
Household Size Shares						
One person	25.1	25.6	27.0	28.4	28.6	28.5
	25.1	25.3	26.5	27.7	27.7	27.5
Two persons	31.8	31.9	32.3	32.6	32.7	32.7
	31.8	31.9	32.2	32.5	32.6	32.6
Three or four persons	32.5	32.1	30.8	29.5	29.3	29.4
	32.5	32.3	31.2	30.1	30.1	30.3
Five or more persons	10.5	10.4	9.9	9.5	9.4	9.4
	10.5	10.5	10.1	9.7	9.7	9.7

The extra 39.5 million people in 2050 increases the number of households in the U.S. by about 9.1 million. The increase in the number of households may seem small given the increase in the population, but there are only 22.6 million more persons over fifteen in 2050. If we exclude the fifteen to nineteen population, then there are slightly less than 18 million more adults in 2050.

The aging of the second-boom can be seen in the first half of table 44. In 2010, the leading edge of the second-boom is twenty-five years old and there is a slight increase in both the total number of households and the number of households headed by persons under the age of thirty-five. All of the increased number of households occurs in the youngest age of head category until 2030, when the leading-edge of the second-boom is thirty-five. Not until 2050 is there any increase in the number of households headed by persons fifty-five or older.

Unlike the total number of households and the age of head variables, there is an immediate effect on the household size variables. The increased number of children in the Fertility scenario causes a drop in the number of single person households and an increase in the number of three/four and five or more person households relative to the Base. The increase in fertility rates causes an increase in the number of children and since children tend to live in households with three or more persons, there is a dramatic increase in the number of households of these sizes.

The changes in the indirect-age variables influence PCE. For example, the increase in the share of households with five or more persons increases spending on education. The increase in the share of households headed by persons under thirty-five causes tenant-occupied rental expenditures to rise, but the fall in the number of single person and two person households has a negative effect on this sector, with the net effect unclear.

Table 45 gives the macroeconomic summary for the two Fertility scenarios. The macroeconomic outlook is approximately the same in both scenarios. The primary difference is the level of nominal GDP. In 2050, both versions of LIFT have lower nominal GDP than the Base scenario, but higher constant dollar GDP. In old-LIFT, nominal GDP is $1,871 billion lower than the Base, but in new-LIFT, nominal GDP is only $509 billion lower. The reason for the difference is the lower price level in old-LIFT. This cause of the lower price level is revealed when we consider average hourly compensation (the last two lines of the table). The last two lines show the percentage difference in average hourly compensation between the Base and Fertility scenarios. In old-LIFT, average hourly compensation is lower than in new-LIFT. This difference occurs because of the higher unemployment in old-LIFT compared to new-LIFT.

Despite being a difference of more than $1 trillion dollars, the difference is actually very small if we consider the level of nominal GDP in both scenarios. By 2050, nominal GDP is approximately $110 trillion in 2050 and the difference in the behavior between old- and new-LIFT represents about a one percent difference in the level of nominal GDP in 2050.

TABLE 45

Fertility Scenario: Macro Summary

	2000	2010	2020	2030	2040	2050
Gross Domestic Product, bil $	3.4	29.4	207.8	683.6	758.3	-1871.0
	1.0	21.4	200.3	778.3	1246.4	-509.5
Gross Domestic Product, bil 77$	1.4	11.8	66.9	166.9	314.8	522.4
	0.3	9.0	62.9	168.9	320.9	525.6
GDP Components, bil 77$						
Personal consumption	0.9	10.3	49.4	131.8	248.3	408.3
	0.3	8.2	45.6	125.2	240.1	404.2
Fixed investment	0.6	3.6	16.4	40.2	78.6	139.0
	0.4	3.0	15.3	48.0	95.0	148.0
Inventory change	0.1	0.2	0.9	1.6	2.7	4.1
	0.0	0.2	0.9	1.8	2.7	3.9
Exports	0.1	0.0	-0.1	-0.9	1.9	5.8
	0.0	0.1	0.3	-0.1	2.1	5.6
Imports	0.2	2.2	11.2	28.5	49.8	81.8
	0.4	2.5	11.2	30.2	54.5	85.4
Price Level and Inflation Indicators						
PCE deflator (77=100)	-0.1	-1.0	-6.0	-17.6	-61.8	-188.3
	0.1	-0.6	-5.1	-14.5	-51.3	-163.1
Total jobs, mil	-0.0	-0.2	1.5	5.1	9.7	15.5
	-0.0	-0.2	1.5	5.3	10.0	15.9
Labor force, mil	0.0	-0.0	1.2	5.3	10.8	18.0
	0.0	-0.0	1.2	5.3	10.8	18.0
Unemployment rate, %	0.0	0.1	-0.1	0.2	0.6	1.2
	0.0	0.1	-0.2	0.1	0.4	1.1
Financial Indicators						
Three month T-bills, %	0.0	-0.0	-0.2	-0.5	-1.1	-1.9
	-0.0	-0.0	-0.1	-0.4	-0.8	-1.7
Federal deficit, bil $	-0.4	-5.3	-62.9	-227.7	-750.6	-2134.2
	-0.2	-4.0	-58.9	-215.2	-674.4	-1857.5
relative to GNP	0.0	0.0	0.3	0.6	1.2	1.9
	0.0	0.0	0.3	0.6	1.1	1.7
Avg Hourly compensation	0.0	0.0	-0.2	-0.5	-2.3	-6.0
(percent difference)	0.0	-0.1	-0.3	-0.5	-1.9	-5.1

Table 46 shows the effect on PCE for the two versions of LIFT. The table gives the difference as a percentage from the Base scenario for each version.

TABLE 46

Fertility Scenario: Detailed Personal Consumption Expenditures

Percent Change from Base

	2000	2010	2020	2030	2040	2050
Personal Consumption Expenditure	0.04	0.39	1.54	3.41	5.43	7.53
	0.01	0.32	1.43	3.25	5.28	7.51
Durable Goods	-0.04	0.27	1.89	4.22	6.64	8.90
	0.32	0.82	2.42	4.60	6.99	9.27
Motor Vehicles and Parts	-0.30	-0.67	0.98	3.68	6.50	8.81
	0.68	0.81	2.47	4.78	7.49	9.78
1 New cars	-0.62	-1.87	0.06	2.85	5.59	7.26
	0.93	0.48	2.37	4.54	7.29	8.81
2 Used cars	0.06	0.78	2.98	6.69	10.89	15.61
	0.42	1.40	3.64	7.03	10.79	15.52
3 New & used trucks	-0.27	-0.55	1.09	3.90	6.87	9.38
	0.87	1.17	2.68	4.99	7.71	10.19
4 Tires & tubes	-0.01	0.20	1.23	3.16	5.29	7.63
	0.14	0.44	1.55	3.54	5.73	8.15
5 Auto accessories & parts	0.01	0.17	1.02	2.76	4.67	6.84
	0.14	0.41	1.33	3.14	5.09	7.35
Furniture & Household Equip.	0.14	0.87	2.29	4.37	6.55	8.68
	0.11	0.76	2.11	4.19	6.33	8.46
6 Furniture,mattresses,	0.33	1.96	4.23	7.02	10.21	12.56
	0.39	2.08	4.31	7.19	10.48	12.82
7 Kitchen, household appl.	0.13	0.73	1.77	3.68	5.96	8.30
	0.45	1.36	2.50	5.23	8.26	10.13
8 China,glass & tableware	-0.09	-0.45	0.36	2.52	4.53	6.87
	0.21	0.17	1.09	3.26	5.19	7.69
9 Radio, tv, records	0.07	0.59	1.86	3.64	5.28	7.16
	-0.24	-0.11	1.01	2.63	4.03	5.91
10 Floor coverings	0.48	2.17	3.81	6.14	9.00	11.36
	0.85	2.90	4.48	6.65	9.32	11.77
11 Durable housefurnishings	0.32	1.52	3.07	5.32	7.96	10.02
	0.74	2.31	3.79	5.86	8.36	10.54
12 Writing equipment	0.00	1.16	4.59	6.85	8.59	9.13
	-0.11	1.05	4.53	6.86	8.96	9.75
13 Hand tools	0.06	0.43	1.56	3.26	4.90	6.47
	-0.02	0.19	1.25	2.93	4.52	6.26

TABLE 46 (Continued)

	2000	2010	2020	2030	2040	2050
Other Durables	-0.19	-0.02	2.18	4.63	7.11	9.58
	0.36	1.01	3.25	5.50	8.01	10.53
14 Jewelry	-0.34	-1.09	0.37	3.02	5.78	8.80
	0.71	1.01	2.46	4.74	7.40	10.46
15 Ophthalmic & orth. goods	-0.03	0.82	2.65	4.52	7.09	10.02
	0.11	1.05	2.93	3.79	6.69	9.09
16 Books & maps	-0.07	-0.10	1.14	2.97	4.28	5.45
	-0.06	-0.09	1.15	3.03	4.38	5.66
17 Wheel goods & durable toys	0.12	0.96	3.08	5.92	8.54	10.74
	0.28	1.18	3.25	6.09	8.73	11.06
18 Boats, rec vech., & aircraft	-0.71	-0.47	3.79	6.08	8.49	10.78
	0.41	1.57	5.69	7.60	9.92	12.23
Non-Durable Goods	0.07	0.69	2.15	4.23	6.63	9.05
	-0.00	0.55	1.96	3.95	6.37	8.98
Food and Alcohol	0.05	0.75	2.28	4.28	6.81	9.40
	-0.09	0.47	1.94	3.89	6.44	9.19
19 Food, off premise	0.12	1.28	3.13	4.79	7.12	9.54
	0.06	1.19	3.04	4.66	7.04	9.59
20 Food on premise	-0.04	0.21	1.42	3.52	5.82	8.10
	-0.23	-0.22	0.93	2.96	5.23	7.66
21 Alcohol, off premise	-0.04	-0.04	1.32	5.20	10.95	17.98
	-0.25	-0.53	0.68	4.53	10.30	17.93
22 Alcohol, on premise	-0.21	-1.51	-1.76	2.32	6.64	9.48
	-1.00	-3.35	-4.17	-0.11	4.34	7.68
Clothing	0.10	0.74	2.25	4.49	6.82	9.14
	0.07	0.63	2.08	4.19	6.45	8.91
23 Shoes & footwear	0.05	0.41	1.66	3.83	6.06	8.32
	-0.15	0.02	1.18	3.24	5.40	7.83
24 Women's clothing	0.17	1.04	2.67	4.86	7.18	9.45
	-0.01	0.62	2.14	4.18	6.41	8.81
25 Men's clothing	0.00	0.34	1.75	4.11	6.53	8.95
	0.33	0.94	2.40	4.66	7.02	9.56
26 Luggage	-0.17	-0.44	0.83	3.44	6.21	9.33
	0.69	1.32	2.60	4.97	7.65	10.80
27 Gasoline & oil	0.05	0.46	1.68	3.84	6.14	8.72
	0.17	0.69	1.99	4.20	6.57	9.26
28 Fuel oil & coal	-0.70	-1.20	0.99	1.72	4.57	5.74
	-1.33	0.18	0.62	-0.79	2.70	6.03
29 Tobacco	0.04	1.02	3.94	9.20	16.49	24.29
	-0.31	0.23	2.91	8.50	16.11	24.16
31 Drug preparations & sundries	-0.27	-0.55	1.09	3.90	6.87	9.38
	0.87	1.17	2.68	4.99	7.71	10.19

TABLE 46 (Continued)

		2000	2010	2020	2030	2040	2050
	Other Non-Durables	0.12	0.66	1.92	3.86	5.88	7.88
		0.09	0.58	1.79	3.67	5.64	7.77
30	Semidurable housefurnishing	0.46	1.55	2.67	4.95	7.69	9.86
		0.63	1.95	3.01	5.10	7.78	10.10
32	Toilet articles & preps	0.10	0.66	1.95	3.88	6.18	8.64
		0.06	0.54	1.78	3.64	5.86	8.42
33	Stationery & writing supp.	0.10	0.59	2.30	4.15	5.44	6.34
		0.03	0.46	2.11	4.04	5.40	6.50
34	Nondurable toys	0.10	0.56	1.71	3.48	5.17	6.96
		0.01	0.34	1.41	3.12	4.75	6.65
35	Flowers, seeds, plants	0.09	0.56	1.75	3.49	5.32	7.28
		0.01	0.36	1.48	3.14	4.88	6.95
37	Cleaning preparations	0.10	0.67	1.91	3.82	5.94	8.29
		0.06	0.53	1.71	3.54	5.60	8.04
36	Lighting supplies	0.06	0.53	1.69	3.49	5.51	7.60
		-0.01	0.36	1.47	3.16	5.13	7.33
38	Household paper products	-0.27	-0.55	1.09	3.90	6.87	9.38
		0.87	1.17	2.68	4.99	7.71	10.19
39	Magazines & newspaper	-0.08	-0.14	1.36	3.49	4.98	6.25
		-0.08	-0.14	1.37	3.57	5.10	6.51
40	Other nondurables--identity	0.04	0.42	1.65	3.61	5.70	7.86
		0.00	0.31	1.50	3.41	5.51	7.79
	Services	0.05	0.24	1.03	2.60	4.28	6.16
		-0.09	0.00	0.75	2.35	4.05	6.07
	Housing	0.19	0.96	1.98	3.53	5.37	7.29
		-0.39	-0.19	0.69	2.17	3.95	6.00
41	Owner occupied space rental	0.16	0.90	1.70	2.78	4.47	6.35
		0.28	1.17	2.04	3.10	4.77	6.75
42	Tenant occupied space rental	0.33	1.38	2.99	5.70	8.18	10.46
		-2.23	-3.66	-2.75	-0.27	1.78	4.02
43	Hotels, motels	-0.32	-1.30	-0.50	1.58	2.73	4.65
		0.46	0.18	1.17	3.13	4.18	6.05
44	Other housing	-0.37	-1.67	-0.84	0.99	1.16	1.25
		0.69	0.58	1.86	3.64	3.75	3.97

TABLE 46 (Continued)

		2000	2010	2020	2030	2040	2050
	Household Operation	0.16	1.02	2.35	4.24	6.60	9.15
		-0.26	0.17	1.35	3.18	5.50	8.20
45	Electricity	0.19	1.17	2.55	4.29	6.45	8.80
		-0.02	0.80	2.16	3.92	6.13	8.63
46	Natural gas	0.17	1.39	3.25	5.87	10.25	15.55
		-0.41	0.27	1.89	4.14	8.18	13.49
47	Water & oth sanitary svc	0.25	2.07	6.26	14.13	28.38	48.58
		0.38	2.45	6.95	15.53	31.33	54.21
48	Telephone & telegraph	0.14	0.85	2.05	3.88	5.90	8.07
		-0.71	-0.93	-0.04	1.64	3.47	5.73
49	Domestic services	-0.00	-0.00	-0.01	-0.00	-0.01	-0.00
		0.00	0.01	0.01	0.01	0.01	0.01
50	Household insurance	0.05	0.49	1.40	2.72	4.44	5.86
		0.09	0.50	1.44	2.65	4.41	6.06
51	Oth hhld operations:repair	0.15	0.74	1.49	2.78	4.40	5.81
		0.19	0.79	1.51	2.72	4.35	5.96
52	Postage	0.16	0.79	1.67	3.17	5.06	7.08
		0.19	0.83	1.68	3.13	5.02	7.18
	Transportation	-0.08	-0.15	0.81	2.67	4.56	6.48
		-0.15	-0.19	0.75	2.42	4.27	6.29
53	Auto repair	-0.04	0.05	1.07	2.92	4.78	6.67
		0.09	0.29	1.33	3.08	4.91	6.90
54	Bridge, tolls, etc	-0.08	0.14	1.63	3.96	6.42	8.86
		0.05	0.33	1.85	4.06	6.56	9.11
55	Auto insurance	0.02	0.21	1.24	3.46	5.62	8.32
		0.08	0.31	1.36	3.55	5.66	8.43
56	Taxicabs	-0.46	-0.65	1.61	3.57	6.67	8.95
		-5.17	-3.35	-0.43	-1.94	1.26	2.52
57	Local public transport	-0.01	0.38	2.27	4.97	8.32	12.28
		-1.41	-2.87	-1.68	0.39	2.84	6.31
58	Intercity railroad	0.00	0.21	1.30	3.35	6.02	9.36
		-0.04	0.16	1.18	3.09	5.65	8.98
59	Intercity buses	-0.09	0.33	3.92	13.30	31.64	46.99
		-0.11	0.09	3.60	11.92	30.63	48.28
60	Airlines	-0.22	-0.88	-0.23	1.50	3.20	4.81
		-0.32	-1.20	-0.64	0.87	2.44	4.24
61	Travel agents,oth trans svc	-0.28	-0.92	0.07	1.64	3.47	4.50
		-0.38	-1.19	-0.24	1.11	2.97	4.24

TABLE 46 (Continued)

	2000	2010	2020	2030	2040	2050
Medical Services	0.06	0.01	0.10	1.01	2.18	3.59
	0.32	0.55	0.81	1.97	3.11	4.47
64 Physicians	0.06	-0.03	0.23	1.47	2.36	3.22
	0.38	0.47	0.54	2.05	2.77	3.79
65 Dentists & other prof svcs	0.02	0.55	2.11	3.28	4.62	5.86
	0.08	0.69	2.35	3.32	3.90	4.98
66 Hospitals	0.12	-0.11	-0.49	0.22	1.39	2.84
	0.47	0.65	0.50	1.48	2.72	4.05
67 Health insurance	0.02	0.21	0.91	2.51	4.63	7.44
	-0.01	0.18	1.00	2.61	5.14	8.13
80 Nursing homes	-0.11	-0.69	-0.79	0.42	2.39	4.50
	0.32	0.24	0.39	2.04	4.01	6.02
76 Education	-0.30	-1.29	0.55	4.46	6.56	9.31
	0.87	0.95	3.08	6.71	8.48	11.05
Other Services	-0.04	-0.15	0.59	2.21	3.99	6.00
	-0.20	-0.44	0.28	1.89	3.71	5.95
62 Laundries & shoe repair	0.12	0.80	2.29	4.59	6.81	8.97
	-0.73	-0.76	0.56	2.78	5.02	7.37
63 Barbershops & beauty shops	0.01	0.45	1.30	2.30	3.80	5.55
	-0.08	0.22	1.06	1.96	3.48	5.27
68 Brokerage,invest. counseling	-0.03	-0.52	-0.05	1.73	3.19	5.01
	-0.97	-2.51	-2.10	-0.02	1.61	3.84
69 Bank service charges	0.04	-0.25	0.45	2.50	4.20	6.26
	-0.79	-1.89	-1.25	1.08	3.01	5.42
70 Life insurance	0.08	0.38	1.51	3.34	5.43	7.93
	0.49	1.23	2.45	4.18	6.24	8.85
71 Legal services	0.07	-0.10	0.18	1.72	3.20	4.87
	-0.61	-1.48	-1.31	0.42	2.00	4.00
72 Funerals, oth pers business	0.06	-0.09	0.37	1.96	3.42	5.00
	-0.57	-1.38	-0.99	0.79	2.33	4.24
73 Radio & tv repair	0.23	1.31	2.35	3.69	5.58	7.13
	0.06	0.92	1.93	3.32	5.38	7.26
74 Movies, theater,spec sports	-0.09	-0.09	0.66	2.47	4.94	7.74
	0.04	0.32	1.04	2.50	4.81	7.85
75 Other recreational services	-0.32	-0.93	0.33	1.96	3.70	5.44
	-0.17	-0.45	0.82	2.17	3.88	5.88
77 Religious & welfare svcs	0.01	-0.05	0.01	1.05	2.88	5.35
	0.28	0.47	0.50	1.31	2.97	5.54
78 Foreign travel by U.S.	0.08	0.58	2.18	4.18	5.94	7.57
	0.02	0.43	1.99	4.17	6.25	8.01
79 Travel in U.S. by foreigner	0.00	-0.00	-0.01	0.04	0.02	-0.01
	-0.00	-0.00	-0.01	0.03	-0.01	-0.03

The Fertility scenario forecast of nearly all of the PCE categories differs between the two versions of LIFT. The discussion here is limited to the categories with the largest differences between the two versions of LIFT. These categories include: all five Motor vehicles and parts categories (PCE1 through PCE5); Jewelry (PCE14); Alcohol, on premise (PCE22); Tenant occupied space (PCE42); and Education (PCE76). Three of these -- Motor vehicles and parts, Jewelry and Education -- increase when changes in the indirect-age variables are incorporated into the forecast and two -- Alcohol, on premise and Tenant occupied space -- fall.

The change in all of these categories can be attributed to the decline in small-sized households (one or two person) and the increase in the larger households. For example, the small-sized households tend to be renters more often than the larger households. The decline in the number of small households (relative to the Base) and the increase in the number of large households indicates a shift towards owner-occupied housing. Similarly, small households tend to spend more on Alcohol than larger households spend and the decline in small-sized households causes spending on Alcohol, on premise to fall relative to the Base when the changes in the indirect-age demographic variables are included in the forecast.[88]

Since the macroeconomic effects of including the indirect-age demographic effects are small, one might wonder if there is any value to including their effects in the forecast. To answer that question, we must look at the forecast at a finer detail than the macroeconomic summary table. While the macroeconomic effects are small, the effects on an industry basis are relatively larger. Table 47 shows the employment by

[88]Alcohol, on premise is alcohol consumed where it was purchased.

several broad industry classifications for the two Fertility scenarios. The first line of the table shows the difference from Base for old-LIFT and the second line shows the difference from Base for new-LIFT.

TABLE 47

Fertility Scenario: Employment by Industry (1000's of jobs)

	2000	2010	2020	2030	2040	2050
Agriculture, forestry, fisher	-0	4	24	65	114	166
	-1	2	22	63	112	166
Mining	0	3	11	25	43	65
	1	3	11	26	45	67
Construction	9	62	184	390	701	1108
	7	57	179	481	896	1234
Nondurables manufacturing	5	29	100	212	343	486
	4	29	99	212	338	480
Durables manufacturing	7	54	172	346	600	878
	13	65	184	379	636	898
Durables, exc. Medical	7	53	170	342	594	870
	12	63	182	375	630	890
Medical Inst. and Opth.	0	1	2	4	6	8
	0	1	2	4	6	8
Transportation	-5	-27	-3	90	205	352
	-9	-34	-10	89	208	353
Utilities	2	13	36	70	109	153
	-1	8	31	66	106	151
Trade	-24	-68	251	1204	2441	4023
	-30	-98	196	1152	2394	3944
Finance, insurance, real estate	2	-5	67	296	562	902
	-34	-83	-36	208	488	819
Services, nonmedical	-32	-196	-92	739	1846	3350
	-10	-144	-32	780	1866	3401
Medical services	-2	-61	-79	81	380	816
	29	20	25	246	580	1056

Table 47 shows the importance of incorporating the indirect-age demographic effects in the forecast. Employment in Medical services is twenty-five percent higher by incorporating these effects. The 240,000 jobs represent a twenty-five percent increase in the number of jobs in these industries. These 240,000 jobs represent about four percent of the number of jobs in the Medical services in 2050 under the new-LIFT Fertility scenario. For those interested in forecasting employment in these industries, the indirect-age demographic effects are non-trivial.

The three scenarios highlight the work done in this study. By modeling Medicare as a price subsidy, we insure that the direct effects of any changes in the program occur in medical PCE and not in categories with high income elasticities. While incorporating the indirect-age demographic has marginal effect on the macroeconomic outlook, the effects by industry can be very important, especially to those interested in industry forecasts.

CHAPTER 8

CONCLUDING REMARKS

The purpose of this study has been to improve the simulation capabilities of a long-term econometric model. These capabilities have been improved by three innovations. The first improvement was in the modeling of the income distribution. Prior to this study, the income distribution model used in LIFT produced forecasts of the income distribution that were theoretically impossible. With the new income distribution model, the Lorenz curves generated by LIFT are valid. As part of this work, we had to find a way of exactly fitting a Lorenz curve to several years of known data while being able to predict Lorenz curves for years in which data were unavailable. By applying the properties of a Lorenz curve, we were able to find a functional form that fit the known data exactly, but also allowed the forecasting and backcasting of the Lorenz curve.[89]

The second improvement involves the new treatment of Medicare as a price subsidy. Prior to this work, changes in the Medicare program affected personal consumption expenditures (PCE) through an income effect, with the greatest impact occurring to PCE categories with large income elasticities. The Medicare scenario discussed in chapter 7 showed that the old treatment of Medicare resulted in a larger change in Automobile-related PCE than in Medical-related PCE. By modeling Medicare as a price subsidy, the overall fit of the system of equations discussed in chapters 4 and 5 improved by over ten percent. The individual fit of PCE categories

[89]Early drafts had us finding a functional form that would "fight with the known data."

accounting for over eighty percent of total PCE improved as well. Nearly as important, chapter 5 also showed that incorrectly modeling a government transfer can result in incorrect forecasts of the effects of a change in the transfer program.

The third improvement is the indirect-age demographics model discussed in chapter 6. Prior to this work, the was no consistent linkage between the size or age structure of the population and the indirect-age demographic variables. As illustrated in the Fertility scenario in chapter 7, with the new demographics model, an increase in the fertility rate will affect the various indirect-age demographic variables in a logical and consistent manner. The Fertility scenario in chapter 7 highlighted the importance of these forecasts by showing the industry and PCE from not consistently modeling these variables.

In the immediate-term, the Medicare as a subsidy work is already in use as a simulation tool. For the short-term continued research concentrate on (1) how well other systems of demand equations incorporate the current work; (2) illustrating the shortcomings of other systems of demand equations for long-run forecasting; (3) policy simulations of interest.

For the medium-to-long run, research will concentrate on endogenizing the exogenous demographic assumptions. Currently, labor participation, fertility and immigration rates are all exogenous. These exogenous rates do not respond to changes in economic conditions. Fortunately, work-in-progress intends to correct this problem, but a solution is not on the immediate horizon. Until the work that will endogenize these assumptions is completed, these forecasts must remain exogenous.

APPENDIX TO CHAPTER 4

Property 1 of a Lorenz Curve

If $L_1(x) \in L$ and $L_2 = L_1(x)^\beta$, then $L_2(x) \in L$, $1 \leq \beta$.

Condition 1: $L_2(0) = L_1(0)^\beta = 0^\beta = 0$.
Condition 2: $L_2(1) = L_1(1)^\beta = 1^\beta = 1$.
Condition 3: $\partial^2 L_2(x)/\partial x^2 = \beta(\beta-1)L_1(x)^{\beta-2}\, \partial L_1/\partial x + \beta L_1(x)^{\beta-1}\, \partial^2 L_1/\partial x^2 \geq 0$ since all of the terms on the left-hand side of the expression must be non-negative.

Property 2 of a Lorenz Curve

If $L_1(x), L_2(x) \in L$, then $\lambda L_1(x) + (1-\lambda)L_2(x) \in L$, $0 \leq \lambda \leq 1$.

Condition 1: $\lambda L_1(0) + (1-\lambda)L_2(0) = 0$.
Condition 2: $\lambda L_1(1) + (1-\lambda)L_2(1) = 1$.
Condition 3: $\lambda\, \partial^2 L_1(x)/\partial x^2 + (1-\lambda)\, \partial^2 L_2(x)/\partial x^2 \geq 0$ since λ, $1-\lambda$, and the second derivative of the two Lorenz curves are all non-negative.

Property 3 of a Lorenz Curve

Let $f(x)$ be some function where $f(1) = 1$, $f(x) \geq 0\ \forall\ x \geq 0$, f', $f'' \geq 0$ and $L(x) \in L$, then $L(x) \cdot f(x) \in L$.

Condition 1: $L(0) \cdot f(0) = 0 \cdot f(0) = 0$.
Condition 2: $L(1) \cdot f(1) = 1 \cdot f(1) = 1$.
Condition 3: $L(x) \cdot \partial^2 f(x)/\partial x^2 + f(x) \cdot \partial^2 L(x)/\partial x^2 + 2(\partial f(x)/\partial x)(\partial L(x)/\partial x) \geq 0$ since all of the terms in the expression must be non-negative.

Property 3A of a Lorenz Curve

If $L_1(x), L_2(x) \in L$, $L_1(x) \cdot L_2(x) \in L$.

The proof of this is trivial and is identical to the proof of Property 3.

Property 4 of a Lorenz Curve

$L(x) = x \in L$. Let $L(x) = x$.
Condition 1: $L(0) = 0$.
Condition 2: $L(1) = 1$.
Condition 3: $\partial^2 L(x)/\partial x^2 = 0 \geq 0$

Conversion from {x,y} Coordinates to {η,π} Coordinates

$$\eta = \frac{1}{\sqrt{2}} (x+y) \qquad (4.2b);$$

$$\pi = \frac{1}{\sqrt{2}} (x - y) \qquad (4.2c).$$

a) $x^2 + y^2 = \eta^2 + \pi^2$ (Pythagorean Theorem)

b) $\pi^2 = x^2 + y^2 - \eta^2$ (rearrangement of a)

c) $\pi^2 = (1-x)^2 + (1-y)^2 - (2^{.5} - \eta)^2$ (Pythagorean Theorem)

d) $x^2 + y^2 - \eta^2 = (1-x)^2 + (1-y)^2 - (2^{.5} - \eta)^2$ (combination of b and c)

e) $x^2 + y^2 - \eta^2 = 1 - 2x + x^2 + 1 - 2y + y^2 - 2 + 2^{1.5} o - o^2$ (expansion of d)

f) $0 = 2^{.5}\pi - x - y$ (simplification of e)

g) Equation 5.3b $\pi = (x + y)/ (2^{.5})$ (rearrangement of f)

h) $\eta^2 = x^2 + y^2 - x^2/2 - xy - y^2/2$ (substitution of g in a)

i) $\eta^2 = x^2/2 + y^2/2 - xy$ (simplification of h)

j) $\eta^2 = (x^2 - 2xy + y^2)/2$ (rearrangement of i)

k) $\eta^2 = [(x - y)/2^{.5}]^2$ (factor right-side of j)

l) Equation 5.3c $\eta = (x - y)/2^{.5}$ (solution to k)

Concavity of the Pollock Function

The first derivative of the Pollock function is given below:

$$\frac{d\pi}{d\eta} = A\frac{d\pi_{base}}{d\eta} + 1.5B\ \eta^{.5}\ (\sqrt{2}-\eta)^{.5} - \frac{.5B\ \eta^{1.5}}{(\sqrt{2}\ -\ \eta)^{.5}} \qquad (4.4).$$

The second derivative of this function is:

$$\frac{d^2\pi}{d\eta^2} = A\frac{d^2\pi_{base}}{d\eta^2} + \frac{B}{4}\{\frac{(\sqrt{2}\ -\eta)^{.5}}{\eta^{.5}} - \frac{\eta^{.5}}{(\sqrt{2}\ -\ \eta)^{.5}} - 3\frac{\eta^{.5}}{(\sqrt{2}\ -\ \eta)^{.5}} - \frac{\eta^{1.5}}{(\sqrt{2}\ -\ \eta)^{1.5}}\}$$

When η equals zero, the value of the term within the parenthesis equals positive infinity since there is a divide by zero in the expression. Thus for all positive values of B, the second derivative is positive at η equals zero and thus the function is not quasi-concave. Thus we know that B cannot be positive. When η equals $2^{.5}$, the value of the term within the parenthesis equals negative infinity. Consequently, for all negative values of B, the second derivative is positive at η equals $2^{.5}$. Thus, we know that B cannot be negative. Since B cannot be negative or positive, it must equal zero. Since, by definition, the value of the function must equal $2^{.5}$ when η_{base} equals $2^{.5}$, A must equal one when B equals zero.

The Gini Coefficient

The Gini coefficient for any function equals:

$$Gini = \frac{Area_A - Area_B}{Area_A} = \frac{\frac{1}{2} - Area_B}{\frac{1}{2}} = 1 - 2\,Area_B \qquad (4A.1);$$

where:

$Area_A$ = Area under 45° line in Figure **XX+1**;
$Area_B$ = Area under Lorenz curve in Figure **XX**+1.

We can determine the Gini coefficient for the Rasche function as follows:

$$Gini = 1 - 2\int_0^1 [1 - (1 - x)^{\alpha}]^{1/\beta}\, dx \qquad (4A.2).$$

$$Let\ U = 1 - (1 - x)^{\alpha}; \qquad (4A.3)$$

$$Then\quad \begin{aligned} dU &= \alpha\,(1 - x)^{\alpha - 1}\, dx \\ dx &= \frac{1}{\alpha}\,(1 - U)^{\frac{1}{\alpha} - 1} \end{aligned} \qquad (4A.4).$$

$$Gini = 1.0 - \frac{2.0}{\alpha}\int_0^1 U^{1/\beta}\,(1 - U)^{-(1 + 1/\alpha)} \qquad (4A.5).$$

This is nothing more than a Beta distribution with parameters $1/\alpha$ and $1 + 1/\beta$.

The Variance and Mean of the Rasche Function

Y is a random variable with a probability density function, f(x).

$$f(x) = \frac{\alpha}{\beta}(1 - x)^{(\alpha - 1)}[1 - (1 - x)^{\alpha}]^{\frac{1}{\beta} - 1} \quad 0 \le x \le 1 \qquad (4A6).$$
$$= 0 \qquad elsewhere$$

And cumulative distribution function, F(x).

$$F(x) = [1 - (1 - x)^{\alpha}]^{\frac{1}{\beta}} \qquad (4a.7).$$

The mean of Y, μ_y is given by:

$$\mu_y = \int_0^1 x\, f(x)\, dx \qquad (4A.8).$$

This can be solved through integration by parts.

Let A = x, then dA = dx

Let dC = f(x)dx, then C = F(x).

Then equation 4 equals:

$$\mu_y = x\,[1 - (1 - x)^{\alpha}]^{\frac{1}{\beta}}\Big|_0^1 - \int_0^1 [1 - (1 - x)^{\alpha}\,]^{\frac{1}{\beta}-1} \qquad (4A.9).$$

The first term in equation (4A.9) equals 1 and the second term in equation (4A.9) is the Beta distribution. This gives us the mean of y:

$$\mu_y = 1 - \frac{1}{\alpha}\int_0^1 U^{\frac{1}{\beta}} (1 - U)^{\frac{1}{\alpha} - 1} = 1 - \frac{1}{\alpha} B(\frac{1}{\alpha}, 1 + \frac{1}{\beta}) \quad (4A10).$$

Where B is the Beta distribution.

The variance of Y, σ_y^2 is given by:

$$\sigma_y^2 = \{\int_0^1 x^2 f(x)\, dx\} - \mu_x^2 \quad (4A.11).$$

The second term in equation (7) equals:

$$\mu_x^2 = 1 - \frac{2}{\alpha}B() + \frac{1}{\alpha^2}B()B() \quad (4A.12).$$

where B() equals B($1/\alpha$, $1 + 1/\beta$).

The first term in equation (7) can be partially solved through integration by parts.

Let $A = x^2$, then $dA = 2x\ dx$;

Let $dC = f(x)\ dx$, then $C = F(x)$.

$$\int_0^1 A\ dC = AC\Big|_0^1 - \int_0^1 C\ dA \quad (4A.13).$$

Substituting into equation (9) we get:

$$= x^2 \left[1 - (1 - x)^\alpha)\right]^{\frac{1}{\beta}} \Big|_0^1 - 2\int_0^1 x \left[1 - (1 - x)^\alpha\right]^{\frac{1}{\beta}} dx \qquad (4A.14).$$

reduces to:

$$= 1 - 2\int_0^1 x \left[1 - (1 - x)^\alpha\right]^{\frac{1}{\beta}} dx \qquad (4A.15).$$

Substituting into (4A.11) and using B() to represent the Beta distribution:

$$\sigma_y^2 = (1 - 1) + \frac{2}{\alpha} \text{B()} - \frac{1}{\alpha^2}\text{B()B()} - 2\int_0^1 x \left[1 - (1 - x)^\alpha\right]^{\frac{1}{\beta}} dx \qquad (4A.16).$$

$$\sigma_y^2 = \frac{2}{\alpha} \text{B()} - \frac{1}{\alpha^2}\text{B()B()} - 2\int_0^1 x \left[1 - (1 - x)^\alpha\right]^{\frac{1}{\beta}} dx \qquad (4A.17).$$

The third term cannot be integrated easily and for this reason, we calculate the value of the integral by direct evaluation using the extended trapezoidal rule. Use of any direct evaluation rule implies that the calculated value of the integral will contain an approximation error. Consequently, the standard deviation used in evaluating the coefficient of error is an estimate.

APPENDIX TO CHAPTER 5

Notation:

P_x^0		Initial Price of Good X (the subsidized good)
P_x^1	$= (1\text{-}s)P_x^0$	Subsidized Price of Good X, where $0<s<1$
P_{else}		Vector of Prices for all Other Goods
M_0		Initial Income
M_1	$=M_0+M_{Tran}$	Income After Income Transfer
$G(P_x,P_{else},M)$		Marshallian or Normal Demand Functions
X_0	$= G(P_x^0,M_0)$	Initial Demand
X_1	$= G(P_x^1,M_0)=\Delta P_x(\partial G/\partial P_x) + X_0$	Demand under Subsidy
X_2	$= G(P_x^0,M_1)=\Delta M(\partial G/\partial M) + X_0$	Demand under Income Transfer
E^m	$= (P_x/X) * (\partial G/\partial P_x)$	Marshallian Own Price Elasticity, Good X
η	$= (M/X) * (\partial G/\partial M)$	Income Elasticity, Good X
α	$= P_x^0X_0/M$	Budget Share, Good X

When will $\Delta X^{Transfer} = X_2 - X_0 > \Delta X^{subsidy} = X_1 - X_0$ when the cost of subsidy equals cost of transfer ?

$\Delta P = \Delta P = P_x^1 - P_x^0 = (1\text{-}s)P_x^1 - P_x^0 = -sP_x^0$ (5A.1)
$\Delta M = -\Delta PX_1 = sP_x^0X_1$ (5A.2)
$X_1 = \Delta P(\partial G/\partial P_x) + X_0 = -sP_x^0(\partial G/\partial P_x) + X_0$ (5A.3)
$X_2 = \Delta M(\partial G/\partial M) + X_0 = -\Delta PX_1(\partial G/\partial M) + X_0 = sP_x^0X_1(\partial G/\partial M) + X_0$ (5A.4)

We exclude the pathological case of a Giffen Good, therefore $X_1 - X_0 > 0$
Thus, $\Delta X_{Transfer}>\Delta X_{Subsidy}$ as $X_2 > X_1$

$sP_x^0X_1(\partial G/\partial M) + X_0 > -sP_x^0(\partial G/\partial P_x) + X_0$ (5A.5)
Simplifying gives us:

$X_1(\partial G/\partial M) > -(\partial G/\partial P_x)$ (5A.6)
Substitute for X_1:

$-sP_x^0(\partial G/\partial P_x)(\partial G/\partial M) + X_0(\partial G/\partial M) > -(\partial G/\partial P_x)$ (5A.7)
Multiply both sides by P_x^0/X_0 and simplify:

$-sP_x^0(\partial G/\partial M)E^m + P_x^0X_0(\partial G/\partial M)/X_0 > -E^m$ (5A.8)
Multiply First Term by $X_0M/X_0M = 1$ and simplify:

$-s\eta\alpha E^m + P_x^0 X_0(\partial G/\partial M)/X_0 > -E^m$ (5A.9)

Multiply Second Term by M/M = 1 and simplify:

$E^m - s\eta\alpha E^m + \eta\alpha > 0$ (5A.10)

An Income Transfer will generate more demand than a price subsidy if the above equation holds.

$\Delta X^{in\text{-}kind} = X_3 - X_0 = \Delta X^{subsidy} = X_1 - X_0$, the cost of subsidy equals cost of transfer and a 1:1 link holds, IFF

$E^m - s\eta\alpha E^m + \eta\alpha = 0$ (5A.10)

For the assumption of a one-to-one link to be true, $\partial G/\partial M = 1.0$ Thus, $\eta = M/X_0$.
Substitute:

$E^m - s(M/X_0)(P_x^0 X_0/M)E^m + (M/X_0)(P_x^0 X_0/M) = 0$ (5A.11)

$$E^m = \frac{-P_x^0}{1 - s\,P_x^0} = \omega(s,\ P_x) \tag{5A.12}$$

Simplify and solve for E^m to give us:

$$E^m = \frac{-P_x^0}{1 - s\,P_x^0} = \omega(s,\ P_x) \tag{5A.13}$$

Except for a few cases, this equation will not hold. Thus, a one-to-one link will not give us the same results as modeling the program as a price subsidy. By imposing a one-to-one link, the size and direction we misstate the effects on PCE of increases in the Medicare program depend on E^m, the subsidy rate and the price of medical care. Since the derivative of equation (5A.13) with respect to both s and P_x is less than zero, we know that as price or the subsidy changed, we would move away from the point where the in-kind transfer was equivalent to a price subsidy.

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