INPUT-OUTPUT ANALYSIS APPLIED TO MRP MODELS WITH COMPOUND DISTRIBUTION OF TOTAL DEMAND

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Abstract

This paper presents the theoretical study about connecting inventory allocation and market area for the goods, defined by customers' travelling decisions, influenced by the expected shortages of goods. Arrivals of customers to the market are Poisson distributed, the order size by arrival of customers is not only one unit of goods but customer can demand any number of products. In this case which is closer to the real world systems, a compound Poisson distribution of cumulative demand has to be considered. In the papers "Inventories in Spatial Models" /3/ and "Inventory Allocation and Customer Travelling Problem in Spatial Duopoly" /3a/ we have emphasized the need for connecting inventory and location analysis. This problem was studied further by H.J.Girlich in his paper "On the Metric Transportation Problems and Their Solution"/4/, which gives us proper foundations for the research described in the paper. The study is influenced by Grubbström-Molinder's work on MRP optimization with the use of Laplace Transforms and Input-Output Analysis /for details see 5-10/. The annuity stream approach has been applied in the environment of spatial duopoly, where the optimal ordering policy depends on the interaction between the prices and shortages of goods in the studied duopolies. Customer travelling problem (**CTP**) was defined, which determines the market area for allocated inventories. In that paper uniform distribution of customer's demand was supposed. In this paper we will upgrade our study with compound Poisson distribution of cumulative demand.

Key words: location, transport, inventory, MRP, Input-Output Analysis, annuity stream, shortage of goods, duopoly, customer travelling problem, compound Poisson distribution

1. Consumer shopping behaviour and local market demand

The benefit that consumers derive from the facilities system can be used as a measure of consumers' welfare. Opportunities available at a certain demand location are often measured in terms of the cost of reaching a critical number of facilities from consumer's location. The cost of reaching the goods from the supply centres depends on spatial distribution of supply centres (the distances from the user location) and the probability of the goods being available in these shops when required. We can assume that the trip chaining generally occurs when the purchase fails at a certain supply centre. Then the trip is continued to another destination where the required goods are supplied. If the purchase also fails at a second destination, this chaining of trips continues until the goods are found. The probability of success at a certain supply point depends on the inventory management at this point.

Having the information about the location of supply points and the probability of success, we can construct, for each separate demand point in the area, the chain of trips with minimal expected cost of travel at a certain level of risk.

2. Consumer behaviour and MRP approach

Various approaches describing production, inventory, transportation and consumption as the transformation of one set of resources to another set of resources and its distribution can be found in literature (for details see /10/). The study of spatial interactions and their influence on optimal inventory policy has not been studied sufficiently. The first such approach has been given in the paper "Inventories in Spatial Models " /3/.

Production-supply units which form a spatial oligopoly, need warehouses to assure the final production stage, if it is located there, and to assure the supply of the final products. In case that the final products are assembled at the location of supply, the components for production of this final products have to be assured with an acceptable shortage level. Their production and supply policy consist of MRP decisions, price and acceptable shortage determination. In the literature there are known two approaches to maximize the supplier's profits: the traditional average cost approach and the more appropriate - annuity stream approach, which is used here.

The basic question at MRP decisions is how to determine the optimal policy for a sequence of production quantities. When stochastic demand depends on spatial oligopoly and the sequences must be decided all at once, it is spatial MRP model which will give us the answer on this question. Though in this paper in the details we will be considering two - level systems which appear in spatial duopoly, the theory developed here is suitable for generalization to multi - level systems in spatial oligopoly. The duopoly results are embedded in annuity stream approach to be able to study the interaction among prices, shortages and ordering policies. In a sense, our treatment which uses the results of /10/ , /1/ and /2/, develops a MRP analysis in continuous time.

Instead of minimizing average costs involved in production inventory processes, the maximization of the net present value *(NPV)* of all payments, including opportunity costs which are the results of shortage of goods, is included in the procedure of ordering policy. According to the suggestion given in /10/, by multiplying the *NPV* by an annuity factor we obtain the hypothetical constant payment stream (the annuity stream) yielding the same *NPV*. If the payments are described in terms of their Laplace transform, the *NPV* is directly obtained by exchanging the complex frequency s for the continuous interest rate ρ.

Detailed coordination of a two - level MRP system is presented by Grubbström and Molinder in /10/. In their paper, for the sake of simplicity, it is assumed that the single final product contains one type or one set of components. In our paper this assumptions stay the same.

The detailed coordination problem in the mentioned literature involves the simultaneous optimization of the ordering intervals, ordering quantities and the initial lags. The shortage of goods which interacts with market area and therefore with demand quantity determined by /3/ because of customers' travelling behaviour, is now included in the optimization procedure considering Compound Poisson distribution of cumulative demand as the main contribution of the presented paper.

3. Customer's travelling problem and its behaviour

In the paper of H.J.Girlich /4/ the influence of shortage of goods on the market area was presented as follows. We assume that customers' locations are continuously distributed with the density of demand

$$r(z) = \text{const} > 0$$

on the area A

$$A = \{z \in R^2 : r(z) > 0\}.$$

Retail of production of i-th firm, located at the point z_i of this area, is selling the final product which is available at i-th firm at an arbitrary chosen moment with probability α_i . This probability depends on MRP policy. The set of all customers' locations supplied primarily from z_i is denoted by A_i , defined as

$$A_i = \{z \in A : EC_i(z) \leq EC_j(z),\ j \neq i\} \qquad (1)$$

where $C_i(z)$ denotes the random cost for a desired quantity of the commodity for a customer with residence z patronizing warehouse i. $EC_i(z)$ is its mathematical expectation (see Girlich /4/).

We assume

$$A = \bigcup_I A_i \qquad A_i \cap A_j = \emptyset\ , \quad i \neq j \quad i = 1,2,\dots,n \qquad (1.a)$$

In this paper the distance between the location of retail i ($z_i=(x_i, y_i)$) and consumer $z=(x,y)$ is assumed to be Euclidean. We still have the same assumption.

We are looking for a rational partition of total demand corresponding to market areas described by (1). Every customer has to patronise one retail. His decision depends on

- the mill price p_i for the commodity at i ,
- the transportation cost,
- the uncertainty α_i at each i.

When a customer arrives to the z_i, there are two possible events:

1. supply unit i is empty……………………...$\{W_i = 0\}$
2. the commodity is available at i……………..$\{W_i = 1\}$

The uncertainty of this oligopoly system can be described by random variables W_1, W_2, W_3, ….W_n, with the state space $\{0,1\}$of each random variable and

$$P(W_i=0) = \alpha_i \in [0,1] \text{ and}$$
$$P(W_i=1) = 1-\alpha_i \in [0,1], \quad i = 1,2,\ldots,n$$

where α_i is the probability of shortage of goods at i-th supply unit. (2)

4. Duopoly of supply for spatial demand under the shortage of goods

Let us simplify this problem for the situation, where there are only two production-supply enterprises with their supply in $z_1=(1,0)$ for the first unit and $z_2=(0,0)$ for the second unit (see Fig.1.). Let us study in this paper Girlich's assumption for policy of delivery:

$$P(\, W_i=1 \mid W_j=0) = 1, \text{ for } i \neq j,\ i = 1,2,\ j = 1,2 \tag{3}$$

This assumption ensures that the commodity is available in the other supply unit in case of a shortage of production in j-th unit, j=1,2. We call it the D_2 model.

We wish to calculate market areas for the enterprise in z_1 and the enterprise in z_2 , to determine demands $\mathbf{d_1}$ and $\mathbf{d_2}$ in a time unit if there are given margins for the total area A and density r(z) which is for the sake of simplicity in this paper constant: r(z)=r

Let $C_i(z)$ denote the cost for a unit of the commodity for a customer with residence z patronising supply unit i. Because of uncertain supply in i, where he is going to buy this product at first travel, C_i is a random variable. If l_i is the Euclidean distance to the i-th supply unit and t_i the corresponding transportation cost factor, we obtain for $i \neq j$:

$$C_i(z) = \begin{cases} p_i + 2t_i l_i(z) & \text{, for } W_i = 1 \\ t_i l_i(z) + t_{ij} l_{ij} + p_j + t_j l_j(z) & \text{, for } W_i = 0 \end{cases} \tag{4}$$

Here

t_{ij}............ is the transportation cost factor for travelling from i to j which arises in the case that the good is not available in i and
l_{ij}......... is the distance from i-th to j- th unit.

Every customer from area A_i is travelling first to z_i and in the case that there is a shortage for this item there, he is continuing his travel to z_j, where (according to our assumption) he gets the product with certainty.

From (4) follows the boundary between A_i and A_j:

$$EC_i(z)=EC_j(z) \tag{5}$$

$$(2-\alpha_i -\alpha_j)(t_i l_i(z) - t_j l_j(z)) = t_{ij} l_{ij} (\alpha_j -\alpha_i) + (p_j-p_i)(1-\alpha_i -\alpha_j)$$

Because the transportation costs are known, boundary depends only on shortage α and final price p of duopolists. Boundary divides A_j and A_i and therefore determines the demand $\mathbf{d_i}$ at $z_{i.}$

Duopolist i determines the value of α_i and p_i so that demand $\mathbf{d_i}$

$$\mathbf{d_i}= r((1-a_i)A_i+\alpha_j A_j) \tag{6}$$

will give the optimal annuity stream ρNPV_i:

$$rNPV_i\,(a_1, a_2, p_1, p_2, T_{2i}, m_i) = \mathbf{d_i}\,(\alpha_1,\alpha_2,p_1,p_2)(p_1 - c_{1i} - c_{2i}) -$$
$$-\frac{\rho}{2}(K_{1i}+K_{2i}) - \frac{1}{T_{1i}}(K_{1i}+\frac{K_{2i}}{m_i}) - \mathbf{d_i}\frac{\rho\ T_{1i}}{2}(c_{1i}+m_i c_{2i}) \tag{7}$$

where we use the following notation:

$\rho\ NPV_i$......................annuity stream of i-th supply unit, i=1,2,
$\mathbf{d_i}$ ....external demand rate for final products in i-th supply unit,
c_{1i}...............................the echelon stock value of final product in i-th supply unit (a value added in the process of manufacturing the item)
c_{2i}.......................... the echelon stock value of component in i-th supply unit, unit (a value added in the process of manufacturing the item)
r............................continuous interest rate,
K_{1i}..........................the set-up cost of final products in i-th supply unit
K_{2i}..........................the set-up cost of components in i-th supply unit
T_{1i}.......................... ordering interval for final products in i-th supply unit

T_{2i}………………………. ordering interval for components in i-th supply unit

$m_i = T_{2i} / T_{1i}$

.

The formula (7) for the annuity stream is derived from the general expression for the annuity stream using MRP theory, input output analysis and Laplace transform given in /1/ which is a special two- level case of general expression of $rNPV$, presented in /10/.

5. D_2- model of equal prices embedded in MRP

Let us study the special case 2.3.1. of the paper presented by Girlich /4/ .

He calls it the D_2- model with equal mill prices and equal transport cost factors.
In this example

$$\alpha_1 \leq \alpha_2 < 1 \tag{8}$$

holds.

From (5) he got

$$a + |z| = l_1(z) \tag{9}$$

where

$$a := (\alpha_2 - \alpha_1) / (2 - \alpha_1 - \alpha_2) \tag{10}$$

and from (8) follows

$$0 \leq a < 1 \tag{11}$$

Let us study MRP problem, presented by /1/ and /10/ on the area XY, where

$$XY = (x_{+1} - x_{-1})(y_{+1} - y_{-1})$$

For boundary the following expression is valid:

$$\partial A_2 = \min\{y_{+1}, y\}, \text{ for } y>0$$

$\partial A_2 = \max\{y_{-1}, y\}$, for $y<0$

where y is determined with the following equation

$$y^2 = (x\text{-}0{,}5)^2\,(1\text{-}a^2)/\,a^2\ \text{-}0{,}25(1\text{-}a^2)\quad , a>0$$
$$x = 0{,}5 \qquad\qquad , a=0 \tag{12}$$

In our case we can write

$$y^2 = (x\text{-}0{,}5)^2\,(1\text{-}a^2)/\,a^2\ \text{-}D$$

where Δ is supposed to be negligible for our problems (x_{-1} is great enough). (see Fig. 1 which presents the boundary of market areas for two duopolists).

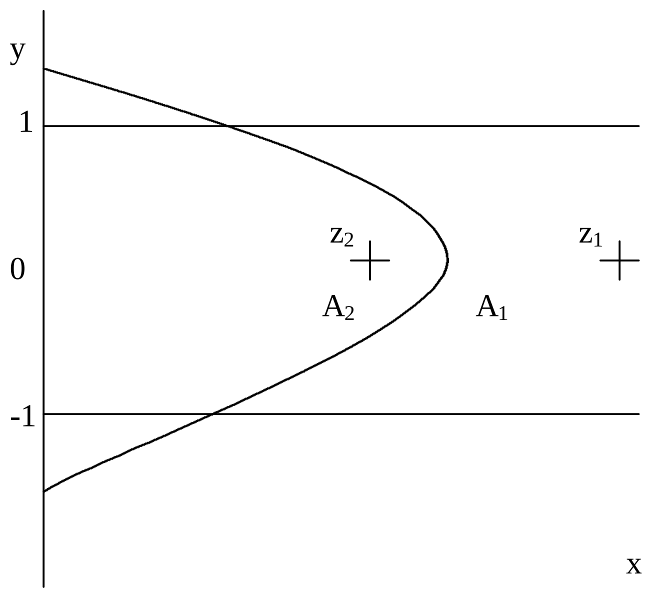


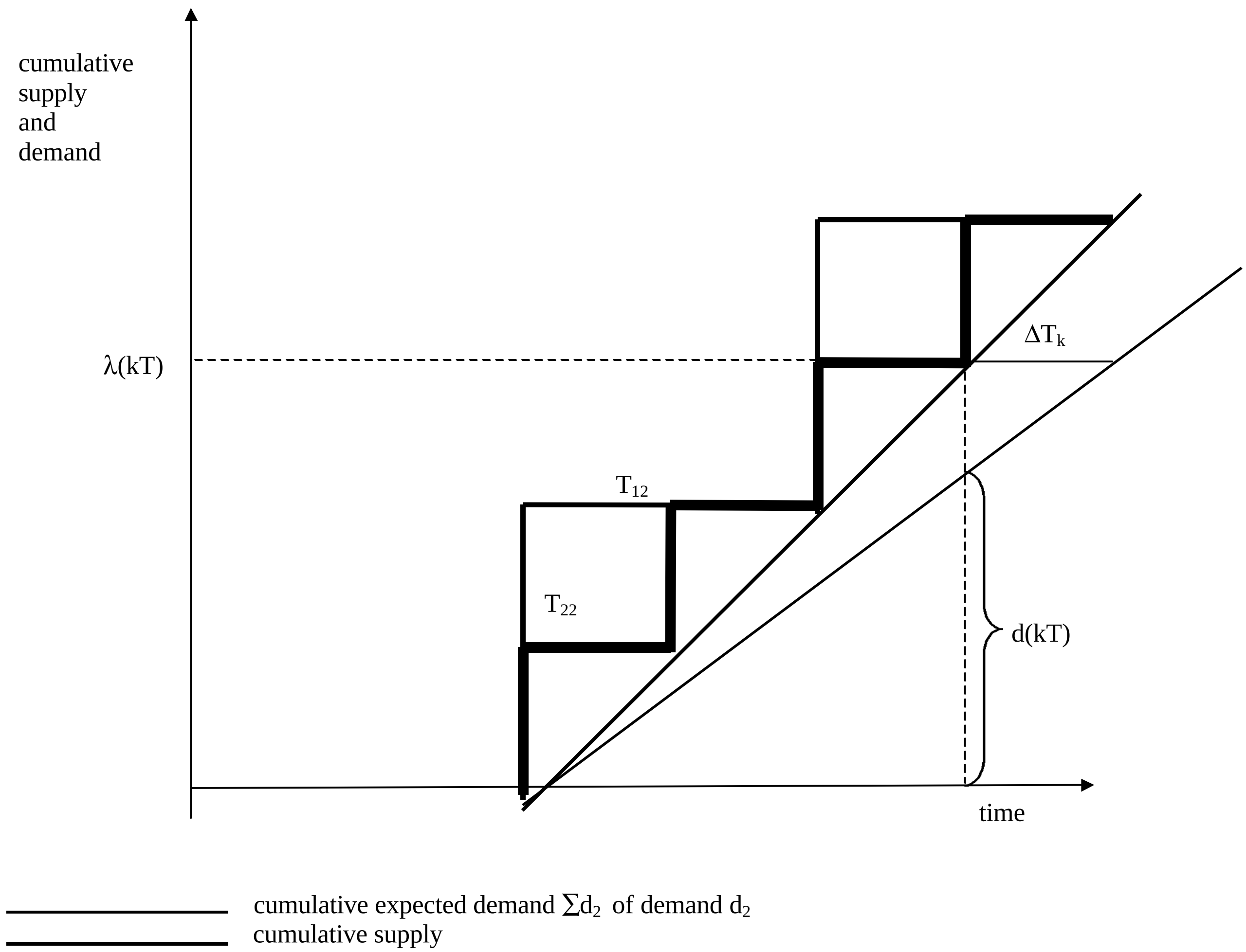
Fig 1. The boundary of market areas for two duopolists.

For the second production - supply unit which knows that the shortage of goods for the first production unit is α_1, in case, that $a>0$ its market area is

- for $|y| \le y_{+1}$

$$A_2(a) = -\sqrt{\frac{1-a^2}{a^2}}\left(\frac{a^2}{4}-\frac{1}{4}-x_{-1}^2+x_{-1}\right) \tag{13}$$

$$d_2(a) = \quad ((1-a_2)\ A_2 + a_1 A_1)r = ((1-a_2)\ A_2 + a_1\,(XY\text{-}A_2))\,r \tag{14}$$



T_{12}……………………… ordering interval for final products in second supply unit
T_{22}……………………… ordering interval for components in second supply unit

Fig. 2: The ordering policy (see details in /10/)

- if there is $|y| > y_{+1}$

$$B(a) = -a\sqrt{1-a^2}\,\left(\frac{1}{4} - \frac{y_{+1}^2}{1-a^2}\right) \qquad (15)$$

$$D(a) = 2\left(|x_{-1}| + \frac{1}{2} - y_{+1}\sqrt{\frac{a^2}{1-a^2}}\right)y_{+1} \qquad (16)$$

$$A_2(\alpha) = B(\alpha) + D(\alpha)$$

The demand in the second example (if there is $|y|>y_{+1}$) has the same expression as in (14). We wish to find a_2 and m_2 at which the annuity stream presented in (7) will be maximal.

Let us for the beginning assume that we do not know much about the distribution of the external demand and we know only upper and lower bound of cumulative demand. In this case we will suppose that the cummulative demand at time t can appear on the interval $((l(t)t - s,\ l(t)t + s))$ with constant probability density and L_2 is projection of *s*.
From this supposition of the demand per unit of time we can determine L_2 (see Fig.2):

$$L_2 = 2a_2T_{12} \tag{17}$$

L_2 is the length of time on the interval T_{12}, where shortage of goods can appear because of increasing demand. In Fig.2 there is drawn the cumulative expected demand $\sum \boldsymbol{d_2}$ and the upper bound of this cumulative demand. In this case the projection of the cumulative demand is supposed to be uniformly distributed on the present belt with the probability ½ which can be assumed only if we have very little information about distribution itself, but we know that cummulative demand cannot exceed upper bound to the right and lower bound to the left. The lower bound can be drawn symmetrically on the other side of the expected demand line.

Correspondingly probability for shortage α_2 at step k, if the demand is Poisson (Po) distributed, is analogous

$$a_2(Po,k) = \frac{1}{lT_{12}} \sum_{x=1}^{\infty} xPo(kl + x)$$

and for compound Poisson distribution

$$a_2(f_S,k) = \frac{1}{lmT_{12}} \sum_{x=1}^{\infty} xf_S(klm + x)$$

6. Why to use Compound Poisson distribution in MRP models

In considered case the total number of customers (or equivalently total demand for products up to time t when each customer requires one product only) is Poisson distributed and the times between events (arivals of customers) are exponentially distributed. The time to the k-th event (the arrival of k-th customer) is Gamma distributed.

In the case that customers demand several products at arrival and not only one, the cumulative demand is properly modelled by Compound Poisson distribution, because the process in this case is compound stochastic process by nature where we have a random sum of random variables as can be seen from the expression for cumulative demand S(t):

$$S(t) = \sum_{i=1}^{N(t)} X_i$$

Here N(t) denotes the number of customers arriving on the interval (0,t], which is itself a random variable.

At the discussed D_2- model we wish to find

$$\max_{a_2, m_2} \{ rNPV(\mathrm{a}\ (a_2), a_2, m_2) \}$$

The annuity stream is expressed by

$$rNPV(\mathrm{a}\ (a_2), a_2, m_2) = \mathbf{d_2}(\mathrm{a})\Big((\mathrm{p}_1 - \mathrm{v}_1) - \frac{r(v_1 + v_2(m_2 - 1))}{2} T_{12} \Big) -$$
$$- \frac{r}{2}(K_1 + K_2) - (K_1 + \frac{K_2}{m_2}) \frac{1}{T_{12}} \qquad (18)$$

where c=v (I-H) , H is input-output matrix (see /1/,/2/ and /10/) and
v is inventory value with components v_1 and v_2.

1. For uniformly distributed projection of cumulative demand in a certain point t we can write

$$rNPV(\mathrm{a}\ (a_2), a_2, m_2) = \mathbf{d_2}(\mathrm{a})\Big((\mathrm{p}_1 - \mathrm{v}_1) - \frac{r(v_1 + v_2(m_2 - 1))}{2} \frac{L_2}{2a_2} \Big) -$$
$$- \frac{r}{2}(K_1 + K_2) - (K_1 + \frac{K_2}{m_2}) \frac{2a_2}{L_2} \qquad (18a)$$

Lemma 1a:
The optimal value of the annuity stream at unknown distributed demand with known lower and upper bound can be estimated by the set of equations

$$(p_1 - v_1 - \frac{r(v_1 + v_2(m_2 - 1))}{4}\frac{L_2}{a_2})\frac{2(1-\alpha_1)}{(2-\alpha_1-\alpha_2)^2}\ \frac{d}{da}\mathbf{d_2(}a\mathbf{)}$$

$$+\frac{r(v_1 + v_2(m_2 - 1))}{4}\frac{L_2}{{a_2}^2}\,\mathbf{d_2(a)} = (K_1 + \frac{K_2}{m_2})\frac{2}{L_2} \tag{19a}$$

and

$$m_2 = \frac{2a_2}{L_2}(\frac{\rho v_2}{2K_2}\ \mathbf{d_2}(a))^{-1/2} \tag{20a}$$

Proof:
The necessary conditions for optimality are:

$$\frac{d}{d\alpha_2}\, rNPV(a(a_2),a_2,m_2) = \frac{\partial}{\partial a}\, rNPV(a(a_2),a_2,m_2)\frac{da}{d\alpha_2} + \frac{\partial}{\partial \alpha_2}\, rNPV_2(a(a_2),a_2,m_2) = 0 \tag{21}$$

with:

$$\frac{da}{d\alpha_2} = \frac{2(1-\alpha_1)}{(2-\alpha_1-\alpha_2)^2} \tag{22.a}$$

$$\frac{\partial}{\partial a}\, rNPV(a(a_2),a_2,m_2) = (p_1 - v_1 - \frac{r(v_1 + v_2(m_2 - 1))}{4}\frac{L_2}{a_2})\ \frac{d}{da}\ \mathbf{d_2(}a\mathbf{)} \tag{22.b}$$

$$\frac{\partial}{\partial \alpha_2}\, rNPV(a(a_2),a_2,m_2) = -(K_1 + \frac{K_2}{m_2})\frac{2}{L_2} + \mathbf{d_2(a)}\ \frac{r(v_1 + v_2(m_2 - 1))}{4}\frac{L_2}{{a_2}^2} \tag{23}$$

and

$$\frac{\partial}{\partial \mathrm{m}_2} rNPV(\,\mathrm{a}\,(a_2), a_2, m_2) = \frac{K_2}{m_2^{\,2}}\frac{2a_2}{L_2} - \mathbf{d_2}(\mathrm{a})\frac{r v_2}{2}\frac{2L_2}{a_2} = 0 \qquad (24)$$

from which follows:

$$m_2 = \frac{2a_2}{L_2}\left(\frac{\rho \mathrm{v}_2}{2\mathrm{K}_2}\ \mathbf{d_2}(\mathrm{a})\right)^{\mathbf{-1/2}} \qquad (25)$$

and also

$$T_{12} = \frac{L_2}{2a_2} \qquad (26)$$

Let us note that ρNPV(a (α_2),α_2,m$_2$) is concave so that solving the necessary optimality conditions is sufficient.

This results give us the maximal estimated annuity stream at optimal decision for uncertainty of availability of final product where there are two players in the competition for customers.

This approach helps us to derive the formulas for more sophisticated model which is closer to the real – world situation, where the cumulative demand has compound distribution. So we can express for the two-level MRP model the annuity stream (for details see Grubbström, Molinder, IJPE 1994) in its extended form as (the mathematical expression of annuity stream is extended to the case of different times between orders$T_{1,i}$ for different steps):

$$rNPV(T_{1,k}(a_2), k = 1,2,\ldots,\mathrm{K}; a(a_2), a_2, m) =$$

$$= r\left\{[p_1, p_2]\begin{bmatrix}\dfrac{l\,(a(a_2))}{r}\\ 0\end{bmatrix} - \right.$$

$$\left. -\left([c_1 T_{1,1}\, l, c_2 m T_{1,1}\, l] + [K_1, K_2]\right)\begin{bmatrix}\displaystyle\sum_{k=0}^{\mathrm{K}} e^{-r\sum_{i=1}^{k} T_{1,i}}\\ \displaystyle\sum_{k=0}^{\mathrm{K}} e^{-rm\sum_{i=1}^{k} T_{1,i}}\end{bmatrix}\right\} \qquad (27)$$

where

$$T_{1,k} = \frac{\sum_{x=1}^{K} x f_X (\sum_{i=1}^{k} T_{1,i}\, l + x)}{l\, a_2 (f_X, k)} \tag{28}$$

and $f_X(x)$ is probability function of chosen probability distribution of X.

Now we have two possibilities:

1. We can choose $T_{1,k} = const$.

or

2. $a_2 = const$

We decided to consider the second possibility: $a_2 = const$.,

so $T_{1,k}$ can be written as:

$$T_{1,k} = \frac{\sum_{k=1}^{K} x f_X (\sum_{i=1}^{k} T_{1,i}\, l + x)}{l\, a_2 (f_X)} \tag{29}$$

Let us note that $a(a_2)$ is a function of customer travelling policy. For D_2 - model of equal prices a is equal to:

$$a = \frac{a_2 - a_1}{2 - a_1 - a_2}, \tag{30}$$

where $a_1 = const$

In the case that for the probability distribution of cumulative demand we choose Poisson distribution Po, the expression (27) for $rNPV$ is the same, only the expressions (28) and (29) change as follows:

$$T_{1,k} = \frac{\sum_{x=1}^{K} x Po(\sum_{i=1}^{k} T_{1,i}\, l + x)}{l\, a_2 (Po, k)} \tag{31}$$

$$T_{1,k} = \frac{\sum_{x=1}^{K} xPo(\sum_{i=1}^{k} T_{1,i}\, l + x)}{l\, a_2 (Po)} \tag{32}$$

Now we want to determine the optimal value of probability of shortage a_2.

Theorem 1:

The necessary optimality conditions are:

$$\frac{d\, rNPV(T_{1,k}(a_2), k=1,2,...,K; a(a_2), a_2, m)}{da_2} =$$

$$= \sum_{k=1}^{K} \frac{\partial rNPV(T_{1,k}(a_2), k=1,2,...,K; a(a_2), a_2, m)}{\partial T_{1,k}} \cdot \frac{dT_{1,k}}{da_2} +$$

$$+ \frac{\partial rNPV(T_{1,k}(a_2), k=1,2,...,K; a(a_2), a_2, m)}{\partial a} \cdot \frac{da(a_2)}{da_2} +$$

$$+ \frac{\partial rNPV(T_{1,k}(a_2), k=1,2,...,K; a(a_2), a_2, m)}{\partial a_2} = 0 \tag{33}$$

and

$$\frac{\partial rNPV(T_{1,k}(a_2), k=1,2,...,K; a(a_2), a_2, m)}{\partial m} \leq 0 \tag{34}$$

where

$$T_{1,k} = \frac{\sum_{x=1}^{K} xf_X(\sum_{i=1}^{k} T_{1,i}\, l + x)}{l\, a_2 (f_X)} \tag{35}$$

In the case of Poisson distribution of cumulative demand X, the necessary conditions of optimality are stil the same and given by Theorem 1, only $T_{1,k}$ is given by

$$T_{1,k} = \frac{\sum_{x=1}^{K} xPo(\sum_{i=1}^{k} T_{1,i}(a_2)\, l + x)}{l\, a_2 (Po)} \qquad (36)$$

where

$$Po(\sum_{i=1}^{k} T_{1,i}(a_2)\, l + x) = \frac{\left(l \sum_{i=1}^{k} T_{1,i}(a_2) \right)^{l \sum_{i=1}^{k} T_{1,i}(a_2) + x} e^{-l \sum_{i=1}^{k} T_{1,i}(a_2)}}{\left(l \sum_{i=1}^{k} T_{1,i}(a_2) + x \right)!} \qquad (37)$$

$x = 1,2,\ldots,K$

Now we consider the general case where the number of customers' arrivals up to time t: N(t) is Poisson distributed. Demand in real world systems is in general for more than one product at a time. In this case we can make a natural choice:
The distribution of cumulative demand is compound Poisson distribution (CPo).
We will answer the question: How does this affect the net present value?
In the case of compound Poisson distribution of cumulative demand the times between the orders $T_{1,k}$, $k = 1,2,\ldots,K$ have the following form:

$$T_{1,k} = \frac{\sum_{x=1}^{K} xf_S(\sum_{i=1}^{k} T_{1,i}(a_2)\, l + x)}{l\, a_2 (CPo)} \qquad (38)$$

where $f_S(x)$ is the probability function of compound Poisson distribution and can be calculated according to recursive formula given in Theorem 1.

The general expressions for necessary conditions of optimality (33)-(34) are still valid considering for $T_{1,k}$, $k = 1,2,\ldots,K$ (36) or (38) instead of (35). So we get the following two Lemmas:

Lemma 1b:

The optimal value of the annuity stream at Poisson distributed demand is determined by the set of equations

$$(\mathrm{p}_1-\mathrm{v}_1-\frac{r(v_1+v_2(m_2-1))}{2}\frac{1}{l\,a_2(Po,k)}\sum_{x=1}^{\infty}xPo(kl+x))\frac{2(1-\alpha_1)}{(2-\alpha_1-\alpha_2)^2}\ \frac{\mathrm{d}}{\mathrm{da}}\mathbf{d_2(}\mathrm{a}\mathbf{)}$$
$$+\frac{r(v_1+v_2(m_2-1))}{2}\frac{1}{a_2(Po,k)^2\,l}\sum_{x=1}^{\infty}xPo(kl+x)\mathbf{d_2(a)}=(K_1+\frac{K_2}{m_2})\frac{l}{\sum_{x=1}^{\infty}xPo(kl+x)} \qquad (19b)$$

and

$$m_2=\frac{l\,a_2(Po,k)}{\sum_{x=1}^{\infty}xPo(kl+x)}(\frac{\rho \mathrm{v}_2}{2\mathrm{K}_2}\,\mathbf{d_2}(\mathrm{a}))^{\mathbf{-1/2}} \qquad (20b)$$

Lemma 1c:

The optimal value of the annuity stream at compound Poisson distributed demand is determined by the set of equations

$$(\mathrm{p}_1-\mathrm{v}_1-\frac{r(v_1+v_2(m_2-1))}{2}\frac{1}{lma_2(f_S,k)}\sum_{x=1}^{\infty}xf_S(klm+x))\frac{2(1-\alpha_1)}{(2-\alpha_1-\alpha_2)^2}\ \frac{\mathrm{d}}{\mathrm{da}}\mathbf{d_2(}\mathrm{a}\mathbf{)}$$
$$+\frac{r(v_1+v_2(m_2-1))}{2}\frac{1}{a_2(f_S,k)^2\,lm}\sum_{x=1}^{\infty}xf_S(klm+x)\mathbf{d_2(a)}=(K_1+\frac{K_2}{m_2})\frac{lm}{\sum_{x=1}^{\infty}xf_S(klm+x)} \qquad (19c)$$

and

$$m_2=\frac{lma_2(f_S,k)}{\sum_{x=1}^{\infty}xf_S(klm+x)}(\frac{\rho \mathrm{v}_2}{2\mathrm{K}_2}\,\mathbf{d_2}(\mathrm{a}))^{\mathbf{-1/2}} \qquad (20c)$$

7. Conclusions

In the paper we have shown how to use MRP and Input - output analysis (the *NPV* model) to investigate the results of customers' behaviour when the supply units compete for customers and they are exposed to uncertain demand which results to possible shortage of goods. Customer's travelling problem and customers' behaviour are here described by equations (1-5), considered by Girlich /4/ and influenced by /3/. On the bases of mentioned results, the optimal ordering policy which includes also the optimal shortage of goods is derived here. The results can be extended also to the other customers' travelling behaviour described by /4/ or even to more general cases.

We have shown that the proper way to handle the model, in the case that more than one product can be demanded by customers at their arrival, is to use Compound Poisson distribution of total demand, while the time to the k-th event is Gamma distributed. The stated Compound Poisson distribution model of total demand is combined with MRP, Input-Output analysis and customer traveling problem stated in this framework, is solved.

8. References

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