INTERINDUSTRY FORECASTING PROJECT UNIVERSITY OF MARYLAND

Research Memorandum No. 2

HOW TO PURIFY AN INPUT-OUTPUT MATRIX

By

Clopper Almon

5 July 1967

HOW TO PURIFY AN INPUT-OUTPUT MATRIX

I. THE SECONDARY PRODUCT PROBLEM

One of the perennial trouble makers in input-output is the secondary product. Examples of secondary products include stampings cut in a plant primarily engaged in rolling aluminum, breakfast cereals made in a plant whose primary product is paperboard, or lumber sawn in a plant whose main product is furniture. 1958 matrix, such products were treated by a transfer. For example, the table shows a \$19 million "sale" of furniture to lumber, which is really a transfer of lumber made in furniture plants back to its home industry. The number of such secondary entries will increase substantially as the number of sectors increases. entries are not only confusing when looking at the table, but they can also distort the outcome of its use. If we asked what would be the impact of an increase in residential construction, the big lumber user, this furniture-to-lumber secondary entry would lead to an immediate increase in the output of the furniture industry and thereby to higher demand for textiles, wire, paint and varnish, and glue. Moreover, because the establishments in the Structural Metals product sector make some furniture, which is transferred to the Furniture sector, the demand for lumber also generates demand for structural steel to "go into" the lumber. Such distortions are probably not large, but they are disconcerting and have led to a desire to "purify" the matrix. This paper describes one

purification rite.

II. THE PURE ACTIVITY MATRIX

The simplest approach postulates that each product is made by the same process, no matter what kind of establishment makes it. The vector of inputs into any establishment is therefore just a linear combination of the pure processes for the items it makes. In symbols

(1)
$$f_i = Mp_i$$
 $i = 1, ..., n$.

where:

 f_i , a column vector, is the transpose of the $i\frac{th}{t}$ row of the primary flow matrix, that is, f_{ij} equals the purchases of product i by establishments in industry j. M is the product mix matrix: m_{ij} = the fraction of product j made in establishments in industry i. (The columns of M sum to 1.0).

 p_i is the transpose of the $i\frac{th}{t}$ row of the pure flow matrix; p_{ij} = inputs of product i into product j.

Equation (1) can be solved for p_i by

(2)
$$p_i = M^{-1}f_i$$
 $i = 1, ..., n.$

The p_i vectors found by this formula will generally have some small negative elements and some small positive entries where the corresponding f_i vectors have zeros. The former are clearly nonsense; the latter are at best dubious and certainly inconvenient, especially as we move toward larger matrices for which we will wish to store only non-zero entries in our computers.

By analysing the solution of (2), we shall find a way to put a stop to this nonsense.

III. THE ITERATIVE INTERPRETATION

In the rest of this paper, we shall drop the subscript i from f and p and understand that the f and p in an equation all have the same i subscript. Equation

(1) may be then written

$$0 = -M_p + f ,$$

and by adding p to both sides we obtain

(3)
$$P = (I - M)_p + f$$
.

The column sums of the absolute values of I - M will be less than 1.0, save in the unlikely case in which less than half of a product is produced by establishments primarily engaged in its production. The iterative process for the solution of (3) will therefore converge. In this process, we take as a first approximation of p,

$$p^{(0)} = f$$

and then define successive approximations by

(4)
$$p^{(k+1)} = (I - M) p^{(k)} + f$$
.

To see the economic interpretation of (4), let us write out the equation for the use of a product, say electricity, in making another product, say j.

The first term on the right of (5) tells us to begin with the electricity purchases by the establishments in industry j.

The second term directs us to remove the amounts of electricity needed for making the secondary products of those establishments, using our present estimate $(p^{(k)})$ of the technology of those products. Finally, the last term causes us to add back the electricity used in making product j in other industries. The amount of electricity added by the third term is exactly equal to the amount stolen, via second terms, from other industries on account of their production of product j:

IV. PUTTING A STOP TO THE NONSENSE

It is now clear how to keep the negative elements out of p. When the "removal" term, the second on the right of (5), is larger than the "primary use" term, the f_j , we simply scale down all components of the removal term to leave a zero balance. Then instead of adding back the "total-stolen-from-other-industries" term, $(l-m_{jj})p_j$ all at once, we added it back bit-by-bit as it is captured. If a plundered industry runs out of electricity with only a third of the total amount of plundering claims satisfied, we simply add only a third of each plundering product's claim into that product's cell in p.

V. IMPLEMENTATION FOR 1958

This procedure has been programmed and applied to the expanded 1958 matrix. The result is available as IFP Listing 2.

This listing shows the programs used for the calculation, and a row-by-row listing of the primary matrix, the purified matrix, and the secondary matrix. The portion of the program which performs the purification is SUBROUTINE PURIFY. Comment statements within this subroutine should make it self explanatory. In particular, it is noteworthy that it requires only non-zero elements of M to be stored. The 1958 matrix had less than 2000 such elements. This subroutine is therefore usable on the 7094 for matrices with up to five times more non-zero elements than the 1958 matrix had.

The primary matrix used in these calculations differs from the primary matrix obtained from the OBE tape (See IFP Research Memo 1) in three ways:

- (1) It has the added detail for the food, non-ferrous metals, and utilities which was published in the <u>Survey of Current Business</u>, April 1966. Secondary flows for this further detail were supplied by OBE.
- (2) OBE calls all the inputs into the two dummy industries, Business Travel and Office Supplies, secondary transfers. We have treated them as primary sales.
- (3) Scrap and By-Products are put into a separate row (number 94) with no corresponding column. Positive entries in this row show het use of scrap or by-products; negative entries show net production. Naturally, the non-negative constraint of the purifying program was not applied to this row.

Comparison of the purified matrix with the primary matrix shows that in the vast majority of cells little change was made.

A few zeros appear in the purified where the primary had entries.

Most of these make good sense. For example, the primary matrix

shows a small purchase by Aircraft from Ships, Trains, Trailers, and Cycles; the purified shows a zero. The purification process found that this purchase was more than accounted for by the purchase of parts needed for the "Ships, Trains, Trailers, and Cycles" made in the Aircraft establishments. Other zeros reveal inaccuracies in the original table. For example, we find a zero for the sale of electricity to Printing and Publishing and to TV and Radio Broadcasting, both manifest absurdities. The trouble is that the purified Business Scrvices electricity use is \$250 million for an output of \$23,269 million, of which 22 percent was advertising transferred in from Printing and Publishing and 6.5% was transferred from TV and Radio. 22 percent of \$250 millon is \$54 million, \$2 million more than Printing and Publishing's primary purchase of electricity. How did Business Services electricity use get up to \$250 million? The first \$186 million was a primary assignment; from that base it was easy to grab all of the electricity input into Printing and TV and Radio to bring the total to \$240; the remaining \$10 were collected from other small secondary producers of Business Services. The source of the difficulty is clearly the \$186 primary assignment of electricity to Business Services. Upon inquiry, we learned that OBE made the electricity allocations in the service sectors in proportion to employment, a procedure acceptable only faute de mieux. The purification procedure tells us that too much got allocated to

Business Services; more should go to, say, Hospitals or Trade.

We have calculated a Value Added row by the purification procedure. Strictly speaking, it may be inconsistent with the detail in the other rows, so we shall later calculate a row which is the residual of product outputs less the purified inputs into them. Meanwhile, the present row bears out the Pythagorean contention of esthetic standards implicit in mathematics, for our calculations reveal that there is no Value Added in TV and Radio; it is all advertising.

RMHY DATE 07/04/67 357/C1/1C1 C.ALMCN - EFN SOURCE STATEMENT - IFN(S) PURIC SUBROUTINE PURIFY (NCNEG) CCMMCN N, TOLER, INCR(100), INCC(3000), S(3000), P(100), FLOW(1100), C(100) LET M(J,I) BE THE FRACTION OF PRODUCT I MADE BY ESTABLISHMENTS IN INCUSTRY J S IS THE NON-ZERC, NON-CLAGONAL ELEMENTS OF M STORED ROW-BY ROW. INCC(I) IS THE COLUMN NUMBER (IN M) OF THE ELEMENT S(I). INDR(J) IS THE NUMBER (IN S) OF THE FIRST ELEMENT OF THE JTH RCW CF M. N IS THE CIMENSICH OF M TCLER IS THE IS MAXIMUM TCLERABLE DISCREPANCY FLCW IS THE RCW OF THE FRIMARY INPUT-OUTPUT MATRIX BEING PURIFIED P BECCHES THE PURIFIED RCh. NCNEG POSITIVE GIVES NO NEGATIVE ENTRIES IN THE PURIFIED MATRIX ASSIGN 30 TC NEXT IF (NCNEG. EC.O) ASSIGN 38 TO NEXT INITIAL P AND C, THE CHANGE VECTOR DC 15 I = 1, NC(I) = 0.

15 P(I) = FLCW(I)M IS THE ITERATION COUNT

M = 018 CC 40 J = 1, N NE AND NU ARE LOWER AND UPPER LIMITS OF ROW J IN S.

NL = INDR(J)

NL = INDR(J+1) - 1IF COLUMN J HAS NO ELEMENTS, SKIP ON TO NEXT COLUMN.

IF (NL.GT.NU) GC TC 4C FIRST WE BRING MATERIALS INTO THE PRIMARY INDUSTRY AS IF WE EXPECTED NO SHORTAGE IN J. THE INDUSTRY WE ARE ROBBING.

SUM BECOMES TOTAL AMOUNT OF ROBBERY. SLM = 0.

DC 20 I = NL, NU

IS = INDC(I)R = P(IS)*S(I)

SLM = SUM + R 20 C(IS) = C(IS) + R

CIC WE STEAL MORE FROM J THAN J HAD/ IF NOT, GC TO 38

GC TC NEXT, (30,38) 30 R = FLCW(J) - SLM

IF(R.GE.O.) GC TC 38 R = 1. - FLOW(J)/SUM

WE SHAL HAVE TO SCALE DOWN OUR PLUNDERING BY THE FACTOR R. DC 35 I = NL, NU

IS = INDC(I)25 C(IS) = C(IS) - R*F(IS)*S(I)

SUM = FLOW(J) SUBTRACT FROM C(.) THE SUM ALLOCATED TO OTHER INDUSTRIES

38 C(J) = C(J) - SLM

40 CONTINUE NEW P IS FLOW + C. NOW COMPARE IT WITH OLD P, STORE NEW P IN P, AND CLEAR C FOR NEXT ITERATION.

INAX = 1DISMAX = 0

CC 50 I = 1.N

C.ALMON	357/C1/	1C1	144 c-186 delektrik (166 til C-188 til Color, A. d. A.) de elektrik beskere i sammen i sam	THE RESERVE AND ADDRESS OF THE PARTY OF THE	DATE	7/1
PURIC			ATEMENT -	· IFN(S) -		
R = FLCW(I) +	CIII				-	
DIS = P(I) - R		Alle alle de la companya de la comp	W-1-18-01-1-1-1-1-1-1-1-1-1-1-1-1-1-1-1-1	<u> </u>		
P(I) = R						
DIS = ABS(CIS)	THE REAL PROPERTY AND ASSESSED ASSESSED.	egerindeling sakislede der sich der sic				
IFICIS.LE.CISM	AX) CC TO	د ر				
45 IMAX = I		A MARCON DESCRIPTION OF THE PROPERTY AND THE RESIDENCE OF THE PROPERTY OF T				
DISMAX = DIS						
50 C(I) = 0.						
WRITE(6,52) IM	AX, EISMAX					
52 FCRMAT(7H IMAX		ISMAX = F1	0.21	en kraderinin. Makairiyê heyeyî ser - mer aranmen meh nadik kadi binan sanan ke ana ser eyêyey	CONTRACTOR AND A SECURE OF THE PARTY OF THE	
M = N + 1		_				
IF(M.GE. 10) G	C TC 6C	o dellik kirik i i i i i i i i i i i i i i i i	alterna mengalagan mengapak dan dalam dalam dan pagaman yang dan sebagai sebagai sebagai sebagai sebagai sebag S	hale selven i i i i i i i i i i i i i i i i i i i		
IF(CISMAX.GT.TO		C 18				
60 RETURN	TO BELLEVIA BELLEVIA AND AND AND AND AND AND AND AND AND AN	. Armoster			The state of the s	
ENC						
						- 1000000
	No. 25 March 184 March Sanda Commission Sanda Commission Sanda Commission Com	· via compania de Mandrea Additiona de montante de mandre de la compania del la compania de la compania del la compania de la compania del la compania de la compania de la compania del la compania				
•						
	The state of the s					
	NO. TO SEE SEE SEE SEE SEE SEE SEE SEE SEE SE	management of the second secon	e roan de la 16 es de decadembro popular enterente propriete a patriolòxica de de comunicación			
	***************************************	e war word of shifts a material of a shifted department of a war two of 1000 to 1000 to 1000 to 1000 to 1000 to			Miles	
	SERVICE AT THE RESIDENCE OF THE PROPERTY OF TH	THE PERSON OF THE PERSON NAMED IN THE PERSON N	Marie Carpenter of against a market dates that the Annie Marie Carpenter of the control of the c	Additional to the control of the con		
•						
	the contract of the contract o	haden da men en en tra a en	THE STATE OF THE THE WAY WHITE INTO A CONTROL OF A VESTELLY STATE A MAKE A	The same is an incommendation and in application. And is also be considered to desire in a sec-	and all all and the second sec	
The second section of the sect	The second second difference of the second s	erickis tribunistisses are enterprised to the second secon		e Andrews - Marie proper come a port from the deliver a participant of the second company of the second compan	Parties a supple control of the cont	
				•		
	mantagat a Managata Calama alama ayan ayan ayan adalah ayan ay					
Me a succession of the success			The second of the second	to the Association (1995) - Martine Committee of the Symposium ballock (1995) - 1995 - 1995 - 1995	* 190 * MACHINE NO. 11 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	
	en hank hiter outh manner on Arreston Militaine after a man an advantagement of some	THE PERSON NAMED IN THE PE		40 m morror militari ilipani mana di seperatura i i del managio di	de Militario Para Companya (Maria Maria Mari	h sour an assess
-		THE STATE OF THE PARTY OF THE STATE OF THE S		The second of th	The second secon	/*: · · · · · · · · · · · · · · · · · · ·
Annual Salara Sa	PARTICIPATE A TRANSPORT OF THE SAME SECTION OF THE PARTY.	SECURITY CONTROL OF COMME. STATE CONTROL STATE COMME AND AND ADDRESS OF COMME ADDRESS OF COMM	**************************************	The state of the s		
Manager, which is a mornished good or properly more representational designation of the state of	om at 11 ark en tallante la er en talla potantat e pajo plinaja je kraljanja e e	reference the second and the second and the second as the second			***************************************	