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MARYLAND INTERINDUSTRY FORECASTING PROJECT

Research Memorandum No. 28

November 16, 1970

Estimation of Capital Stock and Investment Equations

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Equipment investment holds the brightest promise and the hardest problems of any part of the model. Only with an input-output model can we examine side-by-side and consistently the investment spending of all the varied industries and their demands for many kinds of capital equipment. But our forecasting record leaves some room for doubt about the accuracy of these forecasts. We have experimented with a variety of equations. The equations used in The American Economy to 1975 emphasized maintenance of the capital-output ratio. They proved fairly accurate in pointing to a large increase in spending after 1963, but they were dreadful in prediction of timing. We then used equations emphasizing distributed lags and profits. The historical fits were fairly good, but they failed to predict the continued growth in investment by many industries after 1966. We next developed a cost-of-capital measure which further improved these fits but still did not forecast properly the continued growth in investment. The equations explained below regain some of the original emphasis on the capital-output ratio but preserve our advances in distributed lags and cost of capital. We have as yet no forecasting experience with them, but we hope that they will combine the advantages of both approaches.

The equations use several variables which must first be explained.

The first two are capital stock and replacement. We use a perpetual inventory method to arrive at capital stock, that is,

(1)
$$K = \sum_{i=0}^{\infty} R V_{i-i}$$

where K_t is the estimate of the capital stock in year t, V_t is investment in year t, and R_i is the fraction of the investment in one year remaining in use i years later. The pattern for the retention curve, R_i , which is most commonly used is the exponential,

(2)
$$R_i = (1-d)^i$$
 where 0

This specification has the advantage that the forecasting program does not need to "remember" the whole past history of investment to deduce current stock and retirements, Ret t suffices to "remember" only current stock, because

(3)
$$K_t = V_t + (1-d) K_{t-1}$$

and

Ret =
$$K_{t} - (K_{t} - V_{t}) = d K_{t-1}$$
.

This pattern for R, however, is unrealistic. While the <u>value</u> of capital may diminish exponentially, the <u>quantity</u> certainly does not. In an industry where the average life of capital is ten years, nearly all of the equipment bought in one year will be still in use three or four years later. The exponential retention curve, therefore, seriously understates the quantity of capital. Since we mean to use the ratio of capital stock to output to indicate how much investment a given expansion in output will cause, we must use a pattern of R more true to life.

A simple generalization of the exponential can achieve the greater realism, yet remain almost as convenient. We simply create a second, fictitious class of capital, that which has been depreciated but not retired. Retirements flow out of this class at an exponential rate. More precisely, if we let K (t) be the capital in the first, undepreciated class, at time t, and K (t), the stock in the second, then

(4)
$$K_1(t) = V_t + (1-2d) K_1(t-1)$$

$$K_2(t) = 2d K_1(t-1) + (1-2d) K_2(t-1)$$

$$K(t) = K_1(t) + K_2(t)$$

where K(t) is the total stock of capital in service. Notice that the computer forecasting program has to remember only two values, K_1 and K_2 , not the whole past history of investment. Retirements are simply 2d K_2 (t-1), and the average life is 1/d, as before.

There is a natural analogy between this scheme and water running out of a hole in the bottom of one bucket, into a lower bucket, and then out of a hole in the bottom of that bucket. The total quantity of water in the two buckets corresponds to our total capital. Hence, we call it the "two-bucket" stock. Figure 1 shows the kind of retention curve it implies.

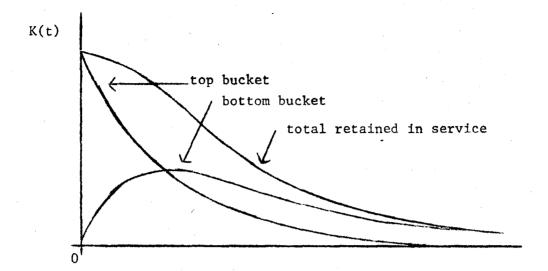


FIGURE 1: RETENTION CURVE OF CAPITAL STOCK

The second concept that distinguishes our equations is the cost of capital. If the production function has a constant elasticity of substitution between capital and labor, then it can be written as

(5)
$$K = a Q r^{-\sigma}$$

where K is the desired stock of capital, Q is output, r is the marginal product of capital, and σ is the elasticity of substitution. Now profit maximizing firms set the cost, s, of an additional unit of capital equal to the after-tax return on it:

(6)
$$s = P_v (1-T)r$$
,

where

- I is the tax rate on corporate profits
- P is the price index of value added by the industry. More vertically, for the ith industry, P is the index of the quantity $\mathbf{p_i} \mathbf{\Sigma} = \mathbf{p_i}$, where $\mathbf{p_i}$ is the price index for product j.

This cost per unit, s, is in turn given by

$$s = P_e(i + d) (1 - Tz - C)$$

where

- i is the rate of interest (the bond rate)
- d is the legal rate of depreciation
- C is the investment tax credit rate (This fraction of equipment investment can be deducted from taxes. It was .075 during most of the 1960's but has now been set to 0.)
- \ensuremath{P} is the price index for equipment used by the industry \ensuremath{e} in question
- z is the present value of the stream of depreciation which flows from a maintained increase in capital stock of \$1.

 It is increased by speeding up depreciation or by lowering the rate of interest.

The term P_{e} (1 - Tz - C) tells how much a firm must actually payafter subtracting tax savings—to get a dollar's worth of capital at the base year prices. Multiplying by (i + d) then gives the yearly cost of this capital. We then combine these two equations, eliminate s, and solve equation for r

(7)
$$r = \frac{P_e(i+d)(1-Tz-C)}{P_v(1-T)}$$
.

For each industry Thomas H. Mayor has calculated the value of r from 1947 to the present. (See MIFP Research Memorandum # 18.)

Net investment, N, should then, by (5), be given by

(8)
$$N_{t} = V_{t} - Ret_{t} = K_{t} - K_{t-1}$$

$$\approx a r^{-\sigma} (Q_{t} - Q_{t-1}) - a\sigma Q_{t-1} r^{-\sigma-1}_{t-1} (r_{t} - r_{t-1})$$

$$= K_{t-1} \left[(Q_{t} - Q_{t-1})/Q_{t-1} - \sigma (r_{t-1})/r_{t-1} \right].$$

In fact, of course, investment reacts to changes in Q and r with a lag distributed over several years, say N years. We therefore replace (8) by

(9)
$$N_{t} = \sum_{i=0}^{n} w K_{t-1-i} \left[\left(\frac{Q_{t-i} - Q_{t-i-1}}{Q_{t-i}} \right) - \sigma \left(\frac{r_{t-i} - r_{t-1-i}}{r_{t-i}} \right) \right]$$

where $\sum_{i=0}^{n} w = 1$ and $w \ge 0$.

The unknowns on the right of (9) are σ and the w_i , which must be estimated by regression. When σ has been picked, the regression is linear in the w_i . We are currently using n=5 and imposing the constraint that w_i , ..., w_i lie on a quadratic polynomial and that $w_i \geq w_i$. On the six w_i , we have therefore three equality constraints and one inequality, plus the requirements that $w_i \geq 0$. The problem therefore fits naturally into the quadratic programming format, and we find that the Dantzig quadratic simplex algorithm works very nicely on it. The computing time runs very little more than for ordinary least squares.

To estimate σ , we may first set it equal to zero, estimate (9), then increase σ by .2, estimate (9) again, and so on until the residual sum of squares is minimized. Because σ is usually between 0 and 1.0, this simple procedure is fairly efficient.

There is no constant term in (9), and consequently the fit can be very bad—all "predictions" could be low or all high—if the wrong length of life is used in calculating the capital stocks. This length of life is therefore implicitly a parameter in this equation, and we can estimate it iteratively also. The first guess is the value given in the Treasury Department's Depreciation Guidelines. With this value, we estimate equation (9) with an intercept term added. If the intercept turns out to be different from zero, it was originally our intention to adjust the length of life and re-estimate until the constant term was negligible.

We found, however, that we were being led to unrealistically short lives. We realized that we put some trust in the Guideline lives, and were not happy with lives of half that length. This "trust" was of the same nature as our convictions that the sum of the weights should equal 1.0, that the constant term should be zero, and that R² should be as high as possible. All of these were prior expectations, and there were trade-offs among the extent of conformity to them. The constant term might be positive to account for the investment called forth by a shift in the composition of output not fully reflected by the increase in output. The sum of the weights might be less than 1.0 if there are increasing returns to scale. And, of course, the average life may differ from the Guideline life either because the Guideline life was wrong in the first place or because replacement practice has changed since it was estimated.

Basically, we had then three prior expectations, no one of which we held with certainty, to reconcile with our desire for a good fit. After some fiddling with the equations we realized that if we hoped to produce understandable and duplicatable results, we would have to put conformity to our expectations explicity into the objective function to be minimized. Specifically, we sought to minimize

(10)
$$V = \sum_{t=1}^{T} (N_{t} - a - \sum_{i=0}^{5} w_{i} x_{i})^{2} + g_{2} (\sum_{i=0}^{5} w_{i} - 1)^{2}$$

$$= \sum_{t=1}^{T} (N_{t} - \overline{N}_{t})^{2} + g_{3} \left(\frac{a}{\overline{N}}\right)^{2} + g_{4} \left(\frac{L-L^{*}}{L}\right)^{2}$$

Subject to

$$w_{i} > 0 \quad i = 0, \dots, 5$$
 $w_{5} < w_{4}$

and w and w on the quadratic curve determined by w $_0$, w $_2$, and w $_4$. Here N is the average net investment, L is the assumed average life of capital, L* is the Treasury Guideline life, and \mathbf{g}_2 , \mathbf{g}_3 , and \mathbf{g}_4 are the weights (gewichte) on the components of the objective function. We have chosen $\mathbf{g}_2 = 10$, $\mathbf{g}_3 = 5$, and $\mathbf{g}_4 = 5$. These choices indicate that we would be indifferent between a one-point (.01) increase in \mathbf{R}^2 and

- an increase of .01 in the sum of the weights when this sum is .95
- a decrease of .01 in the ratio of the intercept term to the average investment (a/\overline{N}) when this ratio is .10.

- a decrease of one percent in the deviation of the assumed average life (L) from the Treasury Guideline life, L*, when this deviation is 10 percent.

Obviously, the choice of the g's is subjective, but the decision to choose them all equal to zero, and thereby overlook information is equally subjective. Subjectivity seems to be unavoidable in econometric forecasting. The most that one can hope for is to be conscious of and explicit about the subjectivity.

The equation (10) for determining the value of objective function is a quadratic function of a and w, but at the same time it is an even more complicated function of σ and L (remember that L affects the value of the X_{ti}). To minimize this objective function we use the same quadratic algoritham as above, but first we must know the values of σ and L. These values for the variables must be picked so as to give the optimum combination. In order to accomplish this, we have set up a search routine to search the (σ, L) plane as shown in Figure 2.

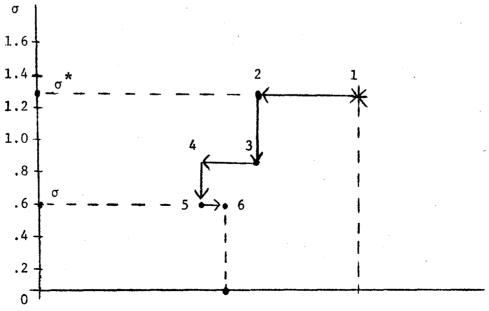


FIGURE 2

Point 1 is found as initial optimum solution (with L unchanged). Next we change L and find the new value of objective function (VOBF). If VOBF decreases, we continue to decrease L until we reach a new minimum value, say at Point 2. At this point we start searching for the new optimum σ which we find at Point 3. This process is continued until at Point 6 we have found the optimal combination of σ and L; that is, at this point any change of either σ or L will lead to a higher VOBF.

The equations which result from this estimation, and plots showing their fit to historical data, are shown in the attached pages. We feel that the results are generally satisfactory for medium term prediction, which is what we really want them for. The graphs show that the equations are not very good at forecasting cyclical turning points, and the generally low Durbin-Watson statistics (P.W.) suggest that adding a forecast of the error, based on the auto correlation, would improve the forecast in the first several years.

The reader will notice a few negative values of R². These come about because we swapped conformity to the other criteria for R². The graphs show that where this happens, the curve is very flat. In looking at these graphs, it should also be born in mind that the standard error in the Census Bureau's data on investment is often five or ten percent for series in this much detail.

The following tables also show the capital stock and the capitaloutput ratio in each industry.

Unfortunately, the tables distributed at the November 19, 1970 meeting will not have been derived from exactly the objective func-

tion set forth above. The VOBF on these pages refers to the value of the objective function as used in that program. As soon as they are available, replacements for the pages will be sent out. We do not anticipate major changes.

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•150	5.759	27.2	.382	. • 407	000	. (76	-152	-228	.304	• :	304	1.065	11.000
k. 1		1 95	5 1956	1957	1958	1 959	1960	1961	1962	1963	1964	1965	1966	1967
EQUIPM.	EXPEND. ACTUA	L 3293.	0 2766.0	2770.0	3370.0	3323.0	2795.0	3013.0	3122.0	3647.0	3900.0	4292.0	4543.0	4543.0
EQUIPM.	EXPEND. PREDI	CT. 2805.	1 2926.3	3104.1	3390.8	3343.7	3149.2	3178.4	3460.7	3752.3	3909.2	4274.3	4181.2	4340.1
1 . CL AS	S CAPITAL STOC	K 19718.	6 19046.6	18459.4	18541.0	18526.7	17953.2	17668.9	17512.1	17874.7	18385.6	19152.5	19981.1	20603.1
2. CLAS	S CAPITAL STOC	K 15634.	9 16346.9	16822.8	17114.4	17371.1	17581.3	17649.6	17653.2	17626.7	17673.7	17810.2	18070.6	18445.5
AVE. IN	STALLATION DAT	E -16.	7 -16.0	-15.3	-14.4	-13.6	-12.8	-11.9	-11.0	-10.0	-8.9	-7 .7	-6.5	-5.4
COST OF	CAPITAL	•230	6 .2458	.2642	. 2457	.2822	.2818	•2772	.2542	.2612	.2539	.2373	. 2288	.2629
CAPITAL	OUTPUT RATIO	1.59	8 1.600	1.595	1.612	1.612	1.539	1.510	1.503	1.478	1.501	1.475	1.519	1.558

SECTOR NUMBER 2 MINING (11-14+16-17)

SIEMA	V . O. F.	CONSI.	RSOR	D. W.	WGHI + I	. WGHI	• Z W G	H1 - 5	WGHI.4	WOHI . 5	WGHI	•P 201	I MPHI2 1	INF . FILE
.450	3 .9 98	•5	.484	1.223	.091	2	69	.323	.253	•059	.0	00	•995	4.500
e de la companya de l		199	5 1956	1957	1958	1959	1960	1961	1962	1963	1964	1965	1966	1967
EQUIPM. EX	KPEND. ACTU	IAL 1113.	0 1356.0	1282.0	940.0	967.0	956.D	943.0	1036.0	994.0	1120-0	1192.0	1356.0	1282.0
EQUIPM. E)	KPEND. PREC	ICT. 1138.	1 1201.6	1169.3	1099.9	1052.5	971.6	1001.3	1043.1	1109.6	1158.4	1208.3	1266.9	1309.4
1 . CLASS C	CAPITAL STO	CK 2771.	1 2946.0	2958.7	2610.3	2428.8	2305.3	2213.4	2245.3	2210.0	2305.7	2417.1	2627.2	2649.D
2. CLASS	CAPITAL STO	CK 2873.	6 2829.9	2879.9	2914.2	2780.5	2624.2	2481.0	2359.6	2307.2	2262.1	2282.6	2346.3	2481.0
AVE. INSTA	ALLATION DA	TE -13.	7 -12.6	-11.5	-10.6	-9.7	-8.7	-7.7	-6.6	-5.5	-4.4	-3.2	-2.1	-1.1
COST OF CA	APITAL	.15	6 .1637	.1940	.2042	.2357	.2328	•2325	.2186	.2094	.1972	.1968	-2015	.2180
CAPITAL OL	JTPUT RATIO	1.29	1.304	1.318	1.247	1.176	1.113	1.060	1.040	1.020	1.001	.981	- 995	.985

SECTOR NUMBER 3 PETROLEUM AND GAS (15)

WGHT .2

WGHT.3

WGHT.4

WGHT.5

WGHT.6 SUM WGHTS AVE.LIFE

9.4

WGHT.1

SIGMA

V . O. F.

CONST.

RSQR

D.W.

	8.7 .	243	.417	•352	. 2	64	. 176	.088	-000	-0	00	.880	10.000
	1 95 5	1956	1957	1 95 8	1959	1960	1961	1962	1963	1964	1965	1966	1967
EQUIPM. EXPEND. ACTUAL	2125.0	2314.0	2445.0	1517.0	1493.0	1671.0	1782.0	1825.0	1985.0	2353.D	2736.0	3269.0	3400.0
EQUIPM. EXPEND. PREDICT.	2604.0	2706.6	2437.8	2289.9	2110.3	2070.9	2113.6	2273.5	2380.2	2438.8	2375.9	2566.7	2774.8
1. CLASS CAPITAL STOCK	10221.2 1	0471.7	10779.5	10073.0	9466.7	9144.7	8982.0	8877.0	8934.3	9325.8	9991.4	11016.7	11913.6
2. CLASS CAPITAL STOCK	9706.4	9810.3	9945.3	10117.4	10108.1	9973.1	9796.9	9621.8	9460.1	9344.6	9340.5	9486.6	9834.3
AVE. INSTALLATION DATE	-17.3	-16.3	-15.2	-14.4	-13.6	-12.7	-11.7	-10.7	-9.6	-8.4	-7.0	-5.6	-4.3
COST OF CAPITAL	.1892	.2065	. 2232	< . 2226	.2523	2507	.2458	.2315	.2218	.2124	.2395	.2664	.2845
CAPITAL OUTPUT RATIO	2.291	2.220	2.269	2.210	2.143	2.093	2.048	1.972	1.917	1.902	1.936	1.953	1.975

SECTOR NUMBER 4 CONSTRUCTION(NEW AND OLD) (18)

SIGMA	V.O.F.	CONST.	RSOR	D. W.	WGHT.1	WGHT	.2 WG	HT.3	WGHT.4	w GHT.5	WGHT	.6 SUP	WGHTS A	VE.LIFE
.000	4.593	1.8	-5.488	-439	.040	-1	30	.211		.344	•0	174 1	1.081	
		1 95	1956	1957	1958	1959	1960	1961	1962	1963	1964	1965	1966	1967
FQUIPM. E	XPEND. ACTUA	991.	1055.0	1008.0	936.0	1077.0	891.0	906.0	996.0	922.0	950.0	911.0	1026.0	960.0
EQUIPM. E	XPEND. PREDI	CT. 845.	841.9	902.0	1012.5	1086.6	1070.8	1025.4	1013.1	1013.4	1011.0	1031.0	1071.6	1098.7
1. CLASS	CAPITAL STOCE	K 2861.	3001.6	3039.6	2982.9	3075.6	2941.4	2857.0	2881.0	2812.4	2784.8	2716.9	2776.9	2737.9
2. CLASS	CAPITAL STOCK	C 2250.	2445.5	2625.2	2760.5	2833.9	2914.5	2923.5	2900.9	2894.1	2865.7	2837.2	2794.5	2788.1
AVE . INST	ALLATION DAT	E -13.	-12.9	-12.0	-11.2	-10.2	-9.3	-8.4	-7.4	-6.5	-5.5	-4.5	-3.4	-2.4
COST OF C	APITAL	.204	.2395	.2465	.2220	.2392	.2436	-2485	.2217	.2324	.2275	.2043	-2010	.1952
CAPITAL O	SUTPUT RATIO	- 241	.249	. 259	. 263	.268	.266	.262	• 26?	.259	.249	-240	.236	.236

1. CLASS CAPITAL STOCK

2. CLASS CAPITAL STOCK

AVE. INSTALLATION DATE

CAPITAL OUTPUT RATIO

COST OF CAPITAL

457.0

465.2

-21.2

-204U

.217

509.3

464.0

-19.6

.2246

.211

505.5

470.7

-18.5

.2500

.212

49 P. 7

476.0

-17.5

.2424

-174

501.4

479.4

-16.3

.2614

.169

508.1

482.8

-15.2

.2571

-151

525.5

486.8

-13.9

.2524

.152

555.6

492.8

-12.6

.2344

-151

576.6

502.8

-11.3

.2378

-153

609.1

514.6

-10.0

.2248

.149

650.3

530.0

.2162

-8.7

.146

718.9

549.7

-7.2

.2186

.154

766.5

577.8

-5.9

.2176

.150

