Interdyme Report #4: The Triangulation Ordering

Douglas S. Meade

The Gauss-Seidel Output Solution

As we have discussed previously, the input-output equations can be written as:

$$X_i = \sum_{j=1}^{n} a_{ij} X_j + F_i \tag{1}$$

where the X_i are the outputs for each sector, the a_{ij} are the input-output coefficients, and F_j are total final demands for each sector. We solve this system of equations by successive approximations using a method called the Gauss-Seidel process. On the first iteration through this process, an initial guess is made for the X_i. Then the guess is improved by successive approximations, using the current solution values for all the other $X_j\,(j \neq i)$ in the process. If we denote the kth approximation X_i by X_i^k, and we have arrived at this kth approximation, we can calculate the $(k+1)st$ approximation by:

$$X_i^{k+1} = \frac{\left(\sum_{j<i} a_{ij} X_j^{k+1} + \sum_{j>i} a_{ij} X_j^k + F_i\right)}{1 - a_{ii}} \tag{2}$$

In other words, as we go through the solution process, we use the current solution values from iteration $k+1$ that we know, and use last (kth) iteration solution values for the industries that haven't yet been calculated in this iteration.

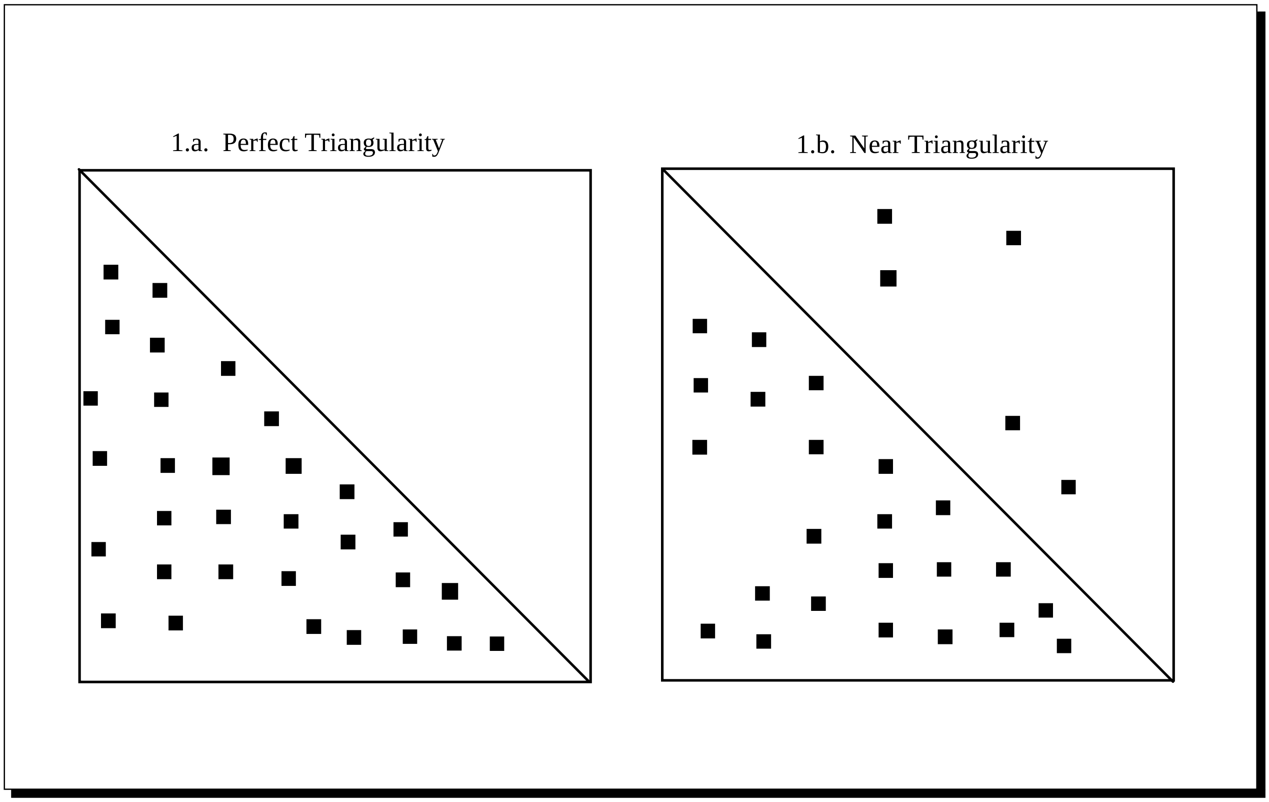
By re-ordering the sequence of the industries in this solution, it is possible to take advantage of the natural relationships of products to their inputs, to improve the speed of the solution. For example, if we order the industries so that automobiles precede steel, which precedes iron ore, then equation (2) will calculate steel output from automobile demand known, and finally will calculate iron ore production knowing the demand for steel. If it were in fact possible to number the sectors so that each product preceded all of its input sectors, then the solution of (2) would only take one iteration. This would be a case of a completely recursive input-output solution. The **A** matrix in this case would be triangular, as shown in Figure 1.a on the next page.

In a real economy, production requirements are simultaneous, so obtaining a perfectly triangular input-output matrix is impossible. However, the process also works well when the **A** matrix is nearly triangular, as shown in Figure 1.b. The more triangular we can make the **A** matrix, the more quickly will the Gauss-Seidel input-output solution converge.

To find an ordering which can achieve a fairly good level of recursivity, we choose as the first industry in the list that which has the highest ratio of final demand to total supply requirements (domestic output plus imports). In the case of the JIDEA model for Japan, this happens to be Construction (67) and Civil Engineering (69) which sell all of their output to final demand construction, and House rental (77), which sells all of its output to consumption.

In this procedure, after picking the first industry, we reclassify all intermediate purchases by that industry as "final demand", and then pick as number 2 that industry which has the highest ratio of this redefined final demand to total requirements, considering only the remaining sectors. The

Figure 1. Illustrations of Perfect Triangularity and Near Triangularity



procedure continues until we reach the last industry. With the equations in the Gauss-Seidel process taken in this order, the input-output solution will usually converge in fewer iterations than using a sequential ordering.[1]

Table 1 shows the list of the first 10 and the last 10 industries in the optimal triangulation ordering for the output solution in the JIDEA model. Note also the industries at the bottom of the list. Office supplies[2] is almost totally purchased for intermediate demand. At the bottom of the list we also have many of the primary materials, such as Pig iron and crude steel (39), Coal and lignite (8), Metal ores (6), Non-metal ores (7) and Crude petroleum (9).

[1] For the JIDEA model, the reduction was about 25% of the total running time, with both real and price side triangulation orderings.

[2] This sector appears to be much like the sector called "Unimportant industry" in the U.S. I-O table. It consists of many small items, many of them office supplies like paper clips and staples, that were small, and not reported in the Census as materials consumed by kind.

The Price Solution

The solution ordering for the price side can also be improved in this way. The price solution for a single sector can be written:

$$P_j = \sum_{i=1}^{n} a_{ij} P_i + v_j \tag{3}$$

where the P_j are the prices for each sector, the a_{ij} are the input-output coefficients as before, and the v_j are unit value added, where $v_j = V_j / X_j$ and V_j is total industry value added, and X_j is real output. The Gauss-Seidel price solution is similar to that for output, but we use the following approximation of industry price:

$$P_j^{k+1} = \frac{\left(\sum_{i<j} a_{ij} P_i^{k+1} + \sum_{i>j} a_{ij} P_i^{k} + v_j \right)}{1 - a_{jj}} \tag{4}$$

The ordering for this solution algorithm is based on the ratio of unit value added to price. If we calculate the triangulation ordering in the base year, all prices are 1.0, so the ordering is simply based on unit value added. The first sector in the optimal triangularization is that with the highest unit value added. In the case of JIDEA, this is House rent (77). Before finding the second industry in the ordering, we first transfer the unit input cost of the first sector to the value added of every other sector that uses that sector as an input. In short, we transfer across the row of the first sector j the amount $a_{ij} P_i$ to each of the other v_i. Then to find the second sector in the ordering, we again find the industry with the highest unit value added, including the transfers just mentioned. We proceed in this way until we reach the last sector.

Table 2 shows the first 10 and last 10 industries in the price side triangulation for JIDEA. The first industries tend to be those with high unit value added. The last industries are those with low unit value added, as well as industries which buy from other industries with low unit value added.

The Program TRIANG

A program has been written for Interdyme which can calculate the real and price side triangulation orderings. This program is called TRIANG.CPP, and works like a normal Interdyme program, except that it loads data only for the base year of the I-O table from the Vam file. This program can be adopted to any individual model by using the proper components to define final demand and value added. The main algorithm for the real side is shown below:

```
fdr = cobr+cohr+cogr+ingr+iprr+venr+expr;  // don't take out imports
for(i=1;i<=NJIDEA;i++)
   rat[i] = 0.0;
for(i=1;i<=NJIDEA;i++) {
   if(fdr[i] > 0.0 && (outr[i]+impr[i]) > 0.0)
      rat[i] = fdr[i]/(outr[i]+impr[i]);
   }
ntr = 0;
while(ntr<NJIDEA) {
   ratmax = -50;
   imax = 0;
   // Find highest share of final demand to domestic use
   for(i=1;i<=NJIDEA;i++)  {
```

```
      if(rat[i]>ratmax)  {
         ratmax = rat[i];
         imax = i;
         }
      }
   Triang[ntr] = imax;
   ntr++;
   if(imax<=0) {
         printf("Problem in triangulation: ntr=%d\n",ntr);
         }
    // Transfer intermediate purchases by this sector to final demand.
   for(i=1;i<=NJIDEA;i++)
      fdr[i] += outr[imax] * A(i,imax);
   fdr[imax] = -10000000.;  // Remove this sector from further consideration
   rat[imax] = -100.;
   }
```

Here is the corresponding algorithm for the price side:

```
for(i=1;i<=NJIDEA;i++)
   rat[i] = 0.0;
for(i=1;i<=NJIDEA;i++) {
   if(va[i] > 0.0 && outr[i] > 0.0)
      rat[i] = va[i]/outr[i];
   va[i] = rat[i];
   }
ntr = 0;
while(ntr<NJIDEA) {
   ratmax = -50;
   imax = 0;
   for(i=1;i<=NJIDEA;i++)  {
      if(va[i]>ratmax)  {
         ratmax = va[i];
         imax = i;
         }
      }
   Triang[ntr] = imax;
   ntr++;
   if(imax<=0) {
         printf("Problem in triangulation: ntr=%d\n",ntr);
         }

   for(i=1;i<=NJIDEA;i++)
      va[i] += A(imax,i);
   va[imax] = -100.;
   }
```

Table 1. Real Side Triangulation Ordering for JIDEA

67	Construction
69	Civil engineering
77	House rent
61	Motorcycles
89	Medical service, health and social security
86	Public administration
98	Personal services
87	Education
14	Tobacco
52	Electric computing equipment and accessories
	.
	.
	.
	.
9	Crude petroleum
31	Coal products
42	Other steel products
6	Metal ores
7	Non-metal ores
8	Coal and lignite
10	Natural gas
39	Pig iron and crude steel
60	Other vehicle
68	Repair of construction
99	Office supplies

Table 2. Price Side Triangulation Ordering for JIDEA

77	House rent
76	Real estate agencies and rent
87	Education
73	Sanitary services
84	Communication
75	Financial and insurance services
93	Renting and leasing of office machines
82	Storage facility services
74	Trade
90	Other public service
	.
	.
	.
99	Office supplies
64	Other transportation
4	Forestry and logging
17	Timber and wooden products
19	Pulp and paper
59	Motor vehicle
39	Pig iron and crude steel
40	Steel
42	Other steel products
60	Other vehicle