Current Price Identities in Macroeconomic Models:

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Abstract

This paper deals with the problem of maintaining consistency in the book-keeping identities at current prices of disaggregated macroeconomic models such as input-output models, even if the quantity and price relations linking supply and demand in the models contain discrepancy terms. Several possible solutions are proposed. Some of them have been implemented in the Danish macroeconometric model ADAM, which is used by the treasury, and the experience gained from such current use is reported.

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Key words: input-output, macroeconomic model, identities

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1. Introduction

Macroeconomic models typically contain a number of bookkeeping identities inherent in the national accounts. These identities are important aspects of the model since they help to maintain the consistency and to assure the reproduction of official national accounts statistics for the historical years. But it is not a matter of course that the consistency of the current price identities can be reproduced in the model. Not even in historical years. This is basically due to the way that national accounts data are deflated, as we shall return to. We devise some enhancements to the standard model to handle such problems.

The most basic identity is the equilibrium condition linking aggregated supply and demand quantities

$$Y + M = C + I + E \qquad (1)$$

where Y is GDP and M imports and together they are total supply. In practice, input-output coefficients are used to provide more detail in the determination of supply components; thus, if we have a separate import matrix we can split the final demand components into a domestically produced part and an imported part

$$Y = a_{YC}C + a_{YI}I + a_{YE}E \qquad (2)$$
$$M = a_{MC}C + a_{MI}I + a_{ME}E \qquad (3)$$

where $a_{Yj}+a_{Mj}=1$, $j=C,I,E$ (though this specific formulation of the i-o model is of the "endogenous imports" type the formulation easily covers the case of "exogenous imports" as well).[2] The restriction that the coefficients must sum to unity ensures that (2) and (3) implies the aggregated condition, so we can write (1) as

$$Y + M = (a_{YC} + a_{MC})C + (a_{YI} + a_{MI})I + (a_{YE} + a_{ME})E \qquad (1a)$$

The coefficients a_{ij} can be fixed at a base year value, or they can be series of observed coefficients. In the first case it is most likely that in years other than the base year, we will find that (2) and (3) do not hold because over time many of the coefficients will tend to drift away from their base year value as the structure of the economy changes. Therefore we will have to add a discrepancy term to (2) and (3) to make them balance. In the latter case, where time series of i-o coefficients are used, (2) and (3) become book-keeping identities with no room for discrepancy terms. For the purposes in the

[2] In this case the variable *E* should be interpreted as net exports, while *Y* and *M* should be considered two types of domestic production.

main text of this paper the latter interpretation is sufficient (the case where (2) and (3) contains discrepancy terms is treated in appendix 1).

Due to the duality between input-output quantity and prices models, the same input-output coefficients can be used to determine the prices on demand components from the prices on supply components, using

$$p_C = p_Y a_{YC} + p_M a_{MC} + u_C \quad (4)$$
$$p_I = p_Y a_{YI} + p_M a_{MI} + u_I \quad (5)$$
$$p_E = p_Y a_{YE} + p_M a_{ME} + u_E \quad (6)$$

Discrepancy terms are included in (4)-(6) since, in practice, we have e.g. $p_C \neq p_Y a_{YC} + p_M a_{MC}$ in the historical data set, even when time series of observed input-output coefficients are used. The reason for the presence of such discrepancy terms is found in the way national accounts are deflated, as we will discuss below.

It is easily verified from (2)-(6) that if the discrepancy terms u_C, u_I, u_E were unrestricted (and nonzero) in model computations then the value of aggregated supply would be different from the value of aggregated demand, i. e. that $p_Y Y+p_M M \neq p_C C+p_I I+p_E E$. Thus, the most basic current price identity of the model would be broken.

In the observed national accounts statistics the discrepancy terms are, of course, restricted in such a way that the current price identities hold. But in model computations such as forecasts non-zero discrepancy terms are likely to generate inconsistencies unless the appropriate restrictions are included in the model equations. This is unfortunate because in practical forecasting it is necessary to specify non-zero discrepancy terms in the forecast period to avoid jumps in the prices between the last historical year and the first forecast year.

In this paper we will derive the conditions on the discrepancy terms that ensure the current price identities of supply and demand. These restrictions are then used to determine a number of the discrepancy terms in the model residually. As an alternative, a technical reformulation involving a general adjustment of demand prices is suggested. We will treat only additive discrepancy terms in the main text, since the analysis of this case is simpler; the analogous, but technically more complicated case with multiplicative discrepancy terms is treated in appendix 2.

The causes of the problem

The standard assumption in i-o price models such as (4)-(6) is that all supplies from the same source are at the same price, independently of which category of demand it is supplied to. Thus, for example, the same price p_Y is applied to all supplies from domestic production, no matter whether they are used in category *C*, *I* or *E*. The economic content of this assumption is that every source supplies a single product, and that the demand elasticity for this product (and therefore the mark-up on marginal cost), must be identical in all uses. In other words, there can be no *price discrimination*. If this assumption actually did hold in practice, there would be no room for discrepancy terms in (4)-(6), and our problem would not exist. Unfortunately, it is in fact very unlikely to hold, since many industries tend to supply their product at different prices to different users. This means that *price discrimination* between different users contributes to discrepancy terms in (4)-(6).

In the simplest input-output and CGE model frameworks, where all parameters are calibrated from a single input-output/SAM matrix in a base year, and where all prices are defined to be equal to 1 in this base year, the problem would not be visible at all in the basic data. However, if we would want to introduce price discrimination in such model calculations, so that e.g. export prices would be able to move away from home market prices, the problem could easily show up anyway.

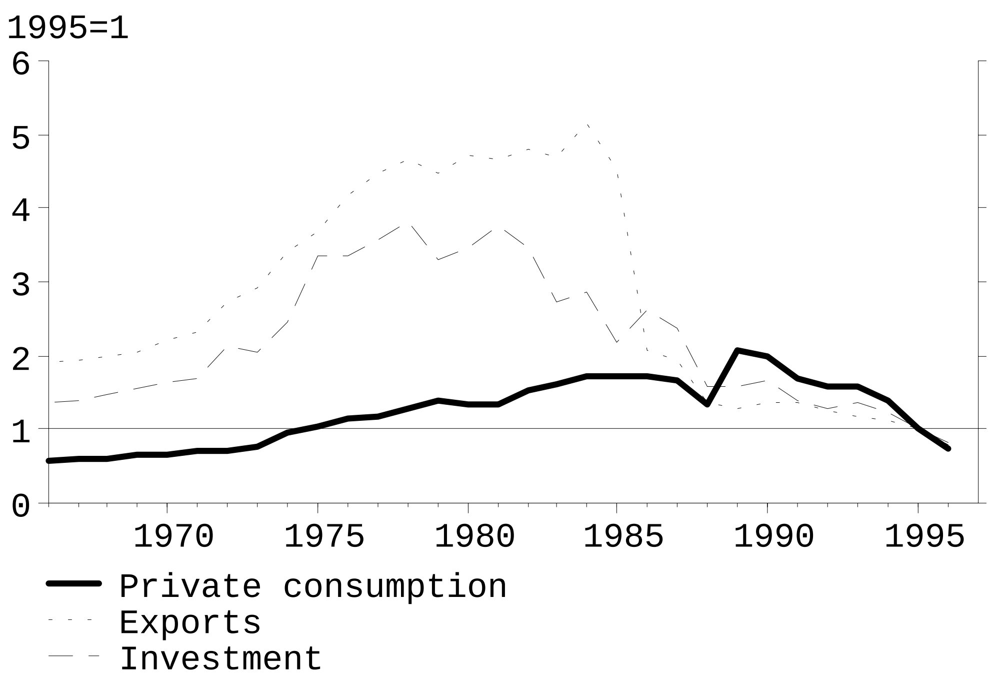
A less tractable, but probably more severe reason for discrepancy terms in (4)-(6) is that each industry in the input-output table does not supply only one product, but several. This is problematic since many of the products supplied from a single industry have different prices, so when the bundles of products are composed differently to different users, prices will necessarily differ between them. This problem arises because the deflation of the national accounts data is carried out at a much more detailed level than the model computations. In Denmark, for example, the deflation of national accounts is carried out at a level of app. 2750 products, while the input-output tables are published at a level of 130 industries only.[3] Thus, on average an industry in the input-output table produces 21 goods. So, if e.g. the domestic supplies to consumption have a different product composition than the supplies to exports, with respect to the deflation level, then the prices for the two supplies are likely to differ, causing discrepancy terms in (4) and (6).

[3] See Thage(1986), Statistics Denmark (1998). We have no wish to pick up the question of commodity vs. industry tables in this context, though; even if the "commodity technology" assumption had been used for the published tables, the deflation would almost certainly still be carried out at a much more detailed commodity level than the "characteristic commodities".

For a simple example, consider the industry 'construction' which typically supplies 'building maintenance' for consumption and 'new buildings' for investment. The two products are deflated using different prices, at least in the Danish national accounts, since maintenance contains almost exclusively labour cost, while the cost of materials contributes much more significantly to the total cost of new buildings. Therefore, the price of supply from construction to consumption tends to grow faster than the price of supply from construction to investment. The production price on construction is some weighted average of the two, the weights depending primarily on the level of new building investment (since building maintenance is very stable).

Figure 1 shows an extreme example of different prices of supplies from an industry.

Figure 1. Prices on supplies from the industry: Manufacturing of computers etc



That is why in integrated models, in which a combination of input-output tables and economic time series is used, *aggregation problems* is likely to be the major cause of discrepancy terms in (4)-(6). In such models, the base year of price index calculations is given by the standards of the available national accounts data set, and we will typically have to use equations like (4)-(6) for years where the price indexes are different from 1. In this case the discrepancy terms u_C, u_I, u_E are directly computable from the data bank as the historical price minus the estimated price, e.g. $u_C = p_C - (p_Y a_{YC} + p_M a_{MC})$, and they are extremely unlikely to be zero in such years.

In practice, aggregation problems and price discrimination are both extremely likely to be significant causes of discrepancy terms in (4)-(6). However, in models based on time series of national accounts data the aggregation problems will probably be the dominating cause, in particular if the base year of fixed price computations is somewhat back in time, and if the aggregation level is so high that the individual product prices within each aggregate are likely to develop differently.

2. Some solutions

The simple solution to the problem of ensuring the aggregated current price identity is to find the necessary constraint on the discrepancy terms and then use this constraint to determine one of the error components residually:

> The value of total supply equals the value of total demand for arbitrary exogenous C, I, E, p_M and p_Y if and only if
>
> $$u_C C + u_I I + u_E E = 0$$
>
> i.e. that the discrepancy terms in the demand price equations, weighted with the appropriate quantities, must sum to 0.

Proof: The condition that the value of total supply equals the value of total demand is equivalent to

$$p_Y Y + p_M M = p_C C + p_I I + p_E E \quad (7)$$

$$= (p_Y a_{YC} + p_M a_{MC} + u_C)C + (p_Y a_{YI} + p_M a_{MI} + u_I)I + (p_Y a_{YE} + p_M a_{ME} + u_E)E$$

$$= (p_Y a_{YC} + p_M a_{MC})C + (p_Y a_{YI} + p_M a_{MI})I + (p_Y a_{YE} + p_M a_{ME})E + u_C C + u_I I + u_E E \quad \Leftrightarrow$$

$$u_C C + u_I I + u_E E = 0 \quad (8)$$

using (4), (5), (6) and (1a).

Please note that the weighting of the discrepancy terms in (8) depends on the quantity variables *C*, *I* and *E*. If these variables are exogenous in the model, as they are here, the user could easily specify the discrepancy terms in such a way that (8) is violated, causing the basic identity (7) to be broken. But even worse, such quantities are usually determined endogenously in a wider model context. In any case, the current price identity (7) can be ensured only if one of the discrepancy terms is determined in the model as a function of the other (exogenous) discrepancy terms using (8). If, for example, we choose the discrepancy term on investment for such a residual determination, we get the equation

$$u_I = -(u_C C + u_E E)/I \tag{9}$$

which should be added to the model; note that u_C and u_E can be set arbitrarily by the user in forecast periods e.g. to implement price discrimination. This model - equations (2)-(6), (9) - is termed the *simple residual price model*.

In practice, this type of residual determination has the disadvantage that the whole load of the adjustment is placed on a single demand price, in this case p_I. This could be a problem, in particular if many categories of final demand are specified in the model and, therefore, many different developments must be balanced in this single demand price. The model user could easily, by accident or unawareness, generate a peculiar residual price.

Therefore, it could be a practical alternative to specify a general correction of all demand prices. This is done by enhancing (4)-(6) with a general correction term, *u* (defined to be 0 in the historical data set):

$$p_C = p_Y a_{YC} + p_M a_{MC} + u_C + u \tag{10}$$
$$p_I = p_Y a_{YI} + p_M a_{MI} + u_I + u \tag{11}$$
$$p_E = p_Y a_{YE} + p_M a_{ME} + u_E + u \tag{12}$$

In model computations, the general correction term *u* is then determined using the identity (7) so that

$$(u_C+u)C+(u_I+u)I+(u_E+u)E = 0 \qquad \Leftrightarrow$$

$$u = -(u_C C+u_I I+u_E E)/(C+I+E) \tag{13}$$

Now, if the user should specify the exogenous discrepancy terms u_C, u_I and u_E in such a way that, without the correction (13), it would lead to a violation of the aggregated value identity (7), then the general correction term u would automatically adjust all demand prices in the opposite direction to keep (7) fulfilled. On the other hand, the user has to accept that the final correction of a price, e.g. p_C, could be different from the correction that was originally intended when the value of u_C was set.

This model - equations (2), (3), (10)-(13) - is termed the *simple general price correction model*.

More sophisticated corrections of demand prices

It should be noted that, though both the residual and the general price corrections ensures the aggregated identity (7), neither of them is sufficient to guarantee the current price identities for the individual components *Y* and *M* in the model. A closer look at this question requires, however, a more general, input-output type formulation of the model.

Therefore, we reformulate the model (2)-(3) as follows

$$Y = C_Y + I_Y + E_Y \qquad (14)$$
$$M = C_M + I_M + E_M \qquad (15)$$

where C_Y, C_M, I_Y, I_M, E_Y, E_M are the individual cells of the input-output quantity matrix, determined by the equations

$$C_i = a_{iC}\, C \qquad i=Y, M \qquad (16)$$
$$I_i = a_{iI}\, I \qquad i=Y, M \qquad (17)$$
$$E_i = a_{iE}\, E \qquad i=Y, M \qquad (18)$$

Equations (14)-(18) are, of course, equivalent to (2) and (3).

Likewise, the price equations are reformulated to determine the price of the individual input-output cells, as

$$p_{Yj} = p_Y + u_{Yj} \qquad j=C, I, E \qquad (19)$$
$$p_{Mj} = p_M + u_{Mj} \qquad j=C, I, E \qquad (20)$$

which means that the final demand price equations become identities given by

$p_C = (p_{YC}C_Y + p_{MC}C_M)/C$ (21)
$p_I = (p_{YI}I_Y + p_{MI}I_M)/I$ (22)
$p_E = (p_{YE}E_Y + p_{ME}E_M)/E$ (23)

Substituting (16)-(20) into (21)-(23) it is easily seen that the only new feature in this price determination is that the discrepancy term u_j in each of equations (4)-(6) is replaced by two "cell-specific" discrepancy terms using the relation

$$u_j = a_{Yj}u_{Yj} + a_{Mj}u_{Mj} \qquad j=C, I, E \qquad (24)$$

Such "cell-specific" discrepancy terms are necessary to ensure that the value of supply is equal to the value of demand for each supply component *Y* and *M*. In the case of *Y* we get that

$$p_Y Y = p_{YC}C_Y + p_{YI}I_Y + p_{YE}E_Y \qquad (25)$$

$$= (p_Y + u_{YC})\, C_Y + (p_Y + u_{YI})I_Y + (p_Y + u_{YE})E_Y \qquad \Leftrightarrow$$

$$u_{YC}C_Y + u_{YI}I_Y + u_{YE}E_Y = 0 \qquad (26)$$

using (19) and (14). This condition is completely analogous to the condition for aggregated consistency, (7), but here it involves only the domestic supplies from *Y*. Of course, a similar condition is required for the imported supplies from *M*.

In general, there will be a condition like (26) for every supply component in the model, which will enable us to determine residually one of the "cell-specific" price discrepancy terms of the corresponding row of the input-output table.

This model - (14)-(23), (26) - is termed the *full residual price model*.

Once again we can avoid the residual determination of a "cell-specific" demand price by specifying a general row correction of the prices from each supply component, in analogy with (10)-(12). This means that (19) and (20) is replaced by

$$p_{Yj} = p_Y + u_{Yj} + u_Y \qquad j=C, I, E \qquad (27)$$
$$p_{Mj} = p_M + u_{Mj} + u_M \qquad j=C, I, E \qquad (28)$$

where u_Y and u_M are defined to be 0 in the historical data set; in model computations, such as forecasts, the general correction of the "cell-specific" prices in each row can be found in analogy with (13) as

$$u_Y = -(u_{YC}C_Y + u_{YI}I_Y + u_{YE}E_Y)/Y \quad (29)$$
$$u_M = -(u_{MC}C_M + u_{MI}I_M + u_{ME}E_M)/M \quad (30)$$

for arbitrary exogenous values of u_{YC}, u_{YI} and u_{YE} set by the user. This model - (14)-(18), (21)-(23), (27)-(30) - is termed the *full general price correction model*.

A fairly complete yet practical solution

The full adjustment methods suggested above determines, in fact, a complete, consistent (implicit) current price i-o matrix, which in turn ensures the current price identities for all supply components. In contrast, the simple adjustment models ensure only the aggregated identity of supply and demand and, therefore, they contain no such matrix. On the other hand, the full adjustments are burdensome in terms of space and effort, since they require the introduction of one error variable per cell of the i-o table; in effect, this is an extra i-o table of variables in the data bank.

There is, however, a kind of compromise solution ensuring a consistent current price matrix while still avoiding the abundance of discrepancy terms in the full adjustment models. This solution is established by redefining the (implicit) current price cells of the i-o matrix in such a way that

$$p_{ij} = p_i + u_i + u_j \quad (31)$$

If such a redefinition should at first sight appear too restrictive to the reader, then please recall that the option of doing no adjustments at all is equivalent to setting $u_i = u_j = 0$ in (31), and that the simple general correction is equivalent to setting $u_i = u$ in (31). So, the compromise solution is still far less restrictive than those options, while we will see that it allows a consistent implicit current price matrix in the model.

It is clear from (31) that some normalisation of u_i and u_j is necessary, since any single number can be added to every u_i and subtracted from every u_j to yield the same p_{ij}. We would suggest using (8) to normalize the demand price errors u_j in the historical data set, which would imply that $u_yY + u_mM = 0$ (using (25) for Y and M and adding the equations); the observed values of u_i and u_j in the historical data set can then be determined from (31), (21)-(23) and (25). In model computations, such as forecasts, the user should be able to set all the demand price errors u_j arbitrarily, since all the u_i can be determined in the model to ensure the current price identities such as (25).

Another possibility: Correction of supply prices

The solutions discussed so far have taken the supply prices p_Y and p_M as given, and therefore, suppressed the effects on aggregated prices from changes in the composition of demand. Such a suppression has the advantage that the economic properties of any determination of supply prices in a wider model context are unchanged, such as e.g. homogeneity with respect to total cost.

On the other hand, in some situations it could be desirable to allow the effects from a change in demand composition to change aggregated supply prices. This would, of course, require that the discrepancy terms of demand price equations had a clear interpretation as caused by price discrimination, rather than by unspecified aggregation problems with no clear interpretation. The best example probably is that we could want to model export prices differently from the home market prices. If competition is harder on the export market, the prices are likely to be lower, and a shift e.g. from home market supplies to export supplies would therefore decrease the aggregated production price.

Though such effects from demand composition to supply prices does not fit very well into the model, since they break the standard assumption of identical prices to all users, they could be accounted for by using the unmodified equations (2)-(6) and then, subsequently, define a modified p_Y to ensure (7). A similar procedure could be applied to each supply component using (14)-(23). They would, however, imply that the discrepancy terms in demand prices could be explained only by differences in the "mark-up" on different markets; ideally, the original supply prices should then reflect (marginal) cost only, not profits. The model would then determine different profits in the various uses. In a wider model context, the operating surplus of industries should be adjusted to conform to the modified prices.

Working with current price cells only

An apparently more radical solution would be to use current price input-output tables only, ignoring the information of fixed price input-output coefficients. This would mean that the model (14)-(23) should be reformulated to use a current price input-output table only, i. e. (using prefix *v* to denote current price input-output cells)

$$Y = (vC_Y + vI_Y + vE_Y)/p_Y \qquad (32)$$

$$M = (vC_M + vI_M + vE_M)/p_M \qquad (33)$$

$$vC_i = p_{iC} a_{iC} C \qquad i=Y, M \qquad (34)$$

$$vI_i = p_{iI}a_{iI}\ I \qquad i\text{=Y, M} \qquad (35)$$
$$vE_i = p_{iE}a_{iE}\ E \qquad i\text{=Y, M} \qquad (36)$$
$$p_{Yj} = p_Y + u_{Yj} \qquad j\text{=C, I, E} \qquad (37)$$
$$p_{Mj} = p_M + u_{Mj} \qquad j\text{=C, I, E} \qquad (38)$$
$$p_C = (vC_Y+vC_M)/C \qquad (39)$$
$$p_I = (vI_Y+vI_M)/I \qquad (40)$$
$$p_E = (vE_Y+vE_M)/E \qquad (41)$$

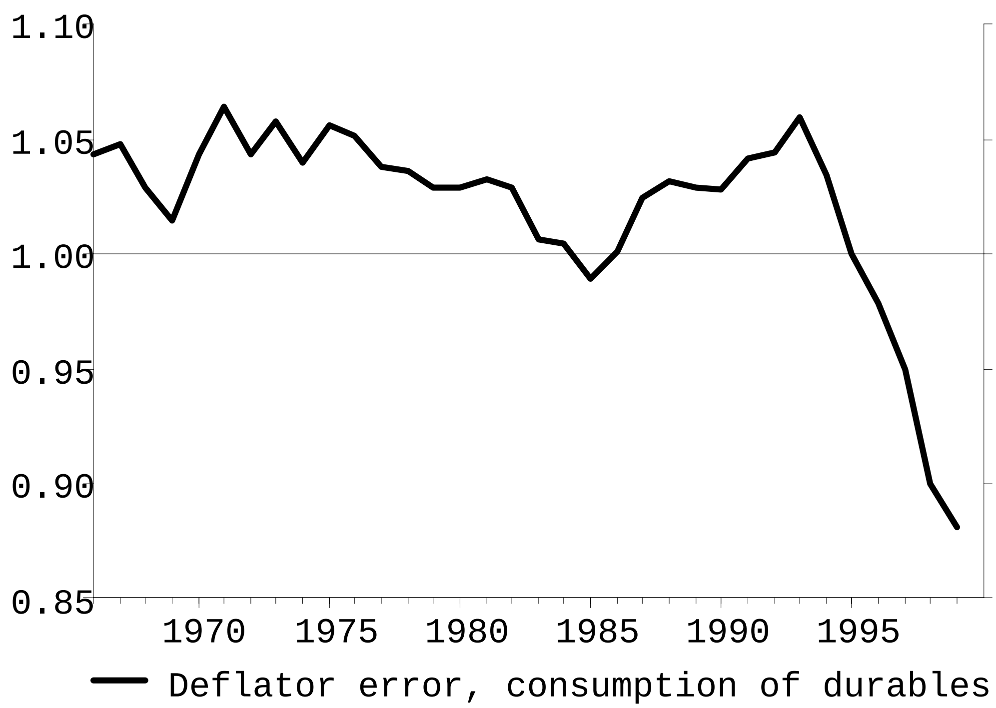
In effect, as some manipulation will show, this solution is equivalent to a redefinition of the input-output coefficients, replacing the "true" fixed price coefficients a_{ij} by new "pseudo-fixed price" coefficients $p_{ij}\, a_{ij} /p_i$. But isn't it a shame to discard the information embodied in the fixed price cells? Well, from economic theory we know that while the nature of the economic system imposes book-keeping constraints on value concepts (at current prices), there is no theoretical reason to expect that common fixed price indexes should satisfy such constraints as, e.g., $Y+M=C+I+E$. And on the other hand there is a wealth of expenditure models determining cost cells of the input-output table as a function of prices and total expenditure, with no need for fixed price cell information. So, if there is a problem in using current price input-output tables only, it may be nothing else than our unwillingness to depart from established professional tradition.

3. Experience

The simple and the “compromise” general price correction models, in their multiplicative versions as listed in appendix 2, have been tested in the Danish model ADAM; the "Annual Danish Aggregate Model" has been used by the government for economic policy analysis, budgeting and forecasting purposes for more than 20 years. The model is in the econometric tradition of Tinbergen and Klein, but it contains an integrated, structural form static input-output system for determination of production and prices, in the way outlined in (2)-(6). This system uses 19 industry branches, 14 types of primary inputs and 27 categories of final demands (the numbers of primary inputs and final demands include 11 components of imports and 7 components of exports, respectively, with commodities broadly by 1-digit SITC).

As an example, the (relative) demand price error on consumption of durables from ADAM is shown in figure 2. The discrepancy term is 1 in 1995, since this is the base year of the national accounts fixed price indexes. It displays, however, considerable drift over time in a way more compatible with a hypothesis of aggregation problems than with any clear hypothesis of price discrimination. The sudden drop below 0.9 at the end of the period is likely to be due to the recently adopted practice of using hedonic computer price indexes in the deflation of the Danish national accounts; while such computer price indexes fall at dramatic rates in the nineties, the conventional price indexes on other durables develop more slowly; in turn, this price split exposes the differences in product composition of the final demand categories in the model, since consumption of durables contains relatively more computers than other demand categories supplied from the same ADAM supply sources.[4]

Figure 2. Relative price error on consumption of durables in ADAM



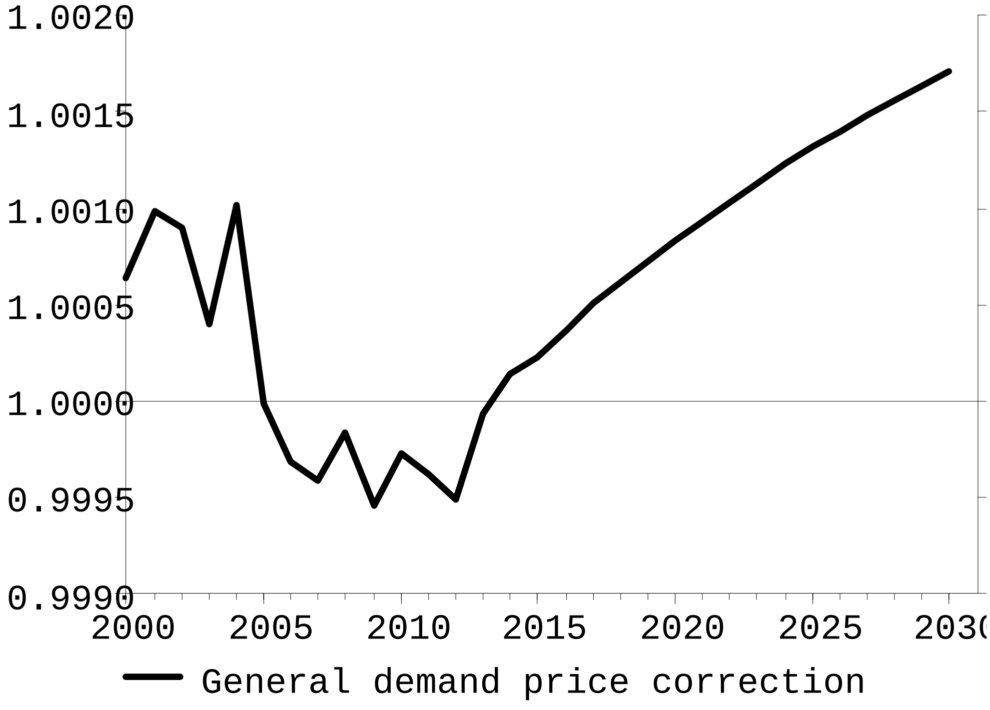
The simple general price correction, in the multiplicative version as in (2.10)-(2.13) in appendix 2, has been implemented in ADAM versions of March 1995 onwards (however, the general correction has been limited to influence only domestic final demand components, to avoid interference with production price formation and with external competitiveness). The established procedure in ADAM forecasts is that the

[4]Except, perhaps, investment in machinery, which shows a similar pattern of price errors.

forecasted value of every (relative) demand price error is set equal to the most recent observed value, to avoid price jumps in the first forecast year. Figure 3 shows a quite typical forecast profile of the general demand price correction factor resulting from this procedure.

In the first year of the forecast period the general correction factor jumps by 0.06 pct from its historical value of 1, due to a twist in the demand components towards components with a relative error lower than 1. Such movements in the first few years of the forecast period are quite typical reflecting the phase of the business cycle embodied in the data for the most recent historical period; this is because the model tends to adjust the cycle fairly quickly towards equilibrium. From app. 2013 the business cycle fades out, and the model solution enters a steady state path, causing the general correction factor to drift slowly, which reflects differences in sectoral steady state growth rates.

Figure 3. A forecast profile of the general demand price correction factor.



Though, as expected, the price movements caused by the general price correction are quite small, the users have found them annoying. One reason for this is that the political demand for inflation convergence embodied in the EMU, in conjunction with the very low rates of inflation in the Euro countries, creates public interest in even small deviations in consumer prices. Another reason is that the model users sometimes want to turn the model "upside down" in order to use flash indicators of export and consumer prices to compute early estimates of domestic inflation; such a procedure becomes technically more difficult when the general price correction is present.

Therefore, the main users have adopted a complicated procedure, which is, in effect, equivalent to the simple residual price model. They have chosen the price on investment in inventories as the residual demand price. In most cases, this price is a relatively harmless one to determine residually; however, since (fixed price) investment in inventories can sometimes be negative, or zero, the correction has to be carefully formulated to function properly in such cases.

The “compromise” general price correction, derived from (2.31) in appendix 2, has been implemented in tests only. It works in a way quite similar to the simple correction, and therefore the simple correction was preferred.

Current work is aimed at changing the formulation of the input-output system to use current price input-output cells only. While simultaneously solving the problem with the current price identities it is expected that this solution will ease the transition to the use of chained quantity indexes recommended by the SNA93/ESA95 manuals.

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Appendix 1: Formulations including discrepancy terms in the quantity relations

If, in addition, the quantity equations contain discrepancy terms, the equations from the main text should be modified as follows, using quantity discrepancy terms q_j and q_{ij}

$$Y = a_{YC}C + a_{YI}I + a_{YE}E + q_Y \quad (1.2)$$
$$M = a_{MC}C + a_{MI}I + a_{ME}E + q_M \quad (1.3)$$
$$u_C C + u_I I + u_E E - p_Y q_Y - p_M q_M = 0 \quad (1.8)$$
$$u_I = (p_Y q_Y + p_M q_M - u_C C - u_E E)/I \quad (1.9)$$
$$u = (p_Y q_Y + p_M q_M - u_C C - u_I I - u_E E)/(C+I+E) \quad (1.13)$$
$$C_i = a_{iC} C + q_{iC} \quad i=Y, M \quad (1.16)$$
$$I_i = a_{iI} I + q_{iI} \quad i=Y, M \quad (1.17)$$
$$E_i = a_{iE} E + q_{iE} \quad i=Y, M \quad (1.18)$$
$$vC_i = p_{iC}(a_{iC}C + q_{iC}) \quad i=Y, M \quad (1.33)$$
$$vI_i = p_{iI}(a_{iI}I + q_{iI}) \quad i=Y, M \quad (1.34)$$
$$vE_i = p_{iE}(a_{iE}E + q_{iE}) \quad i=Y, M \quad (1.35)$$

All other formulae, including (26), (29) and (30), are unchanged; though, of course, in this case C_i, I_i and E_i are determined by the modified formulae (1.16)-(1.18).

Appendix 2: Formulations with multiplicative discrepancy terms

In this appendix the formulae are quite analogous to those in the main text, except that multiplicative discrepancy terms are used instead of additive discrepancy terms. Such multiplicative discrepancy terms are perhaps the most common type in practice. The treatment of them is slightly more technical. Only formulae that differ from the main text are shown, and they carry the same numbers as their analogues, preceded with a '2.'.

$$p_C = (p_Y a_{YC} + p_M a_{MC})\, k_C \qquad (2.4)$$
$$p_I = (p_Y a_{YI} + p_M a_{MI})\, k_I \qquad (2.5)$$
$$p_E = (p_Y a_{YE} + p_M a_{ME})\, k_E \qquad (2.6)$$

> The value of total supply equals the value of total demand for arbitrary exogenous C, I, E, p_M og p_Y if and only if
>
> $$(k_C-1)p_C C/k_C + (k_I-1)p_I I/k_I + (k_E-1)p_E E/k_E = 0$$
>
> i.e. that the deviations of multiplicative discrepancy terms in the demand price equations from 1, weighted with the appropriate uncorrected demand components at current prices, must sum to 0.

Proof: The condition that the value of total supply equals the value of total demand is

$$p_Y Y + p_M M = p_C C + p_I I + p_E E \qquad (2.7)$$

$$= (p_Y a_{YC} + p_M a_{MC})\, k_C C + (p_Y a_{YI} + p_M a_{MI})\, k_I I + (p_Y a_{YE} + p_M a_{ME})\, k_E E \quad \Leftrightarrow$$

$$0 = (k_C-1)p_C C/k_C + (k_I-1)p_I I/k_I + (k_E-1)p_E E/k_E \qquad (2.8)$$

(using (2.2)-(2.6) and collecting terms).

The formula determining the residual discrepancy term, analogous to (9) is simple, but tedious and it is not shown here. Instead we will show the simple form of the multiplicative general price correction model. First a general correction term k, defined to be 1 in the historical data set, is added to (2.4)-(2.6):

$$p_C = (p_Y a_{YC} + p_M a_{MC})\, k_C\, k \qquad (2.10)$$

$p_I = (p_Y a_{YI} + p_M a_{MI})\, k_I\, k$ (2.11)
$p_E = (p_Y a_{YE} + p_M a_{ME})\, k_E\, k$ (2.12)

In model computations the general correction term *k* is determined using (7) which, after some term collection, yields the unsurprising formula

$$k = (p_y Y + p_M M)/(p_C C + p_I I + p_E E) \qquad (2.13)$$

obviously ensuring the aggregated identity (7).

To ensure all the current price identitites the extended framework of (14)-(23) is needed. In the multiplicative case we need only to modify (19) and (20) as

$p_{Yj} = p_Y\, k_{Yj}$ j=C, I, E (2.19)
$p_{Mj} = p_M\, k_{Mj}$ j=C, I, E (2.20)

In the case of the supply component *Y* we find that (25) yields

$$p_Y Y = p_{YC} C_Y + p_{YI} I_Y + p_{YE} E_Y$$
$$= p_Y k_{YC} C_Y + p_Y k_{YI} I_Y + p_Y k_{YE} E_Y \quad \Leftrightarrow$$

$$k_{YC} C_Y/Y + k_{YI} I_Y/Y + k_{YE} E_Y/Y = 1 \quad \Leftrightarrow$$

$$k_{YI} = (Y - k_{YC} C_Y - k_{YE} E_Y)/I_Y$$

A similar condition applies to supply component *M*.

The multiplicative version of the full general price correction model is found from (14)-(18), (21)-(23) and the modified formulae

$p_{Yj} = p_Y\, k_{Yj}\, k_Y$ j=C, I, E (2.27)
$p_{Mj} = p_M\, k_{Mj}\, k_M$ j=C, I, E (2.28)

where k_i are defined to be 1 in the historical data set. In model computations, k_i will ensure the identity of supply and demand for supply component *i*. From (25), (2.27) and (2.28) we can determine such correction factors for *Y* and *M* using

$k_Y = Y/(k_{YC} C_Y + k_{YI} I_Y + k_{YE} E_Y)$ (2.29)
$k_M = M/(k_{MC} C_M + k_{MI} I_M + k_{ME} E_M)$ (2.30)

The multiplicative analogue of the compromise solution (31) becomes

$$p_{ij} = p_i \, k_i \, k_j \qquad (2.31)$$

It is clear from (2.31) that some normalisation of k_i and k_j is necessary, since either of them can be multiplied and the other one divided with some scalar number to yield the same p_{ij}. We would propose using (2.8) for such a normalization of k_j. This is, however, tedious. Instead, we would suggest the following algorithm: In step one, compute k_j for k_i=1, which is equivalent to the use of (2.4)-(2.6); then compute k_i as in (25) using (2.31). Repeat this procedure iteratively using the new k_i as start values, until k_i and k_j converges. In effect, this is the well-known biproportional rAs procedure used here to determine an implicit, consistent current price matrix; however, only the factors k_i and k_j need to be used in the model, since the current price cells can be derived from (2.31) subsequently. The procedure makes (25) and (21)-(23) hold in the historical data set. In model computations, such as forecasts, the k_j factors should be set exogenously by the user, while the factors k_i should be determined in the model as in (25) to preserve all current price identities.