Embodying Embodiment in a Structural, Macroeconomic Input-Output Model

This draft: September 30, 2002

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Abstract:

In this paper, I develop a regression-based system of labor productivity equations which account for capital-embodied technological change and I incorporate this system into IDLIFT, a structural, macroeconomic input-output model of the U.S. economy. Builders of regression-based forecasting models have long had difficulty finding labor productivity equations that exhibit the "Solowian" property that movements in investment should cause accompanying movements in labor productivity. The production theory developed by Solow and others dictates that this causation is driven by the effect of traditional capital deepening as well as technological change embodied in capital. Lack of measurement of the latter has hampered the ability of researchers to properly estimate the productivity-investment relationship. Recent research by Wilson (2001) has alleviated this difficulty by estimating industry-level embodied technological change. In this paper, I utilize those estimates to construct capital stocks adjusted for technological change and then use these adjusted stocks to estimate Solow-type labor productivity equations. It is shown that replacing IDLIFT's former productivity equations, based on changes in output and time trends, with the new equations results in a convergence between the dynamic behavior of the model and that predicted by traditional (Solowian) production theory.

Keywords: Equipment-Embodied Technological Change, Input-Output, Productivity, Forecasting

JEL Codes: C3, C5, O3

* This paper was adapted from Chapters 4 and 5 of my dissertation at the University of Maryland. Much of the research for this paper was conducted while I was a researcher at the INFORUM research center. I would like to thank the staff of INFORUM, particularly Doug Meade, and its director (and my advisor) Clopper Almon for all of their support and advice.

1. Introduction

The hypothesis that much of technological progress is embodied in new capital goods, and therefore investment in new capital is necessary to foster productivity growth, is an old one -- tracing its roots at least as far back as Smith's *Wealth of Nations*, which attributed its source to the division of labor: "The invention of all those machines by which labour is so much facilitated and abridged, seems to have been originally owing to the division of labour" (Smith, 1776, p.9).[1] The basic hypothesis was refined and extended over time by Karl Marx, Joseph Schumpeter, and Robert Solow, among others. Yet, obtaining independent measures of the rate at which embodied (or "investment-specific") technological change has progressed has long eluded us. Absent knowledge of this rate, it is impossible to correctly measure the *productive capacity* of the economy's capital stock.

The concept of the *productive capacity of capital*, or simply *productive capital* for short, is the theoretically correct (in terms of traditional production theory) concept of capital to be used in production and productivity analyses. The productive capital stock, combined with information on the degree to which capital is being utilized, tells us the flow of capital services used in the production process. The flow of capital services to production is generally considered one of the main determinants of labor productivity. In particular, the production theory developed by Solow and others in the 1950's and 1960's assumes that labor and capital have some degree of complementarity so that increases in capital increase the productivity of labor.[2] Thus, modeling long-run labor productivity growth relies on good measures of productive capital (as well as utilization rates). And, of course, modeling long-run productivity growth is *a*, if not *the*, key element to any forecasting model.

Yet surprisingly little empirical research has focused on the measurement of capital and the implications of mismeasurement for modeling productivity.[3] The situation appears to be changing, however. Thanks in part to the rapid advances in equipment technology which have exacerbated and exposed the shortcomings of the current ways of measuring capital, researchers interested in productivity analysis and forecasting can no longer ignore these shortcomings in their empirical work.

Wilson (2001), for instance, made important strides toward quantifying the contribution that technological advances in equipment have on productive capital and productivity (and more importantly, on their growth rates). Wilson first develops a production-side approach to

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[1]See Scherer (1999), Chapters 2-4, for a discussion of the history of economic thought relating to technological change (particularly that which is embodied in machinery) and long-run productivity growth.

[2]Hereafter, this theory will be referred to as the "Solowian" production theory and labor productivity equations which exhibit the property of capital increases causing labor productivity increases will be referred to as "Solow-type" equations.

[3]Unless otherwise indicated, *capital* will hereafter refer to *productive capital*.

estimating equipment-embodied technological change.[4] The method estimates embodied technological change for U.S. manufacturing industries directly from observed production, input, and investment decisions at the plant level using the Longitudinal Research Database of the U.S. Census Bureau. Specifically, he estimates a gross-output production function in which the equipment capital input is a parameterized stream of all past investments net of physical depreciation. The vintage weights in this stream, or perpetual inventory, imply an estimable rate of embodied technological change.

The empirical results are shown in the top panel of Table 1. The estimates of embodied technological change generally clustered around 5-10%. 15 of the 24 industries had positive estimates with 9 being significant. Of the 9 negative estimates, 5 were significant. These estimates seem quite reasonable in their ordering, but the presence of negative estimates (which are at best counter-intuitive and at worst nonsensical) and of unrealistically high estimates for producers of Computers and Communications Equipment as well as the relative imprecision of the estimates leaves some skepticism regarding the usefulness of these estimates.

Evaluating the reasonableness of the estimated rates of embodied technological change and deriving analogous rates for nonmanufacturing industries, for which longitudinal plant-level data is not currently available, was the motivation for Wilson (2002). In that study, Wilson constructs an index capturing the extent of research and development directed at the various capital goods that constitute a given industry's capital stock. Specifically, he combines (and adjusts) data from the National Science Foundation and the Commerce Department to construct a weighted average of the R&D done on the equipment capital that an industry purchases for 62 industries that span the U.S. private economy. This industry-level index of capital-embodied R&D is shown to have a large, positive correlation with the manufacturing estimates of embodied technological change.

The estimated relationship in manufacturing between embodied technological change and the index of embodied R&D, along with the index values for nonmanufacturing industries, is used to impute nonmanufacturing rates of embodied technological change. These rates are shown in the bottom panel of Table 1. They range from 0 to 11%. It should be noted that the estimated coefficients in the imputation regression have large standard errors, thus the imputed rates have correspondingly large standard errors associated with them. Nonetheless, the magnitudes and the cross-sectoral ranking of these rates of embodied technological change are quite reasonable. For instance, the lowest imputed rates of embodied technological change are found in mining industries which, not surprisingly, invest mainly in mining equipment which has historically experienced very little R&D. The highest rate is in Communications Services which invests mainly in R&D-intensive equipment such as telecommunication equipment and computers.

In the current paper, I use the results of the above research to analyze the effect on the IDLIFT model of replacing the former labor productivity equations, which contain no influence

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[4]It should be noted that the general production-side approach and the estimation of embodied technological change for aggregate U.S. manufacturing was a collaborative effort with Plutarchos Sakellaris (see Sakellaris and Wilson (2000)). Wilson (2001) extended these empirical results to the industry-level and presented a separate method of deriving rates of embodied technological change for non-manufacturing industries.

from investment, with Solow-type equations, estimated using correctly measured productive capital stocks. Section 2 briefly describes the structure, particularly of the labor productivity equations, of the IDLIFT input-output simulation and forecasting model which is maintained by the INFORUM research center.[5] In this section, I also discuss the shortcomings of the current labor productivity equations that motivated the proposed changes suggested in this paper. Then, armed with a full set of industry-level estimates of capital-embodied technological change spanning the economy from the aforementioned research, I construct *quality-adjusted*[6] equipment capital stocks in Section 3. The estimation of several alternative sets of productivity equations utilizing the constructed capital stocks is described in Section 4. In Section 5, Base forecasts are generated for both the current version of the model and the new, rival version. Two alternative scenarios, or "shocks," are then introduced to each model and the deviations from base are analyzed. Section 6 concludes.

2. Brief Overview of the IDLIFT model

A. The structure of IDLIFT

Since its founding in 1967 by Clopper Almon, Inforum has been building, and encouraging others to build, regression-based structural macroeconomic models based on input-output relationships between industries. The Inforum modeling philosophy differs from that of other large-scale macro models primarily in the input-output structure underlying the model.

Inforum's main model of the U.S. economy is IDLIFT, which is presently in the process of replacing its predecessor, LIFT (Long-term Interindustry Forecasting Tool).[7] In this section, I will discuss the general structure of the IDLIFT model as it currently stands. For a discussion of how IDLIFT differs from the LIFT model and planned future changes to the model (aside from those proposed in this paper), see Meade (1999).

The IDLIFT model forecasts output, employment, prices, exports, imports and interindustry flows for 97 commodity sectors; personal consumption expenditures (PCE) for 92 categories; equipment investment by 55 industries, construction spending for 19 categories; and the components of value-added for 51 industries. In addition, the model provides a full accounting of the macroeconomy. Macroeconomic variables such as the personal savings rate or the 3-month Treasury bill rate are estimated econometrically. Others are determined according to national accounting identities and still others are given to the model exogenously.

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[5]INFORUM stands for Interindustry Forecasting at the University of Maryland. It is a non-profit research center founded by Clopper Almon in 1967 which provides industry-level and macroeconomic forecasting and policy analysis. Douglas Meade has been largely responsible for the development of IDLIFT.

[6]In the context of the capital, the terms *quality* and *embodied technological change* should be thought of as synonymous.

[7]The "ID" in IDLIFT stands for Interdyme, the C++ framework developed at Inforum for building interindustry dynamic macroeconomic models (LIFT was built using Fortran).

The overall structure of the model is based on the national accounting system embodied in the U.S. national income and product accounts (NIPA). There is a real side and a price side. On the real side, each component of final demand (i.e., the usual C+I+G+X-M) is modeled at the various levels of disaggregation mentioned above using structural behavioral equations. The disaggregate, sectoral equations are estimated individually (as is the case with the labor productivity equations) or as a system (such as a demand system for consumption equations) using mainly industry-level time series data. Bridge matrices convert each of these final demand components from their particular level of disaggregation to the 97-sector commodity level. Sectoral (gross) output is then determined according to the fundamental input-output equation:

$$q = Aq + f \quad , \tag{1}$$

where $\boldsymbol{q}$ is a 97×1 vector of output, $\boldsymbol{A}$ is the intermediate coefficient matrix (also called the input-output matrix or the requirements matrix), and $\boldsymbol{f}$ is the vector of final demand:

$$\begin{aligned} f_{97\times1} = H^{c}_{97\times92}c_{92\times1} + H^{eq}_{97\times55}eq_{55\times1} + H^{s}_{97\times19}s_{19\times1} \\ +i_{97\times1} + x_{97\times1} - m_{97\times1} + g_{97\times1}. \end{aligned} \tag{2}$$

The subscripts indicate the dimension of each matrix or vector. Here $\boldsymbol{c}$ denotes the consumption vector, $\boldsymbol{eq}$ denotes equipment investment by purchaser, $\boldsymbol{s}$ structures investment (construction) by type of structure, $\boldsymbol{i}$ inventory change, $\boldsymbol{x}$ exports, $\boldsymbol{m}$ imports, and $\boldsymbol{g}$ government spending.[8] $\boldsymbol{H^j}$ is the bridge matrix for component j. All of the variables in equations (1) and (2) should rightly have time subscripts as well, including the $\boldsymbol{A}$ and $\boldsymbol{H}$ matrices which vary according to trends in the across-the-row totals. A detailed discussion of the equations or systems that forecast the components of the final demand vector is beyond the scope of this paper but is available in Meade (1999).

Given the forecasted vector of output ($\boldsymbol{q}^*$), employment (number of jobs) by sector is computed as:

$$n^* = q^* \left(\frac{1}{(q/\ell)^*} \right) \left(\frac{\ell}{n} \right)^* \tag{3}$$

where ℓ is hours worked. An asterisk indicates that a variable is forecasted by the model. For instance, ℓ is not a variable in the model *per se* (it is determined by identity once $[\boldsymbol{\ell/n}]^*$ and $\boldsymbol{n}^*$ are forecasted), but the average hours per job ($\boldsymbol{\ell/n}$) and labor productivity ($\boldsymbol{q/\ell}$) are. Employment forecasts ($\boldsymbol{n}^*$), together with forecasts of the labor force, determine the unemployment rate, a key variable in the model. Aside from being of obvious interest in its own right, the unemployment rate affects many macroeconomic and industry equations on both the real and the income side of

[8]In the model, government spending is actually decomposed into 5 components such as state and local spending, defense spending, etc. The macro-level of these components are generally exogenous to the model; the exogenous macro values are shared-out to the 97 sector level using the sectors' shares of that component of government spending from the most recent year of available data.

the model. By extension, then, it is evident that labor productivity is a key driver of the model (both through its effect on the model's unemployment rate and through its own direct presence in many model equations).

On the income/price side of the model, prices at the 97-sector level are determined according to equations modeling the markups over unit intermediate and labor costs. Given this forecasted price vector $\boldsymbol{p}$ (91×1), value added by commodity sector (v) is calculated as a residual using the dual of the fundamental input-output equation:

$$p = pA + v \quad . \tag{4}$$

The components of value added (corporate profits, inventory valuation adjustment, capital consumption adjustment, net interest income, rental income, indirect taxes, government subsidies, and the big one: labor compensation) are each modeled separately. The forecasted values of the capital income components (everything except labor compensation) are then scaled to be consistent with equation (4) and the markup forecasts. Hourly labor compensation is modeled as a function of the growth in M2/GNP, the growth in *labor productivity*, and a supply shock (it is then multiplied by the forecast of the labor hours requirement, ℓ, from the real side). So labor productivity has an important influence on the income side of the model as well.

B. The Problem and the Need for Change

With its considerable influence on labor compensation on the income side and employment and the savings rate on the real side, it should be evident by now that labor productivity is one of the most important variables in the IDLIFT model as well as virtually any other large-scale structural macro model. Currently, the IDLIFT model's labor productivity equations are determined essentially by time trends and the difference between industry output and its previous peak, and does not contain any factor inputs as explanatory variables:

$$\ln(q^i / l^i) = \beta_0^i + \beta_1^i t_1 + \beta_2^i t_2 + \beta_3^i qup + \beta_4^i qdown \tag{5}$$

where: t_1 = a linear time trend starting in the first year of data;

t_2 = a second time trend, starting in 1972;

$qup_t = dq_t$, when $dq_t > 0$, 0 otherwise;

$qdown_t = -dq_t$, when $dq_t < 0$, 0 otherwise;

$dq_t = \ln(q_t) - \ln(qpeak_{t-1})$;

$qpeak_t = q_t$, if $q_t > qpeak_{t-1}(1\text{-}spill)$, otherwise $= qpeak_{t-1}(1\text{-}spill)$;

$spill$ = depreciation rate of capacity;

and i indexes the 55 industries.

Inforum has long had difficulty building into its models a sensible relationship between investment and labor productivity. Given that labor productivity is the key driver of the long-run output growth behavior of the model, the lack of an influence from investment or capital stock is lamentable. Virtually any Solow-type growth model attributes a substantial share of output growth to the growth of capital. Its omission from Inforum models, IDLIFT in particular, is due neither to a disbelief in Solowian production theory nor to a lack of effort.

Many valiant attempts have been made over the years to develop and estimate productivity equations based on firm optimization behavior that incorporate the effects of changes in capital stock. These attempts have generally been foiled by one of two problems. First, in industry-level time-series regressions (with which the IDLIFT equations are typically estimated), the capital coefficient is often found to be negative and/or near-zero (particularly in service sectors). Second, because the investment equations in IDLIFT have always been of a *flexible accelerator*-type nature (i.e. driven largely by current and lagged changes in output), the introduction of investment (via capital stock) into the productivity equations provided a seed for the explosion of output in the model's forecast. That is, any exogenous positive shock to the model caused output to grow, which caused investment to grow, which caused labor productivity to grow, which caused output to grow (mainly through productivity's increasing of the wage rate which lowers the savings rate which thus spurs consumption, the largest component of final demand),...*ad infinitum*. The model has lacked a supply constraint (such as a nonconvex adjustment cost in the investment equations) to put the brakes on investment and stabilize output. For these reasons, IDLIFT's labor productivity equations (as well as those of other Inforum-type models) have heretofore remained essentially a series of time trends.

The maintained hypothesis has been that one of the key problems with finding a successful Solow-type equation has been mismeasurement of capital due to unobserved changes in embodied technology. It is well-known that classical measurement error causes an attenuation bias (toward zero) on the coefficient associated with the mismeasured independent variable. In fact, the problem is even worse. The measurement error in equipment capital that is caused by ignoring embodied technological change is not random; it is systematically related to the intertemporal investment distribution. The error will be greater the more an industry's capital is comprised of recent vintages. Recent investment will be positively correlated with other factor inputs such as labor. This will lead to an upward bias in the estimated labor elasticity. Furthermore, if constant returns to scale are imposed, this positive bias in labor elasticity implies a lower capital elasticity (in a value-added production function).

Thus, in order to correct this measurement problem, in the next section I construct quality-adjusted capital stocks using the estimated rates of embodied technological change in Table 1. In Section 4, I estimate various labor productivity equations, some of which attempt to avoid the measurement error either by using the quality-adjusted capital stocks or by including the stock of embodied R&D along with the unadjusted capital stock as an independent variable.

3. Constructing Quality-Adjusted Capital Stocks

As I stated above, quality-adjusted capital stocks are needed in order to properly estimate labor productivity equations that are of a Solow-type specification. In addition, for a structural model that forecasts labor productivity based partially on forecasted capital, a capital stock formula must be built into the model such that capital can be updated in each future period using the model's forecast of investment. The capital stocks I construct in this paper are defined according to the usual perpetual inventory formula that aggregates current and past vintages of investment into a current real capital stock according to some weighting scheme (i.e., a distributed lag):

$$K_t = \sum_{s=1}^{T} I_{t-s} \cdot \underbrace{\left[\left(\frac{1}{p_{t-s}} \right) D_{t,t-s} (1+\gamma)^{t-s-t_0} \right]}_{Weight(t-s)} \qquad (6)$$

where K is the capital stock (either equipment or structures), I is nominal investment, p is the price deflator, γ is the rate of embodied technological change, and $D_{t,t-s}$ captures physical depreciation (i.e., wear and tear). $D_{t,t-s}$ gives the fraction of vintage t-s capital still in production in year t. In most capital stock data constructs, the quality (technology) change component is generally considered to be included in either the measure of depreciation (making it *economic* depreciation rather than *physical* depreciation) or the price deflator.[9] Unfortunately, most data sources of investment price deflators and economic depreciation do not adequately adjust for quality change in equipment. Thus, it is important to decompose the vintage weights in equation (6) into the three separate components of physical depreciation, quality change, and price change.

Using the definition of capital in equation (6), I construct separate industry-level capital stocks for structures and equipment. For structures, I assume that technological change is negligible and thus $\gamma = 0$. For equipment, the rates of embodied technological change come from Wilson (2001) (see Table 1). As for the price deflator, Hornstein and Krusell (1996) show that if embodied technological change is measured independently, one should deflate investment by a consumption deflator. I measure physical depreciation in structures as the inverse of the weighted average of the service lives of the structures assets owned by the industry. The weights are the industry's shares of capital in each asset type constructed from the capital flows tables supplied by the U.S. Bureau of Economic Analysis (BEA).

The Board of Governors of the Federal Reserve (FRB) and the U.S. Bureau of Labor Statistics (BLS) construct capital stocks using a methodology for capturing physical depreciation based on stochastic service lives and a nongeometric, "beta-decay" function. In Sakellaris and Wilson (2000), we back out the implied industry-level physical depreciation patterns, $D_{t,t-s}$, for equipment from the FRB capital stocks. However, the fact that the equipment stocks will need to be forecasted introduces a complication into how they must be constructed. The physical depreciation schedules constructed in Sakellaris and Wilson (2000) are functions of both year and age. In order to "forecast" physical depreciation for future years, one must make some assumption regarding how $D_{t,t-s}$ will vary over t in the future.

What is needed is a time-invariant physical depreciation pattern to apply to the forecasted investment flows. One would also like this pattern to match as closely as possible the FRB physical depreciation schedules since these schedules were used in estimating γ with the plant-level data. Thus, I use the average (over years and industries) age profile from those schedules.

The average profile is shown in Figure 1 by the line labeled "Actual." It has a reverse-S shape. The method by which this pattern of physical depreciation was imposed upon historical

[9]See Gort and Wall (1998) or Hulten and Wykoff (1981) for discussions of the distinction between economic and physical depreciation.

and forecasted values of investment is described in Appendix A.

4. Alternative Labor Productivity Equations

In this section, I perform a general-to-specific modeling exercise involving several alternative specifications for labor productivity equations. I begin with a set of general specifications and evaluate their performance in terms of average fit (over all sectors) and the signs and magnitudes of the coefficient estimates. Based on this evaluation, this set of specifications was pared down to a smaller set of candidate specifications. A series of modifications is applied to each specification which is then reestimated. The modifications are to exclude materials as a factor input, to use an alternative method of adjusting for capacity utilization, and to allow for disembodied technological change. Finally, a single specification is chosen for each industry based on fit and the sensibility of the coefficients obtained from each modification.

This approach of estimating a number of specific equations that are special cases of a more general model and choosing a single equation for forecasting based on economic and statistical criteria, is similar to the general-to-specific modeling approach recommended by Hendry (2000).[10]

A. Equations in Log-Levels and Including Materials

In this subsection, I describe the results of estimating 11 alternative specifications for a labor productivity equation. Each specification is estimating for each of the 55 industries in the IDLIFT investment sectoring scheme.[11] Table 3 shows the functional forms of the 11 specifications and provides a guide to how they differ from one another. With the exception of the current IDLIFT specification, all of the specifications are derived from a standard Cobb-Douglas Solow-type production function:

$$Q_{it} = M_{it}^{\theta} L_{it}^{\beta} J_{it}^{\alpha} S_{it}^{\eta} \tag{7}$$

where Q is real output, M is real materials input, L is labor hours, J is equipment capital services, and S is structures capital services.

The first specification (labeled A) is simply the standard Cobb-Douglas production function in logs, rearranged so as to have (log) output per hour on the left-hand side. The next specification (B) proxies for unobserved variation in capital utilization using the energy-capital ratio following Sakellaris and Wilson (2000). The utilization rate of equipment is assumed to be an increasing function of the energy-equipment ratio (likewise for the utilization rate of

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[10] General-to-specific modeling is also known as the LSE methodology. For references to this literature, see Hendry (1997), Hendry (1995), Hendry and Clements (1996), Hoover and Perez (1999), Ericsson and Marquez (1998), and Cook and Hendry (1993). For a critique of general-to-specific modeling, see Faust and Whiteman (1997).

[11] Actually industries 6 (Construction) and 55 (Scrap and used equipment) are omitted due to lack of data.

structures). It is assumed that in order to increase utilization by 1%, one must increase the energy-equipment ratio by τ%. The special case $\tau = \infty$ means that there is no variation in utilization; $\tau = 1$ means energy use is perfectly proportional to capital services; and $\tau = 0$ means an infinitesimal change in the energy-equipment ratio will fully adjust utilization to the desired level.

The third specification (C) begins with the standard Cobb-Douglas form but imposes constant returns to scale (RTS). Then, I combine constant RTS and the adjustment for capacity utilization for the fourth specification (D). The fifth specification (E) is the current IDLIFT labor productivity equation, equation (5) above. The next four specifications (F, G, H, I) are exactly the same as the first four except the equipment capital stock is not adjusted for embodied technological change (i.e., J is constructed with γ set equal to zero).

Figures 2 through 4 summarize the results of estimating these 11 equations for all of the 55 sectors in IDLIFT (spanning the U.S. private economy). Given that data mismeasurement is generally considered to be more serious in nonmanufacturing industries and that the estimated rates of embodied technological change used for constructing equipment stock in these industries are imputed, I also look separately at the results just for nonmanufacturing sectors.[12] In the following discussion, I will generally focus on the results for all sectors, though I will point out things that are substantially different in the nonmanufacturing subset.

Figure 2 shows the average adjusted-R^2 over all sectors for regressions corresponding to each specification above. Figure 3 gives the average estimated factor elasticities for each specification. The percentage of estimated elasticities that are positive for each specification is shown in Figure 4. Several important findings are apparent from the figures. First, I find that for the most part adjusting equipment capital for quality using the γ's from Table 1 substantially improves the fit and sensibility (in terms of average value and positivity of estimated factor elasticities) of the labor productivity equation in comparison to using an unadjusted equipment capital stock. Second, despite some loss of fit, imposing constant RTS seems to greatly improve the sensibility of the estimates. The beneficial effects that imposing constant RTS has on α and η, in terms of increasing the percentage that are positive and raising their average values closer to *a priori* expectations based on income shares, seem to easily outweigh the cost of a slightly lowered fit. Lastly, controlling for utilization using the energy-capital ratio improves the fit and raises the estimated elasticities of structures, but it reduces the elasticities of equipment.

Based on these findings, it seems reasonable to drop from our consideration all but specifications C and D. That is, we can feel comfortable hereafter imposing constant RTS and adjusting equipment capital by constructing the stock according to the γ's in Table 1. Furthermore, adjusting for utilization seems to provide a slight improvement in fit, so I will retain specification D for now despite its tendency to produce outlying unrealistic capital elasticities.

B. Equations Omitting Materials

I next analyze how the regression results for these two specifications change if we remove materials. It is often the case in production function or productivity regressions that materials

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[12]Figures analogous to Figures 2 through 7 for the nonmanufacturing subset are not reported but are available from the author upon request.

(intermediate inputs) dominate the explanatory power of the independent variables and obscure the effects of the other inputs. This domination by materials appears to be the case in our regressions as well, evidenced by the very high average coefficient estimate and enormous mexval statistics (marginal explanatory power, not shown) for the materials elasticity (Θ). Furthermore, all but specification E (the current IDLIFT equation, which does not include materials) have very high adjusted-R^2's.

Another problem with including materials in aggregate or industry-level production regressions is that data on materials is often inadequately measured. The measures on real materials used in the above regressions are constructed by taking the column sum of a constant dollar input-output flow matrix. That is, real materials for industry j is $m_{jt} = \sum_i a_{ijt} q_{jt}$ where a_{ijt} is element (i,j) in the intermediate coefficient matrix (*A* in equation (1)). The problem here is that we do not observe the true input-output coefficients, a_{ijt}, in most years (at least in the U.S. data). Specifically, we only have data on a_{ijt} on a quinquennial basis as the BEA only constructs an input-output table every 5 years. Coefficients for years in between are simply interpolated between benchmark-year coefficients and are therefore essentially determined by q_t. Thus, shocks in q_t, which affect the dependent variable in a productivity regression and are part of the regression disturbance term, are transmitted to the regressor $(m_t-\ell_t)$ causing an upward bias in the estimator of its coefficient.[13]

Therefore, I re-ran the regressions corresponding to C and D omitting the $\theta(m-\ell)$ term. These new *sans*-materials specifications will hereafter be referred to as C' and D'). The omission of materials can be justified theoretically by assuming the Leontief conditions for separability of materials and value added as is frequently done in the literature (e.g., Basu (1996) and Wilson (2000)). That is, $Y = min[M, F(J,S,L)]$. Assuming firms are optimizing, this implies $dlog(Y) = dlog(F(J,S,L))$. The $F(\,)$ function can be any of equations (1)-(11) after omitting the term $\theta(m-\ell)$.

Figures 5 through 7 summarize the results of these regressions (ignore for now the specification labeled D", this will be explained below). As with the previous regressions, I repeat the regressions for a nonmanufacturing subset to check for robustness. Except in the cases mentioned below, the nonmanufacturing subset yielded similar results to those of the full sample.

As expected, the adjusted R^2's fall, though not by much, when materials are left out (see Figure 5). Again the fits are higher when capital utilization is adjusted for (compare D' to C'). But the specification which does not adjust for utilization (C') yields a much higher percentage of positive α's, though it results in a somewhat lower percentage of η's that are positive. This result does not appear to be the case in nonmanufacturing though, where specification D' yields more positive coefficients for both elasticities.

As for the average coefficient estimates, the average estimated elasticities for specification C' for the full sample (though not in the nonmanufacturing subset) are almost exactly as theory would predict. The generally accepted expectations for the elasticities of output with respect to labor and capital are 2/3 and 1/3, respectively, when output is value added and 1/3 and 1/6 when output is gross output (with materials responsible for the other ½). These numbers are based on

[13] In fact, exactly the same problem is true for our measures of real energy expenditures which are also constructed via slow-moving input-output coefficients multiplied by industry output.

the shares of national income going to labor and capital. The capital share is further broken down, generally, to be 2/3's equipment (which includes embodied R&D) and 1/3 structures. Thus, one would expect our estimates of the output elasticities with respect to each input to be somewhat close to these values. This means that when materials are included, we would expect $\alpha \approx (1/6)*(2/3)=2/18 = 0.111$, $\eta \approx (1/6)*(1/3)=1/18 = 0.056$, $\beta = 0.33$ and $\Theta \approx 0.5$. When materials are excluded, we expect $\alpha \approx 2/9 = 0.222$, $\eta \approx 1/9 = 0.111$, and $\beta = 0.66$. According to the average estimates obtained thus far, these *a priori* expectations are met more closely by the regressions which do <u>not</u> include materials and which do not adjust for utilization using the energy-capital ratio.

However, before abandoning the theoretically attractive notion of accounting for variations in capacity utilization, there is one other approach we can try for measuring utilization.

C. *Alternative Adjustment for Unobserved Variation in Capacity Utilization*

Besides using the energy-capital ratio, another method that has been suggested to control for unadjusted variation in factor utilization is what is actually used in the current IDLIFT equation. Industry-level variation in utilization is captured by including the terms *qup* and *qdown* as defined in equation (5). The method first measures capacity with the previous peak level of industry output less some "depreciation." The absolute value of the percentage difference between current output and capacity is then included as a regressor, with positive and negative differences treated asymmetrically. The rationale behind this method is that when current output is being stretched beyond the previous peak level, the economy will be pushing up against capacity constraints, and when output is much below the previous peak, there is excess capacity not being utilized.

To evaluate the efficacy of this method, I estimate an equation identical to C' but with *qup* and *qdown* as additional independent variables. Call this specification D''. The results of estimating this specification for each industry are shown in Figures 5 through 7. Compared to using the energy-capital ratio as a proxy for utilization, specification D'' yields slightly lower fits but far more reasonable capital elasticities. Compared to results for specification C', the specification that does not account for variation in utilization, those for specification D'' are quite similar in both fit and capital elasticities. This makes specification D'' a rather attractive option relative to C' given the theoretical appeal of accounting for variation in capital utilization.

One concern, however, with specification D'' is the possibility of reverse causation (i.e. simultaneity, or what Almon (1998) refers to as the "umbrella effect"[14]) since industry-level output is part of both the dependent variable and the regressors *qup* and *qdown*. If there is any measurement error in output, this may bias the coefficients on *qup* and *qdown* as well as artificially inflate the R^2's. The bias can be seen formally by assuming that there is an i.i.d. measurement error in q: $q^{measured} = q^{true} + \nu$, where $\nu \sim N(0, \sigma^2)$. So our regression equation (D'') becomes:

[14]Almon (1998) cautions against the use of "umbrella" variables, which in econometric parlance are simply endogenous variables, as explanatory variables. The name comes from the analogy to using "the number of people carrying umbrellas to explain rainfall." (p. 97).

$$(q^{measured} - \ell)_t = c^0 + c^1 t + \alpha(j-\ell)_t + \eta(s-\ell)_t$$
$$+b^0 qup_t^{measured} + b^1 qdown_t^{measured} + u_t \tag{8}$$

Notice that ν will be contained in the dependent variable as well as *qup* and *qdown* resulting in spurious correlation between these two regressors and the dependent variable. The bias on the estimator of b^0 will be positive and that of b^1 will be negative.

To evaluate the seriousness of this problem, I perform a mixed empirical-Monte Carlo estimation procedure. In this procedure, I specify the data generating process (DGP) for the true dependent variable as:

$$(q-\ell)_t^{true} = 2 + 0.01*t + 0.17*(j-\ell)_t + 0.16*(s-\ell)_t$$
$$+0.1*qup_t^{true} - 0.1*qdown_t^{true} + \varepsilon_t \tag{9}$$

where $\epsilon_t \sim N(0, 4\times10^{-06})$; the variance of 4×10^{-06} implies the standard deviation of the i.i.d. shock to true productivity is 0.002. The 0.01 and -0.01 assumed coefficients represent the true relationship between *qup* and *qdown* and labor productivity, i.e. absent any spurious correlation due to the presence of measurement error in q. I further assume that σ, the standard deviation of the measurement error in log output is 0.05, or one-half of one percentage point. Using the DGP specified in equation (9), I construct the "true" dependent variable, then regress it on *t*, *(j-ℓ)*, *(s-ℓ)*, $qup^{measured}$, and $qdown^{measured}$ each measured with actual historical time series. I repeat this procedure 2000 times and calculate the mean and standard deviation for each coefficient.[15] This says that the standard deviation in the measurement error of log output is assumed to be one half of one percent, which should be as large as is realistically possible.

The coefficient means and standard deviations are shown in Table 4. The estimated biases are all extremely close to zero. Thus, even assuming a very large variance for the measurement error in q, coefficient bias due to the presence of *qup* and *qdown* does not appear to be a problem.

Therefore, it seems reasonable to drop from our consideration the specification which attempts to adjust for unobserved variations in capital utilization using the energy-capital ratios (specifications D') due to its propensity to yield nonsensical capital elasticities and to the fact that including *qup* and *qdown* as explanatory variables seems to be a powerful alternative way of adjusting for utilization. This leaves specifications C' and D'' as promising alternative options to the current IDLIFT equation (specification E). However, there is still one other modification we must do to these specifications before deeming them superior to the current equation.

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[15] I arbitrarily choose the "Printing and Publishing" industry for the historical data. The choice of industry should not affect the coefficient means (and therefore their biases) but may affect the standard deviations since the sample variance of a variable helps determine the variance of its coefficient (and, of course, the sample variance of a variable will be different across industries). To be sure, I repeated the procedure with a 2nd industry and obtained similar estimated biases.

Also, I include a time trend in this regression because the specifications I settle on in the end include a time trend.

D. Allowing for Disembodied Technological Change

It is possible that there is some spurious positive correlations between labor productivity and the factor inputs due to the fact that these variables are all trended upward. In other words, the above equations should probably also contain a Hicks-neutral productivity (or disembodied technology) term that is sure to be highly trended.

The results of estimating equations C' and D'' with a single linear time trend added are as follows. The adjusted R^2's for both of these specifications are now slightly better than that of the current IDLIFT equation (specification E) at 0.866, 0.867 and 0.853 for specifications C', D'', and E, respectively. The average estimated capital elasticities decrease somewhat due to the introduction of the time trend though they are still reasonable. For specification C', the average α falls from 0.22 absent the time trend to 0.01 with it, while the average η rises from 0.15 to 0.17. Similarly, the percentage of α's that are positive falls from 80% to 52% and the percentage of η's that are positive rises from 52% to 63%. For specification D'', α falls from 0.22 to 0.08 on average with the inclusion of the time trend and the average η remains at 0.18. The positivity of α falls from 80% to 59% and that of η drops from 61% to 57%. The results are quite similar in the nonmanufacturing subset.

From the results of this round of regressions, the most promising specification appears to be D'' with a time trend. C' with a time trend also seems to be reasonable, though the average equipment elasticity is probably too low and the equipment elasticity is somewhat less likely to be positive under C' relative to D''. Compared to the former IDLIFT equation, these specifications have as good a fit and obviously have far more economic appeal. Most importantly, they capture the productivity gains due to capital deepening (which, given how capital was constructed here, includes embodied technological change). Therefore, one of these two specifications, along with the coefficients found from estimating them, are used for each of the 55 sectors and can now be incorporated into the IDLIFT model. For a particular industry, which specification is used is chosen on a case-by-case basis, as described below, based on the criterion of best fit and most realistic coefficients. For the sake of clarity, let us explicitly write out the final specifications:

Specification C' $$q - \ell = c^0 + c^1 t + \alpha(j - \ell) + \eta(s - \ell) \tag{10}$$

Specification D'' $$q - \ell = c^0 + c^1 t + \alpha(j - \ell) + \eta(s - \ell) + b^0 qup + b^1 qdown \tag{11}$$

E. Determining Industry-Specific Labor Productivity Equations

In the previous subsections, I evaluated many possible specifications for a general empirical model of labor productivity based on the criteria of average fit and the economic realism of the coefficients. The results of that evaluation have enabled us to now focus our attention on a small number of specifications in determining the "best" one for each particular industry (rather than simply the best on average). Obviously, the specification that yields the best results on average may not necessarily yield the best results for a particular industry. The choice of specification must be made on an industry-by-industry basis.

For each industry, I compare the results of estimating specifications C', D'', and E. For a small number of industries, it was clear that the lagged values of the equipment and structures stocks had more explanatory power (with reasonable coefficients) than the current values and, thus, the lagged stocks were used instead. The improved explanatory power afforded by using lagged stocks can be explained by the industry having a time-to-build requirement greater than one year and/or by the presence of substantial learning-by-doing effects. For most industries, even the best specification yielded one or more unrealistic coefficients. For these industries it was necessary to "softly constrain" the coefficient estimates to lie inside a realistic range. "Soft constraining," also known as "Theil's mixed estimation" or "stochastic constraints," is a Bayesian regression technique that allows one to combine *a priori* theoretical beliefs on parameter values with the values estimated using the data. A soft constraint essentially adds artificial observations (or a fraction of an observation) in which the constraint holds with certainty. The *a priori* expectation for parameter values and the number of artificial observation to add are chosen by the econometrician. I only imposed soft constraints if the unconstrained estimated coefficient was outside the range of [0,0.4] for either capital elasticity (α and η), [0,1] for the coefficient on *qup,* and [0,-1] for the coefficient on *qdown*. The theoretically-based, *a priori* expected parameter values that I used as soft constraints were 0.18 for the elasticity of output with respect to the equipment stock, 0.17 for the structures elasticity, 0.5 for the coefficient on *qup*, and -0.5 for the coefficient on *qdown*.[16]

Table 5 shows the number of industries for which each of the four specifications was chosen (second column) as well as the number, within each specification, that required soft constraining (third column). Recall that the regressors in specification C' are a constant, time trend, log of the equipment-labor ratio, and the log of the structures-labor ratio. Specification D" includes these same regressors in addition to *qup* and *qdown*. Specification E is the traditional (current) IDLIFT labor productivity equation. Let the specification which is equivalent to specification C' but with lagged capital stocks be denoted specification X.

Specification D" was chosen in exactly one half of the industries. Overall, all but five industries required some type of soft constraint(s). In nearly all cases, the soft constraints were quite weak, amounting to only a fraction of an artificial observation. Thus, the equation fits suffered very little due to the use of soft constraints.

5. Simulation Results

The equations summarized in Table 3 are incorporated into IDLIFT through a series of new computer routines which take forecasted values of equipment investment, structures investment, and output and generate values for productivity, hours, and employment, which then get fed back into the model. The structure of the IDLIFT model had to be modified considerably in order to accomplish this task. The modifications are described in Appendix C.

With these new, alternative routines incorporated into the model (along with the estimates for the productivity equations), one can produce a base forecast that is stable, i.e. a forecast that does not cause any variable to spiral out of control. In addition, these new routines were programmed into the model in such a way as to allow the model to calculate productivity, hours, and employment using both the new set of equations and the old set of equations. The model

[16] The rationale behind these *a priori* values for capital elasticities is explained in subsection B above. The *a priori* values for the coefficients on *qup* and *qdown* were chosen simply to be at the halfway point of their respective plausible ranges.

user can specify which set of equations he or she would like to feed back into the model. That is, the user can have the model calculate productivity and hours using the new equations but have those calculated values in no way affect the rest of the model, and the same for the old equations. This allows one to generate a base forecast for both the old model (i.e. the model set to have the current IDLIFT equations' forecasts feed back into the model) and the new model (having the new equations feed back into the model).[17]

Since what we are interested in is how the behavior of the two models differs in response to changes in economic activity, such as variations in equipment investment, comparing the two base forecasts to one another is of little interest. What will be of interest to us in this section is comparing and contrasting the responses of each model to some exogenous shock to the system. The behavior of each model in response to such an experiment is the only way to illuminate the effect of changing the IDLIFT's productivity equations. Since the key difference between the two models is the presence of a direct influence of capital stock on productivity in the new model, the interesting shocks to investigate will naturally involve investment.

Moreover, given IDLIFT's dependence on many exogenous, user-supplied assumptions ("fixes"), one cannot fairly compare a forecast from the old model with one from the new model. The existing fixes, which either override or modify the endogenous forecasts produced by the model's equations, were specified in such a way as to produce the most sensible forecast using the *current* model. Alternatively, these fixes could be specified so as to optimize the sensibility of the new model. However, having each model have its own optimal fixes would confuse the differences in the models' results due to different productivity equations with those due to different sets of fixes. Yet, many of these fixes must be given values for the model to run at all, therefore turning off all fixes is not an option either. Thus, I run both models using the fixes in place for the most recent semi-annual Inforum forecast using IDLIFT (see Inforum (2001)). One important exception is the exclusion of all fixes on industry-level productivity, industry-level employment, and the aggregate unemployment rate. Thus, again, comparison of the two models must be between the models' *differences* from their own base forecast to a simulation forecast in which a shock was imposed, and not between the models' base forecasts.

To produce base forecasts, I ran each model out to 2015. 1997 was the last year of historical data for most of the industry-level variables in the model, yet much of the aggregate data is available through 2000 (or at least through 1998 or 1999) and this data is imposed on the model through fixes (with the exception of the unemployment rate as mentioned above).[18] The new functions generally result in lower labor productivity and thus higher hours and employment in the base forecast. This result is true even if the output of these functions is not fed back into the model, but it is stronger when feedback does occur. However, this difference in productivity between the base forecasts is largely due to fixes that act to boost productivity in the current model and thus is not very revealing.

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[17]The model using the new, alternative productivity equations will be referred to as the "new" model in this section while the old/current/pre-existing IDLIFT model will be referred to as the "old" model.

[18]For instance, NIPA data is available on aggregate equipment investment and residential and nonresidential structures through 2000.

A. Response to a Permanent Investment Shock

For each model, I evaluate the response of the model to a shock in equipment investment. Specifically, with a set of fixes on equipment investment, I override the models' forecasted vectors of equipment investment with the investment vector from the base forecast plus 2%. That is, for each industry I multiply the equipment investment values from the base forecast by 1.02 and force the model to use these new values in all of the functions that make use of equipment investment. Because aggregate equipment investment is known (from NIPA data) through 2000, I impose this fix just for the years beyond 2000.

Figures 8 through 17 graph, for each model, the deviations over the forecast period of key macro variables relative to each model's base forecast. In both models, real GDP rises by about a quarter of a percent relative to the base in the first year in which the 2% higher equipment investment is imposed. From then on the models diverge substantially. The old model falls to near the base level in the second year, oscillates between 0.05% and 0.2% over base through 2008, then seems to settle at about 0.08% over base. The new model also comes back down closer to base in 2002 but then rises relative to base almost monotonically until the end of the forecast where it stands at 0.31% over base. This Solowian response of real GDP, i.e. higher and less variable, to permanently higher equipment investment is what one would have expected and hoped for from the new model.

The increase in labor productivity induced by higher investment also reduces unit labor costs and this reduction lowers the GDP deflator. The GDP deflator rises in the old model in response to the demand stimulus of higher investment. Because of this, the deviation from base in *nominal* GDP is actually higher in the old model. The different responses of the price level also has an effect on the Treasury bill rate: the deviation from base is generally lower and less volatile in the new model. The lower interest rates in the new model cause, in part, a smaller deviation in the savings rate.

In both models, the unemployment rate goes down relative to base due to the substantial demand stimulus caused by the increase in investment. However, the deviation is smaller on average in the new model because its increase in labor productivity has an immediate negative effect on employment. This Ricardian (or Luddite) effect of labor productivity having a short-run negative relationship with employment would have occurred in the old model as well had labor productivity increased substantially, which it did not.[19] This difference in labor productivity deviations can be seen in Figure 17. Labor productivity in the new model grows steadily to almost 0.4% above its base level by the end of the forecast. This is compared to the old model in which productivity oscillates until it converges to about 0.04% over base. In short, in the new model, the long-run effect of investment on productivity is ten times what it was in the old model.

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[19]In Ricardo's later works, he developed the notion that the introduction of machinery can, under certain circumstances such as the sudden introduction of a new type of machinery, have an adverse effect on employment. In his *Notes* to Malthus's *Principles,* he states:

> *It might be possible to do almost all the work performed by men with horses, would the substitution of horses in such case, even if attended with a greater produce, be advantageous to the working classes, would it not on the contrary very materially diminish the demand for labor?*

The deviations in labor productivity by industry for the new model are shown in Table 6 below, along with each industry's estimated elasticity of output with respect to equipment capital stock. Listed in parentheses after each industry name is the label identifying the equation type used for that industry. As one would expect, the largest deviations can be found in industries which have the largest elasticities of equipment stock. The correlation between this elasticity and the deviation in labor productivity is approximately zero in 2001 but rises to 0.96 by 2015.

B. Response to a Temporary Investment Shock

Next I impose a one-time shock on each model of 10% higher equipment investment (relative to that which is forecast by the model's equations) in 2001. Determination of equipment investment returns to each model's investment equations from 2002 on. The shock is assumed to take place in every industry. Figures 18 through 29 show the deviations relative to the base for the same macro variables as in the earlier figures as well as equipment investment (Figure 28) and quality-adjusted equipment stock (Figure 29).[20] Both models have an initial response of between 1.2 and 1.3 percent in real GDP. After oscillating for several years, the old model returns nearly to its base level. The new model, however, quickly reaches a steady state at approximately three-tenths of a percent above its base. As with the previous experiment, the GDP deflator's deviation is lower in the new model than in the old model as one would hope for. The GDP deflator converges to the base level over time in the old model whereas it falls steadily relative to the base in the new model. Interest rates deviations move similarly in the two models though they are somewhat less volatile in the new model. The same is true for their savings and unemployment rates. In both models, unemployment initially drops dramatically in response to the shock, then jumps dramatically, and finally begins to converge to its base level around 2005. The new model has less of a drop and subsequent jump because the positive demand stimulus of raising investment is partially offset by the negative effect on employment that the increase in productivity has in the short-run (the Ricardian effect), though this is dominated by the stimulus as can be seen in Figure 24.

As expected, labor productivity in the old model, after oscillating for several periods, returns to its base level by 2010 and stays there whereas productivity in the new model, after also oscillating for a few years, is permanently above its base levels. This permanent increase in productivity in response to a temporary increase in investment is the key difference in the behavior of the two models. In the old model, a one-time jump in aggregate investment only affects labor productivity by directly increasing every industry's final demand, which directly increases their output, which increases their *qup* which increases their labor productivity. The next year, when equipment investment comes back down, output will likely be below its previous peak making *qdown* go up which will lower labor productivity. This cycle will fade away over time returning labor productivity to its base level.

In the new model, on the other hand, labor productivity in every industry jumps initially because of both the jump in *qup* <u>and</u> the jump in the equipment stock. In the following year, productivity comes back down due to the jump in *qdown* in the following period but this decline

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[20]Equipment investment here is not adjusted for embodied technological change. Also, note that though quality-adjusted equipment capital is shown for both models in Figure 29, it only has an effect on the other variables (as well as its own future values through the investment equations) in the new model.

is offset somewhat by the still-present higher level of equipment stock.

There is also a strong and long-lasting positive effect on equipment investment itself from the initial shock. This effect has two causes. First, the 2001 jump in investment causes the following year's desired capital stock (constructed and used in the model's investment equations) to rise which increases the forecast of investment for that year which then increases desired capital and investment for the next year, and so on. Second, the increase in final demand in 2001 raises the 2000-2001 change in output. Distributed lags in the change in output are part of the model's investment equations. Thus, the increased change in output in 2001 directly increases investment for the following four years (there are four lags of output change in the investment equations).

The continuing though depreciating presence of that extra 10% of equipment purchased in 2001, combined with the long-lasting increase in equipment investment due to the positive feedback from the initial demand stimulus, keeps the quality-adjusted equipment stock about 2% above its base level from 2005 through the end of the forecast (see Figure 29). The physical depreciation and obsolescence of the extra 10% of vintage-2001 equipment begins to dominate any positive feedback remaining from the initial stimulus by 2009 and a very slow decline in the equipment stock begins. Shortly thereafter, labor productivity thus begins to decline very slowly.

The labor productivity deviations from the base forecast of the new model are shown for each industry in Table 7 below, along with each industry's estimated elasticity of output with respect to equipment capital. As was the case with the permanent shock, the largest deviations are in industries with large elasticities of equipment stock. The correlation between the estimated elasticity and the deviation in productivity is -0.07 in 2001 but rises to 0.82 by 2015.

6. Conclusion

The preceding experiments show that the introduction of the new labor productivity equations into IDLIFT do have substantial effects on the general equilibrium behavior of the model. With the new equations operating, the macroeconomic variables of the model exhibit behavior in response to changes in investment that is more in line with that predicted by the Solow growth model. Importantly, we do not see the model spiral out of control in terms of output or prices when the new equations are introduced as was feared due to the lack of a supply constraint in the investment equations. In general, the macroeconomic situation of the economy is *permanently* and substantially improved by an increase in equipment investment, even if it is only a one-time shock, according to the new model. In contrast, the macroeconomy of the IDLIFT model without the new equations exhibits a smaller long-run benefit due to a permanent investment increase and little or no long-run benefit from a temporary increase. The permanent and reasonable response of the new model to increases in investment is one of the main contributions of this paper.

This Solowian response in the new model was accomplished through the use of new labor productivity equations that account for both traditional capital deepening (more units of capital per worker) and embodied technological change (higher quality units per worker). It is important to note, however, that these new equations were not simply chosen *ad hoc* and forced into the model but rather were shown to fit the historical data more closely that the preexisting IDLIFT equations.

References

Almon, Clopper. *The Craft of Economic Modeling*. Fourth Edition ed. College Park, MD: Interindustry Economic Research Fund, Inc., 1998.

Almon, Clopper. "New Developments in Input-Output." Paper presented at the A.S.A. Meeting, New York, 1969.

Basu, Susanto. "Procyclical Productivity: Increasing Returns or Cyclical Utilization." *Quarterly Journal of Economics* 111 (1996): 719-51.

Cook, S., and David F. Hendry. "The Theory of Reduction in Econometrics." *Poznan Studies in the Philosophy of the Sciences and the Humanities* 38 (1993): 71-100.

Ericsson, Neil R., and Jaime Marquez. "A Framework for Economic Forecasting." *Board of Governors of the Federal Reserve System, International Finance Discussion Paper* 626 (1998).

Faust, Jon, and Charles H. Whiteman. "General-to-Specific Procedures for Fitting a Data-Admissible, Theory-Inspired, Congruent, Parsimonious, Encompassing, Weakly-Exogenous, Identified, Structural Model to the DGP: A Translation and Critique." *Carnegie-Rochester Conference Series on Public Policy* 47, no. 0 (1997): 121-61.

Gort, Michael, and Richard Wall. "Obsolescence, Input Augmentation, and Growth Accounting." *European Economic Review* 42 (1998): 1653-65.

Hendry, David F. *Dynamic Econometrics*. Oxford: Oxford University Press, 1995.

Hendry, David F. "The Econometrics of Macro-Economic Forecasting." *Economic Journal* 107 (1997): 1330-1357.

Hendry, David F. *Econometrics: Alchemy or Science?* Oxford: Oxford University Press, 2000.

Hendry, David F., and Michael P. Clements. “Multi-Step Estimation for Forecasting.” *Oxford Bulletin of Economics and Statistics* 58 (1996): 657-84.

Hoover, Kevin D., and Stephen J. Perez. "Data Mining Reconsidered: Encompassing and the General-to-Specific Approach to Specification Search." *Econometrics Journal* 2, no. 2 (1999): 167-91.

Hornstein, Andreas, and Per Krusell. "Can Technology Improvements Cause Productivity Slowdowns?" *NBER Macroeconomics Annual 1996.* Editors Julio J. Rotemberg and Ben S. Bernanke. Cambridge, MA: MIT Press, 1996.

Hulten, Charles R., and Frank C. Wykoff. "The Measurement of Economic Depreciation." *Depreciation, Inflation, and the Taxation of Income From Capital.* Editor Charles Hulten.

Washington, D.C.: Urban Institute, 1981,81-125.

INFORUM. *Inforum Forecast Spring 2001*. College Park, MD: IERF, Inc., 2001.

Marx, Karl, and Friedrich Engels. *The Communist Manifesto*. Editor Samuel H. Beer. New York: Appleton-Century-Crofts, 1955.

Meade, Douglas S. "IDLIFT -- Recasting the Inforum Model of the U.S. Economy." Paper presented at the INFORUM World Conference, 1999.

Sakellaris, Plutarchos, and Wilson Daniel J. "The Production-Side Approach to Estimating Embodied Technological Change." University of Maryland Working Paper, no.00-05, 2000.

Scherer, F.M. *New Perspectives on Economic Growth and Technological Innovation*. Washington, DC: Brookings Institution Press, 1999.

Smith, Adam. *An Inquiry into the Nature and Causes of the Wealth of Nations*. Editor Edwin Cannan. New York: Modern Library, 1937.

Wilson, Daniel J. "Estimating Returns to Scale: Lo, Still No Balance" *Journal of Macroeconomics* 22, no. 2 (2000): 285-314.

Wilson, Daniel J. *Capital-Embodied Technological Change: Measurement and Productivity Effects*, Ph.D. Dissertation, University of Maryland, 2001.

Wilson, Daniel J. "Is Embodied Technology the Result of Upstream R&D? Industry-Level Evidence" *Review of Economic Dynamics* 5, no. 2 (2002): 342-362.

Appendix A – Method of Imposing Reverse-S-Shaped Depreciation Pattern

To impose the reverse-S-shaped pattern of physical depreciation on historical and forecasted values of investment, what is needed is a function with a minimal number of parameters that can mimic this reverse-S shape. I found such a function in the "cascading buckets" concept which is frequently utilized by users of the G regression software package (the package used to estimate the time-series labor productivity equations in Section 4). A cascading buckets system is a combination of several "bucket" functions. A single bucket is created by the use of the *@cum* function in G. The statement, $k_t = @cum(k_t, i_t, z)$, defines the variable kt by the following equations:

$$k_0 = 0 ; \quad k_t = (1 - z) \cdot k_{t-1} + i_t \quad \forall t > 0$$

The reverse-S shape can be obtained by a "cascading" of two or buckets, i.e. by having the outflow of the first bucket (here, $z \cdot k_{t-1}$) be the inflow (here, i_t) into the next bucket, then the outflow of the second bucket be the inflow into a third bucket, and so on.... The final function is the sum of these buckets.

In fact, even more variety of shape can be obtained by letting the inflow into the lower (i.e. second, third, ...) bucket "splatter out" or "miss" some of the lower bucket so that only $(z-\epsilon) \cdot k_{t-1}$ actually flows into it (and $\epsilon \cdot k_{t-1}$ is lost). Allowing some "splatter" turns out to be quite necessary for fitting the average physical depreciation schedule because without the splatter there would be no decrease in efficiency over the first N-1 years, where N is the number of buckets (i.e. without splatter, nothings falls out of the bucket system until there is no longer a lower bucket to catch the last bucket's outflow). A decrease in efficiency beginning in the first year is a property of the age-efficiency schedule I am trying to fit.

Using the following three-bucket system, I was able to very closely replicate the age profile implied by the average physical depreciation schedule shown in Figure 2-2:

b1 = *@cum*(b1, *drop*, *A*)
b2 = *@cum*(b2, b1[1]*B, *C*)
b3 = *@cum*(b3, b2[1]*A, *C*)

where *drop* is a variable that is one at age 0 and zero thereafter and the notation [1] indicates a lag of 1 period. Allowing $B < A$ results in some of the outflow from b1 to splatter out or miss b2 allowing for efficiency loss immediately after the first year. I performed a grid search to find the parameters *A*,*B*, and *C* which resulted in the lowest sum of squared errors (SSE). The values *A*=.14, *B*=.129, and *C*=.3 led to a SSE < 0.001. Figure 1 shows the fitted values from this cascading bucket versus the actual depreciation schedule. Clearly, the fit is extremely close. This three-bucket system with the above parameter values became the $D_{t,t-s}$ used in the definition of the equipment capital stock given in equation (6). Now, rather than *drop* going into the first bucket, the actual equipment investment (adjusted for embodied technological change) flows in:

vi = (*eqicu/pced*)*$(1 + \gamma)^{t-t(0)}$
b1 = *@cum*(b1, vi, 0.14)
b2 = *@cum*(b2, b1[1]*0.129, 0.3)
b3 = *@cum*(b3, b2[1]*0.14, 0.3)
J = b1 + b2 + b3

where *eqicu* is equipment investment in current dollars, *pced* is the PCE deflator, *vi* is vintage equipment investment adjusted for embodied technological change assumed to take place at the rate γ, and *J* is the resulting quality-adjusted equipment capital stock.

Appendix B – Construction Categories

1. 1 unit residential structures
2. 2 or more unit structures
3. Mobile homes
4. Additions & alterations
5. Hotels, motels, dormitories
6. Industrial
7. Offices
8. Stores, restaurants, garages
9. Religious
10. Educational
11. Hospital & institutional
12. Miscellaneous NR bldg.
13. Farm buildings
14. Mining exploration shafts & wells
15. Railroads
16. Telephone & telegraph
17. Electric light & power
18. Gas & petroleum pipes
19. Other structures
20. Highways & streets
21. Military facilities
22. Conservation
23. Sewer systems
24. Water supply facilities
25. Brokers' commission

Appendix C

Incorporating the Alternative Estimated Equations into IDLIFT

Incorporating the new labor productivity equations into the IDLIFT model turned out to far more complicated than it would seem at first. The task at hand was to use the new labor productivity equations to determine productivity and employment, at the 97-sector level of aggregation, which can feed back into the model. The model can then use the productivity and employment forecasts to help calculate various other components of the model such as the unemployment rate and hourly labor compensation.

The first complication was how to deal with having labor hours, which are calculated *using* the productivity equations, on the right-hand side of the productivity equations. There are at least three options for handling this problem. The first is to algebraically rearrange each of the specifications containing hours on the right-hand side so that output is on the right-hand side instead of labor and then estimate the equations in this form. For example, specification C' can be rearranged from:

$$q - \ell = c^0 + c^1 t + \alpha(j - \ell) + \eta(s - \ell) \tag{A1}$$

to:

$$q - \ell = \left(\frac{1}{1-\alpha-\eta}\right)\left\{c_0 + c_1 t + \alpha j + \eta s - (\alpha+\eta) q\right\} \tag{A2}$$

I tried this approach and found that the capital elasticities implied by the estimated coefficients were far less sensible than those estimated directly. As in Section 2 above, one could impose soft constraints to force the coefficients into a range that would imply reasonable capital elasticities. However, the constraints would have to be much stronger, i.e., the trade-off between *a priori* expectations of parameter values and those estimated by the data would have to lean far more towards the former. Another option would be to program the equations into the model with hours on the right-hand side, supply the model with starting values (a guess) for hours, let the productivity equations calculate new values for hours, and then let the model iterate until it converges. The third option is to use the estimated equation coefficients found in Section 2 above, but use them in the algebraically rearranged forms of the specifications (such as equation (A2) above) which have output on the right-hand side. This option requires no iterative procedure since output has already been calculated earlier in the model and thus this is the option I used.

The next issue that needed to be dealt with was how to get forecasted values of structures investment at the 55-industry level, the level of disaggregation at which the productivity equations were estimated. Previously, the IDLIFT model generated only *equipment* investment by 55 industry and structures investment by *type*. The 25 types/categories of construction are listed in Appendix B. Rather than developing new structures investment equations by industry in IDLIFT, similar to the existing equipment investment equations, I instead exploited the fact that there is (approximately) a clear one-to-many mapping from some construction types to the industries that purchase those types. For instance, construction of "Farm buildings" (construction type 13) can be clearly attributed to the "Agriculture, forestry, and fisheries" investment industry (industry 1). This assumption can be supplied exogenously to the model through what is known as a "fix." Fixes are supplied by the model user and override or modify the equation results of endogenous variables. Thus, I fix structures investment in industry 1 to "follow" construction of farm buildings, starting from the last year of historical data for structures investment by industry (1997). That is, structures investment in year t, S_t, is determined by equalizing S_t/S_{1997} to C_t/C_{1997} for all t>1997, where C is construction in the corresponding type. Similarly, for cases where one type is associated with many industries, such as "Industrial" construction which is attributable to all of the manufacturing industries, I fix structures investment in each industry to follow the model's forecast for construction in that type. Again, industry structures investment does not *equal* the value of construction in that type; rather, it starts with the last historical data point and then moves forward at the same ratio of forecast year value to last data value that is the case in the forecasts of construction by type. For two industries (which each have very little investment in structures anyway), no clear match could be made to a construction type and so structures investment in those industries was assumed to simply follow aggregate nonresidential construction from their last data point on.[21]

Now, with forecast values for structures and equipment investment by 55 industry, one can calculate structures and quality-adjusted equipment capital stocks to be used in the productivity equations. This is done in the C++ routine, DANBKT.CPP, which is shown below, along with the other new routines. The routine takes in forecasted values of structures and equipment investment along with the exogenously supplied rates of embodied technological change and produces stocks. The stocks of structures are calculated using the traditional perpetual inventory method with depreciation rates computed as the reciprocal of the mean service life of structures in that industry (provided by the BEA). The quality-adjusted equipment capital stocks are calculated using the estimated rates of embodied technological change and the cascading bucket system, as described in Section 3.

The routine DANPROD.CPP then takes in these stocks along with the model's forecasted values of output by 55 industries (which are aggregated from the 97-sector level) and the coefficient estimates for the productivity equations (including the estimate of ρ, the autocorrelation coefficient) and calculates both productivity and hours for each industry. Since other stages of the model require productivity and hours at the 97-sector level, these had to be disaggregated to that level. To split 55-industry hours to the 97-sector level, I used a one-to many mapping key. The shares used to split one industry to many sectors were taken from the 97-by-1 hours vector forecasted by the old IDLIFT productivity equations. Thus, the old productivity equations were left operational in the model solely for the purpose of providing time-varying shares for this disaggregation. Productivity at the 97-sector level was then calculated by simply dividing the output (already generated by the model at this level) by the 97-sector level hours. Employment at the 97-sector level was calculated by dividing hours by the model's forecasts of average annual hours per worker. The disaggregation and the calculation of productivity and employment can be seen in the routine REMPLOY.CPP.

[21]The two industries are Construction (6) and Air transportation (40).

Table 1 - Estimates of Embodied Technological Change

Sector	Rate (std. error)
Manufacturing	
Food & Tobacco	-0.056 (0.021)
Textiles and knitting	0.098 (0.030)
Apparel	0.004 (0.025)
Paper	-0.064 (0.027)
Printing & publishing	-0.053 (0.023)
Chemicals	-0.004 (0.024)
Petroleum refining & Fuel Oil	0.017 (0.039)
Rubber & Plastic products	0.084 (0.026)
Shoes & leather	-0.046 (0.052)
Lumber	0.007 (0.023)
Furniture	-0.056 (0.028)
Stone, clay & glass	0.006 (0.026)
Primary metals	0.080 (0.029)
Metal products	-0.005 (0.022)
Industrial Equipment, except computers & office equipment	0.031 (0.024)
Computers & other office equipment	2.927 (0.202)
Electrical equipment except communications and elec. components	0.049 (0.029)
Communication equipment	0.141 (0.044)
Electronic components	0.766 (0.059)
Motor vehicles & parts	-0.064 (0.028)
Non-motor vehicle transportation equipment	0.098 (0.033)
Scientific Instruments	-0.023 (0.034)
Other instruments	0.087 (0.039)
Miscellaneous manufacturing	0.029 (0.032)
Nonmanufacturing	
Agriculture, forestry, and fisheries	0.009
Metal mining	0.027
Coal mining	-0.006
Natural Gas and Crude Petroleum extraction	0.013
Non-metallic mining	-0.001
Construction	0.026
Railroads	0.024
Air transport	0.109
Other transportation	0.058
Communication services	0.112
Electric utilities	0.056
Gas utilities, and water and sanitary services	0.033
Wholesale trade	0.066
Retail trade, and restaurant and bars	0.044
Finance and Insurance	0.065
Real Estate	0.065
Hotels, and personal and repair services (exc. auto)	0.061
Business services	0.078
Automobile services	0.063
Movies and amusement parks	0.037

Table 1 continued on next page...

Medical services	0.062
Education, social services, membership organizations	0.062

Table 2. Notation Guide

Variable	Abbreviation	Elasticity of Output with respect to the variable
Real Output (in parenthesis: log of real output)	Q (q)	--
Real Materials, including Energy (log)	M (m)	Θ
Labor Hours (log)	L (ℓ)	β
Real Equipment Stock (log)	J (j)	α
Real Structures Stock (log)	S (s)	η
Real Energy Expenditures (log)	E (e)	--
Utilization Rate of Equipment	U^J	--
Utilization Rate of Structures	U^S	--
Elasticity of Energy:Capital Ratio w.r.t. Utilization	τ	--

Table 3

Guide to Alternative Specifications for Labor Productivity Equations

Description	Equation
A. Standard Cobb-Douglas Production Function in logs.	$q-\ell=b_0+(\beta-1)\ell+\theta m+\alpha j+\eta s$
B. Same as (A) but adjusting to control for utilization using energy. This equation is derived from assuming $U^J=\left(E/J\right)^{\frac{1}{\tau}}$ and $U^S=\left(E/S\right)^{\frac{1}{\tau}}$ (where U^J and U^S are the utilization rates of equipment and structure respectively) and replacing *j* and *s* in specification A above with $log(U^J J)$ and $log(U^S S)$.	$q-\ell=b_0+\left(\frac{\beta-1}{\beta}\right)q+\left(\frac{\theta}{\beta}\right)m+\left(\frac{\alpha(\tau-1)}{\beta\tau}\right)j+\left(\frac{\eta(\tau-1)}{\beta\tau}\right)s+\left(\frac{\alpha+\eta}{\tau}\right)e$
C. Same as (A) but with constant returns to scale (RTS) imposed. This equation is derived by setting $\beta+\theta+\alpha+\eta=1$.	$q-\ell=b_0+\theta(m-\ell)+\alpha(j-\ell)+\eta(s-\ell)$
D. Same as (A) but with constant RTS *and* adjusting for utilization using energy.	$q-\ell=b_0+\theta(m-\ell)+\left(\frac{\alpha(\tau-1)}{\tau}\right)(j-\ell)+\left(\frac{\eta(\tau-1)}{\tau}\right)(s-\ell)+\left(\frac{\alpha+\eta}{\tau}\right)(e-\ell)$
E. Current IDLIFT equation. Note: *qup* and *qdown* are defined in equation (5).	$\ell - q = b_0 + a_1{*}t + a_2{*}t^2 + a_3{*}qup + a_4{*}qdown$
F. Same as (A) but with *J* <u>not</u> adjusted for embodied technological change (i.e., *J* is constructed with =0 for all sectors).	
G. Same as (B) but with *J* <u>not</u> adjusted for embodied technological change (i.e., *J* is constructed with =0 for all sectors).	

Table 3 continued...

H. Same as (C) but with *J* <u>not</u> adjusted for embodied technological change (i.e., *J* is constructed with =0 for all sectors).
I. Same as (D) but with *J* <u>not</u> adjusted for embodied technological change (i.e., *J* is constructed with =0 for all sectors).

Table 4. Mixed Empirical-Monte Carlo Results

Coefficient	True value	Mean Estimate	Std. Deviation	Estimated Bias
c^0	2	1.99092	0.23680	-0.00908
c^1	0.01	0.00968	0.00842	-0.00032
α	0.17	0.18120	0.28050	0.0112
η	0.16	0.15852	0.03292	-0.00148
b^0	0.1	0.10118	0.35719	0.00118
b^1	-0.1	-0.08530	0.37322	0.0147

Table 5. Specification Choice

Specification	Number of industries	Number requiring soft constraints
C’	19	18
D"	27	25
E	4	2
X	4	4
Total	54	49

Table 6. Deviations in Labor Productivity (Permanent Shock)

Industry	Equipment Elasticity	Percent Deviations from Base 2001	2005	2010	2015
1 Agriculture, forestry and fisheries (C’)	0.288	0.02	0.54	0.83	0.97
2 Metal mining (X)	0.094	-0.30	-0.09	0.00	0.03
3 Coal mining (C’)	0.295	-0.05	0.52	0.87	1.05
4 Crude petroleum and natural gas (C’)	0.359	-0.04	0.69	1.05	1.21
5 Non-metallic mining (D")	0.065	0.02	0.06	0.10	0.12
6 Construction (D")	0.318	0.08	0.56	0.87	1.02
7 Food and tobacco products (D")	0.206	0.06	0.32	0.48	0.57
8 Textile mill products (D")	0.060	0.09	0.08	0.13	0.14
9 Apparel and other textile products (D")	0.131	0.02	0.13	0.22	0.28
10 Paper and allied products (D")	0.133	0.09	0.11	0.21	0.27
11 Printing and publishing (D")	0.140	0.22	0.09	0.17	0.22
12 Chemicals (X)	0.239	-0.12	0.34	0.63	0.76
13 Plastics and synthetic materials (X)	0.207	-0.20	0.18	0.44	0.54
14 Petroleum refining (D")	0.025	0.03	0.01	0.03	0.05
15 Rubber and miscellaneous plastics (D")	0.081	0.31	0.08	0.13	0.14
16 Footwear and leather products (C’)	0.326	0.03	0.35	0.63	0.79
17 Lumber and wood products (C’)	0.054	-0.04	0.06	0.12	0.12
18 Furniture (D")	0.026	0.36	-0.04	-0.03	-0.02
19 Stone, clay and glass products (D")	0.058	0.05	0.06	0.11	0.14
20 Primary iron and steel (D")	0.095	0.16	0.09	0.15	0.16
21 Primary nonferrous metals mfg. (D")	0.055	0.17	0.06	0.09	0.10
22 Metal products (D")	0.064	0.15	0.05	0.09	0.11
23 Engines and turbines (C’)	0.123	-0.16	0.04	0.13	0.16
24 Agricultural, construction & mining mach. (C’)	0.061	-0.18	-0.11	-0.07	-0.06
25 Metalworking machinery (E)	N/A	0.56	0.00	0.00	0.01
26 Special industry machinery (E)	N/A	0.15	0.01	0.00	0.02
27 General and miscellaneous industrial mach. (C’)	0.062	-0.19	-0.07	-0.01	0.03
28 Computers and office equipment (C’)	0.125	-0.23	-0.04	0.04	0.08
29 Service industry machinery (D")	0.083	0.43	0.08	0.11	0.12
30 Electrical industrial equipment and app. (D")	0.088	0.40	0.03	0.08	0.10
31 Household appl., elec lighting & wiring (D")	0.306	0.29	0.48	0.72	0.81
32 Audio, video and comm. equipment (C’)	0.073	-0.17	-0.06	0.00	0.03
33 Electronic components (D")	0.215	0.01	0.32	0.45	0.49
34 Motor vehicles and equipment (C’)	0.086	-0.12	0.02	0.10	0.12
35 Aircraft and parts (C’)	0.195	0.00	0.25	0.37	0.40
36 Ships and other transportation equipment (D")	0.074	-0.03	0.02	0.05	0.06
37 Instruments (C’)	0.158	-0.07	0.12	0.22	0.26
38 Miscellaneous manufacturing (D")	0.297	0.06	0.44	0.61	0.65
39 Railroad transportation (X)	0.029	-0.08	-0.04	-0.02	-0.01
40 Air transportation (C’)	0.330	0.11	0.82	1.09	1.16

Table 6 continued on next page...

41 Trucking and other transport (C’)	0.094	-0.07	0.05	0.11	0.13
42 Communications services (C’)	0.183	-0.02	0.37	0.48	0.50
43 Electric utilities (C’)	0.349	0.08	0.66	0.93	1.02
44 Gas, water and sanitary services (C’)	0.095	-0.07	0.05	0.09	0.11
45 Wholesale trade (D")	0.089	0.10	-0.05	0.03	0.06
46 Retail trade, restaurants & bars (E)	N/A	0.02	0.00	0.00	0.00
47 Finance and insurance (D")	0.036	0.04	0.02	0.01	0.03
48 Real estate and rental (E)	N/A	0.04	0.00	0.00	0.00
49 Hotels, repairs except auto (D")	0.132	0.11	0.14	0.20	0.22
50 Business and professional services (D")	0.214	0.15	0.08	0.25	0.32
51 Automotive repair and services (D")	0.031	0.11	0.01	0.04	0.06
52 Movies and amusements (D")	0.161	0.14	0.23	0.31	0.34
53 Health services (C’)	0.348	0.17	0.71	0.92	0.98
54 Educational and social services and NPO (C’)	0.147	0.01	0.23	0.29	0.31

Table 7. Deviations in Labor Productivity (Temporary Shock)

Industry	Equipment Elasticity	Percent Deviations from Base 2001	2005	2010	2015
1 Agriculture, forestry and fisheries (C’)	0.288	0.32	0.74	0.97	0.96
2 Metal mining (X)	0.094	-1.46	-0.02	0.06	0.08
3 Coal mining (C’)	0.295	-0.15	1.07	1.49	1.63
4 Crude petroleum and natural gas (C’)	0.359	0.04	1.11	1.47	1.56
5 Non-metallic mining (D")	0.065	0.08	0.20	0.16	0.16
6 Construction (D")	0.318	0.45	1.12	1.32	1.37
7 Food and tobacco products (D")	0.206	0.32	0.42	0.51	0.54
8 Textile mill products (D")	0.060	0.47	0.17	0.12	0.10
9 Apparel and other textile products (D")	0.131	0.07	0.23	0.21	0.22
10 Paper and allied products (D")	0.133	0.42	0.17	0.22	0.22
11 Printing and publishing (D")	0.140	1.12	0.27	0.20	0.21
12 Chemicals (X)	0.239	-0.56	0.55	0.63	0.61
13 Plastics and synthetic materials (X)	0.207	-1.01	0.40	0.41	0.37
14 Petroleum refining (D")	0.025	0.12	0.02	0.02	0.04
15 Rubber and miscellaneous plastics (D")	0.081	1.03	0.18	0.14	0.13
16 Footwear and leather products (C’)	0.326	0.19	0.52	0.74	0.79
17 Lumber and wood products (C’)	0.054	-0.21	-0.02	0.14	0.13
18 Furniture (D")	0.026	1.83	0.16	-0.01	-0.01
19 Stone, clay and glass products (D")	0.058	0.25	0.15	0.12	0.11
20 Primary iron and steel (D")	0.095	0.79	0.20	0.17	0.16
21 Primary nonferrous metals mfg. (D")	0.055	0.86	0.23	0.12	0.09
22 Metal products (D")	0.064	0.78	0.16	0.11	0.11
23 Engines and turbines (C’)	0.123	-0.81	0.15	0.18	0.17
24 Agricultural, construction & mining mach. (C’)	0.061	-0.86	0.03	0.04	0.04
25 Metalworking machinery (E)	0.000	2.70	0.24	-0.02	0.00
26 Special industry machinery (E)	0.000	0.72	-0.05	0.00	0.00
27 General and miscellaneous industrial mach. (C’)	0.062	-0.94	0.03	0.04	0.06
28 Computers and office equipment (C’)	0.125	-1.13	0.21	0.20	0.17
29 Service industry machinery (D")	0.083	2.16	0.29	0.16	0.15
30 Electrical industrial equipment and app. (D")	0.088	1.94	0.15	0.14	0.14
31 Household appl., elec lighting & wiring (D")	0.306	1.30	0.87	0.80	0.77
32 Audio, video and communication equipment (C’)	0.073	-0.85	0.04	0.06	0.06
33 Electronic components (D")	0.215	0.02	0.43	0.38	0.35
34 Motor vehicles and equipment (C’)	0.086	-0.65	0.13	0.16	0.17
35 Aircraft and parts (C’)	0.195	-0.01	0.41	0.39	0.37
36 Ships and other transportation equipment (D")	0.074	-0.15	0.36	0.13	0.12
37 Instruments (C’)	0.158	-0.36	0.31	0.32	0.31
38 Miscellaneous manufacturing (D")	0.297	0.28	0.78	0.66	0.57
39 Railroad transportation (X)	0.029	-0.41	-0.03	-0.01	-0.01
40 Air transportation (C’)	0.330	0.51	1.13	1.07	0.95
41 Trucking and other transport (C’)	0.094	-0.36	0.07	0.10	0.08
42 Communications services (C’)	0.183	-0.08	0.48	0.48	0.42
43 Electric utilities (C’)	0.349	0.49	1.03	1.12	1.09
44 Gas, water and sanitary services (C’)	0.095	-0.28	0.08	0.15	0.15
45 Wholesale trade (D")	0.089	0.02	0.08	0.09	0.09
46 Retail trade, restaurants & bars (E)	0.000	0.10	0.02	0.00	0.00

Table 7 continued on next page...

47 Finance and insurance (D")	0.036	0.17	0.00	0.04	0.07
48 Real estate and rental (E)	0.000	0.20	0.00	0.00	0.00
49 Hotels, repairs except auto (D")	0.132	0.58	0.22	0.20	0.19
50 Business and professional services (D")	0.214	2.33	1.03	0.41	0.38
51 Automotive repair and services (D")	0.031	0.71	0.01	0.07	0.07
52 Movies and amusements (D")	0.161	0.76	0.38	0.34	0.33
53 Health services (C’)	0.348	0.93	0.92	0.96	0.87
54 Educational and social services and NPO (C’)	0.147	0.14	0.22	0.28	0.26

Figure 1, Gridsearch Results of Matching FRB Physical Depreciation Patterns

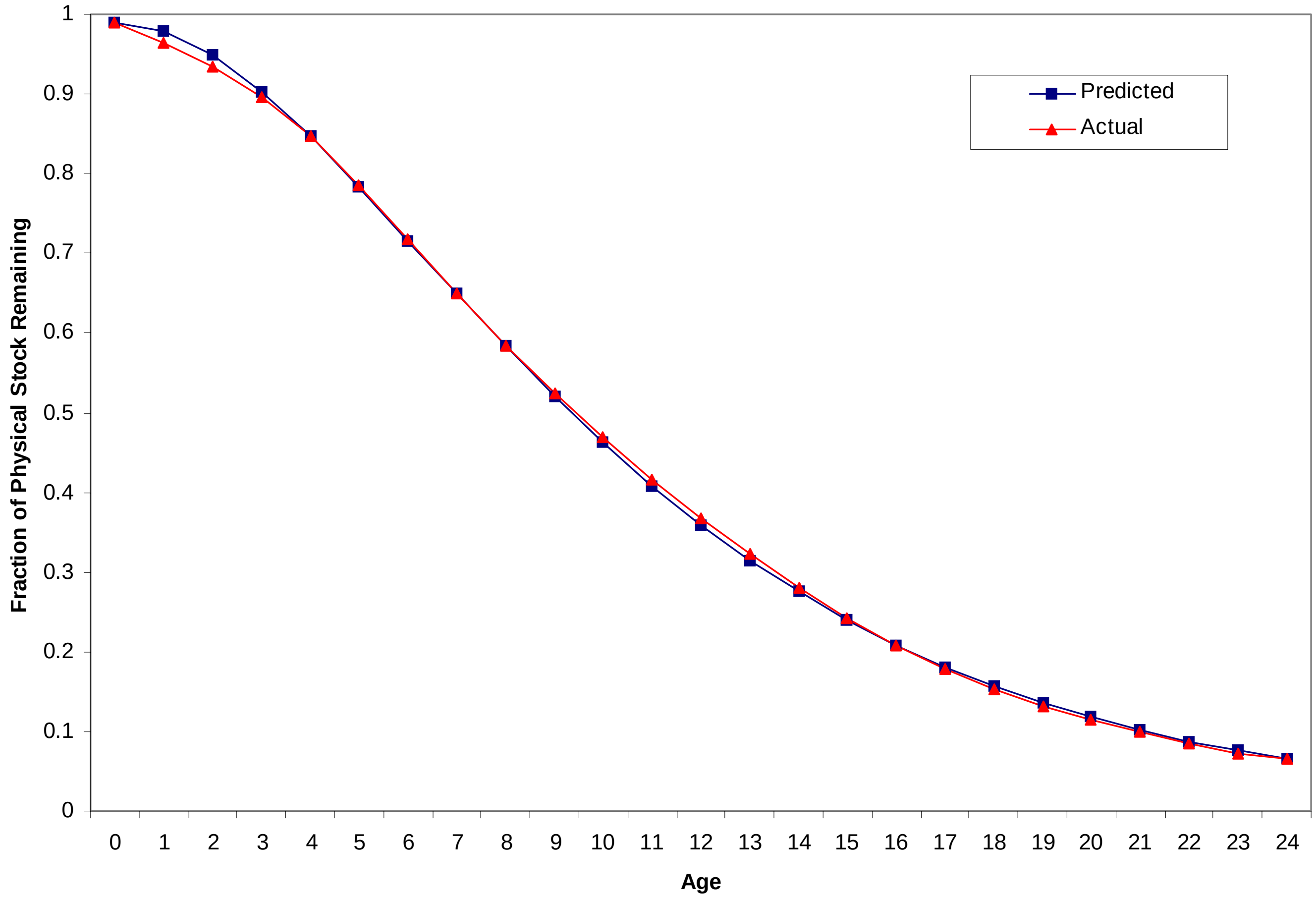


Figure 2, Average Adjusted R-squared

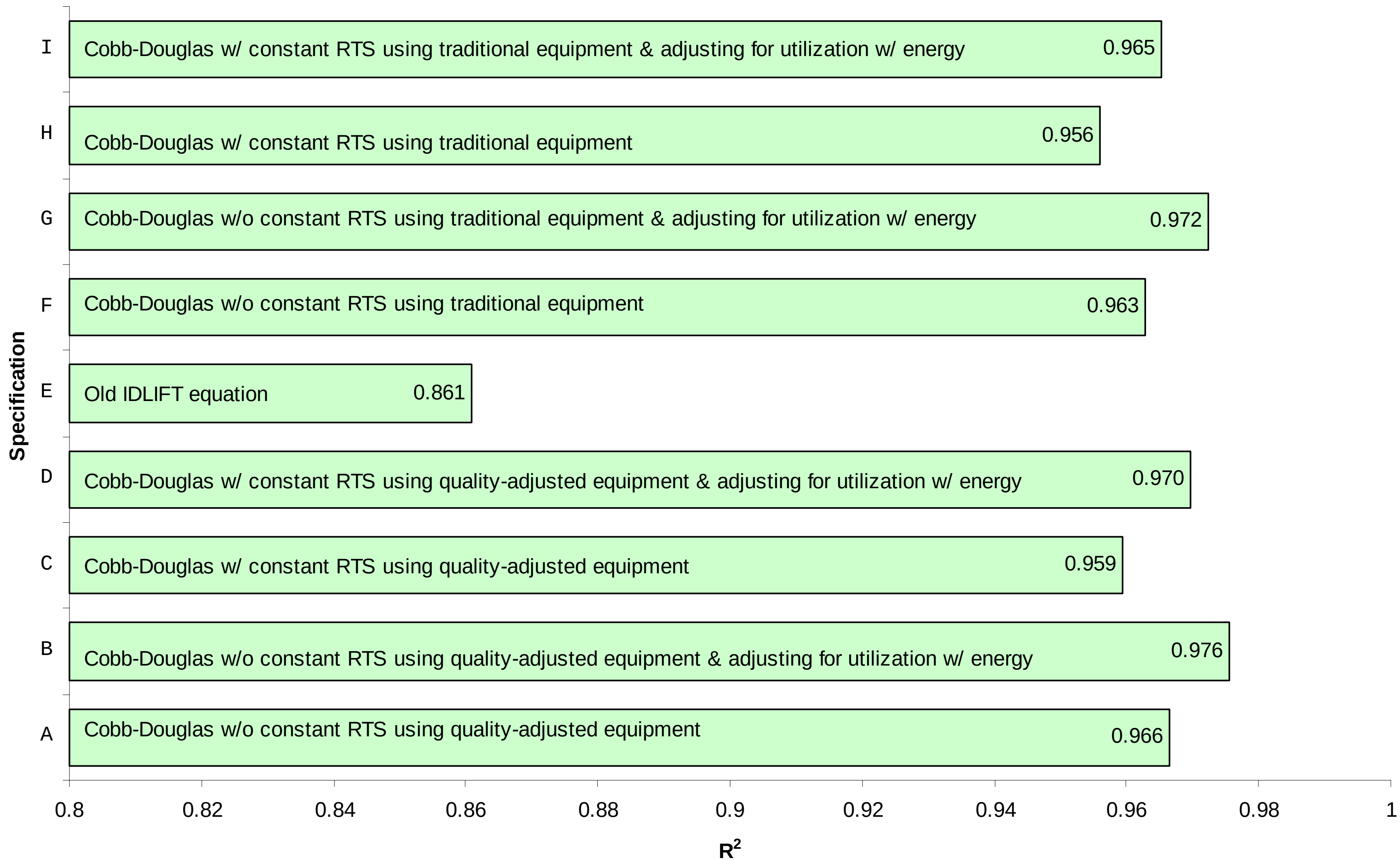


Figure 3, Average Elasticities

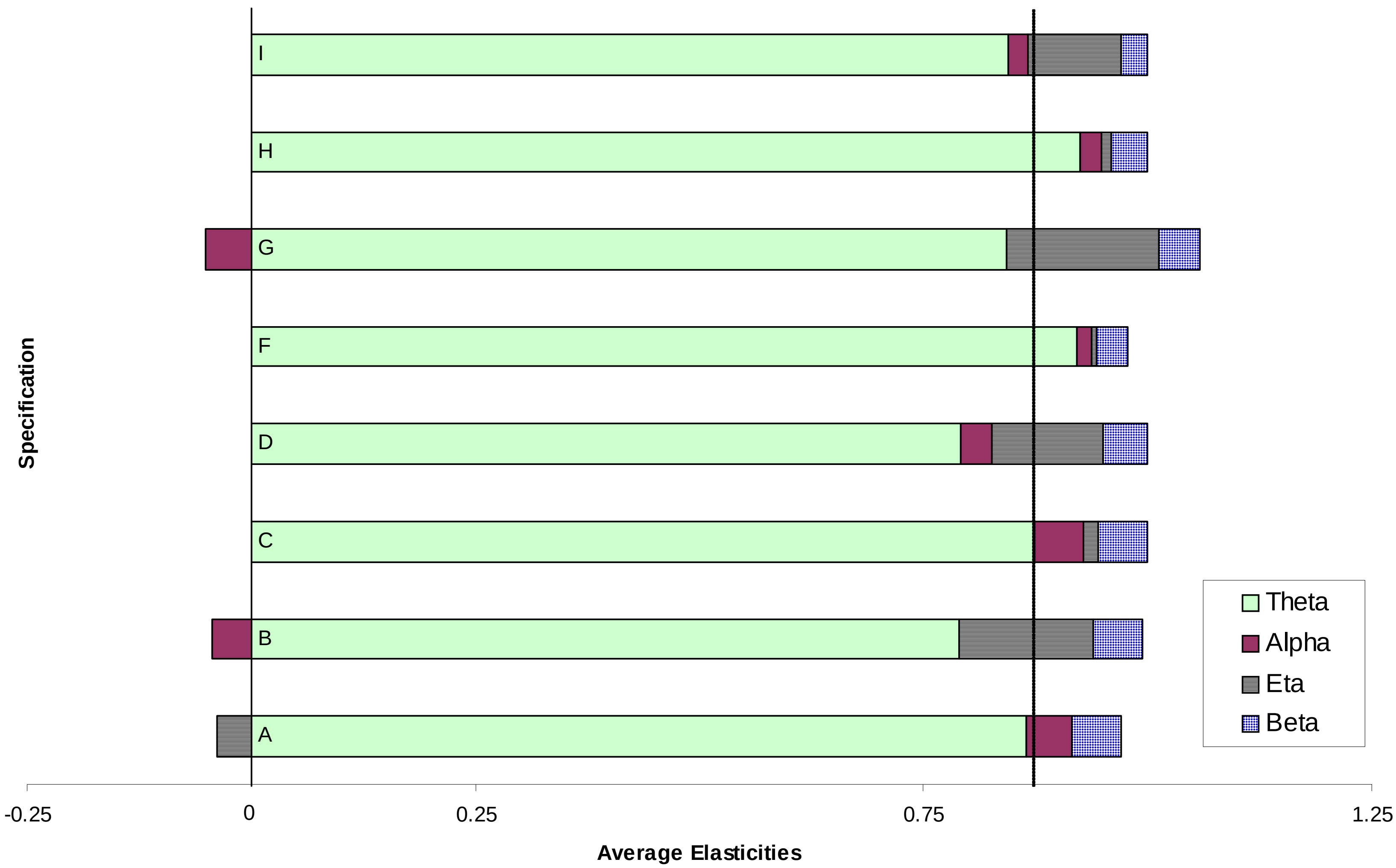


Figure 4, Percent of Elasticities that are Positive

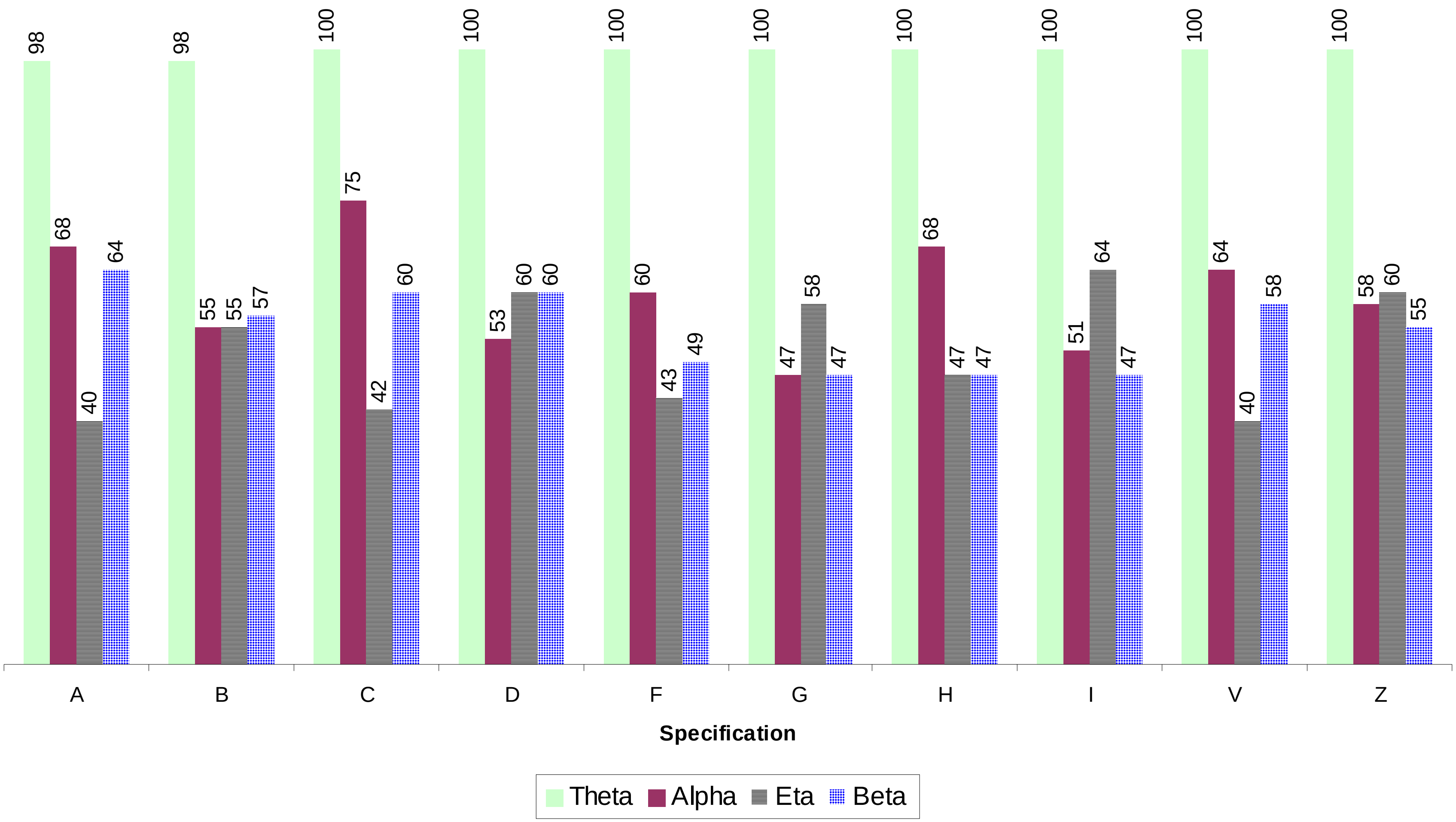


Figure 5, Average Adjusted R-squared -- Specifications Not Including Materials

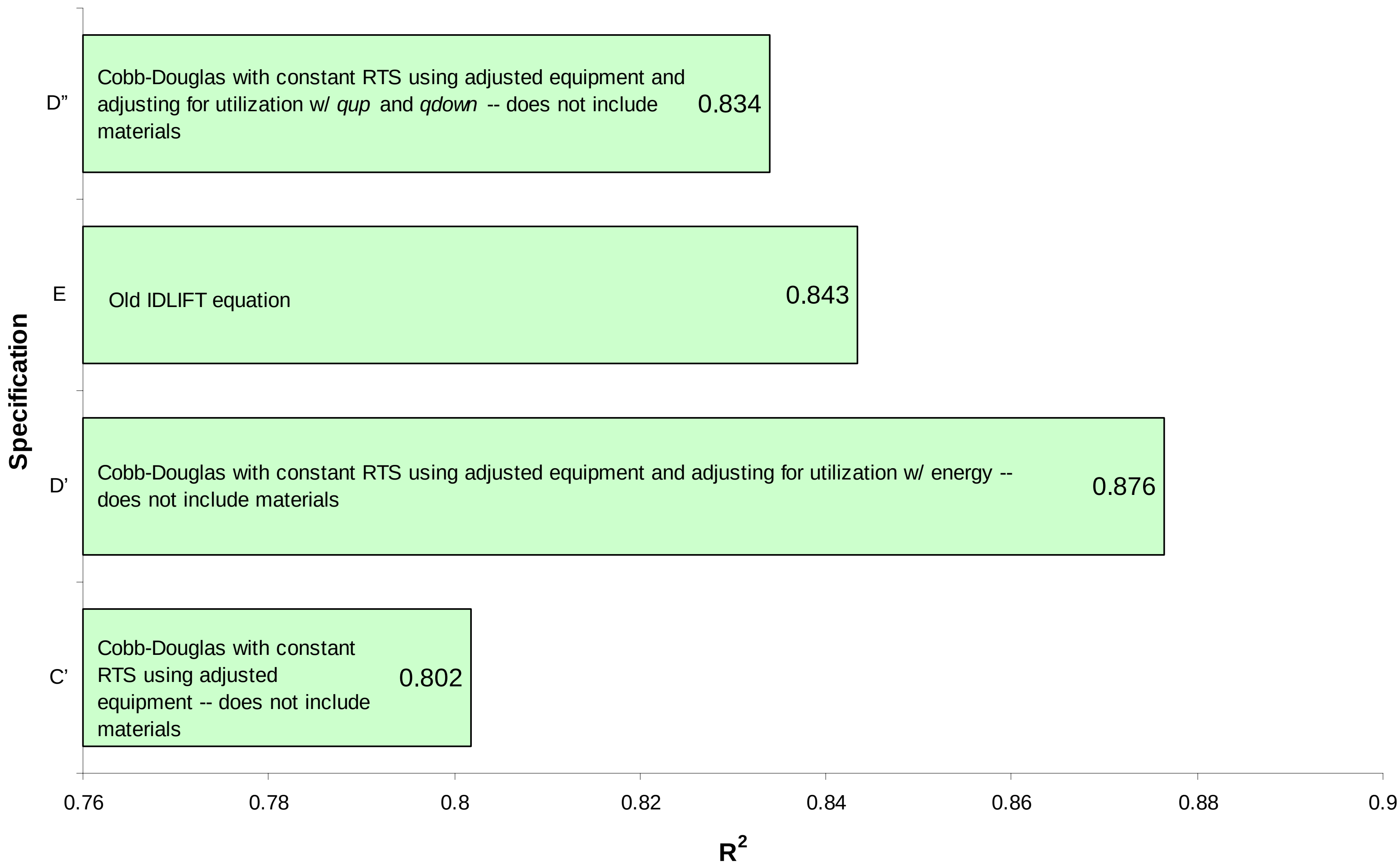


Figure 6,   Average Elasticities -- Specifications Not Including Materials

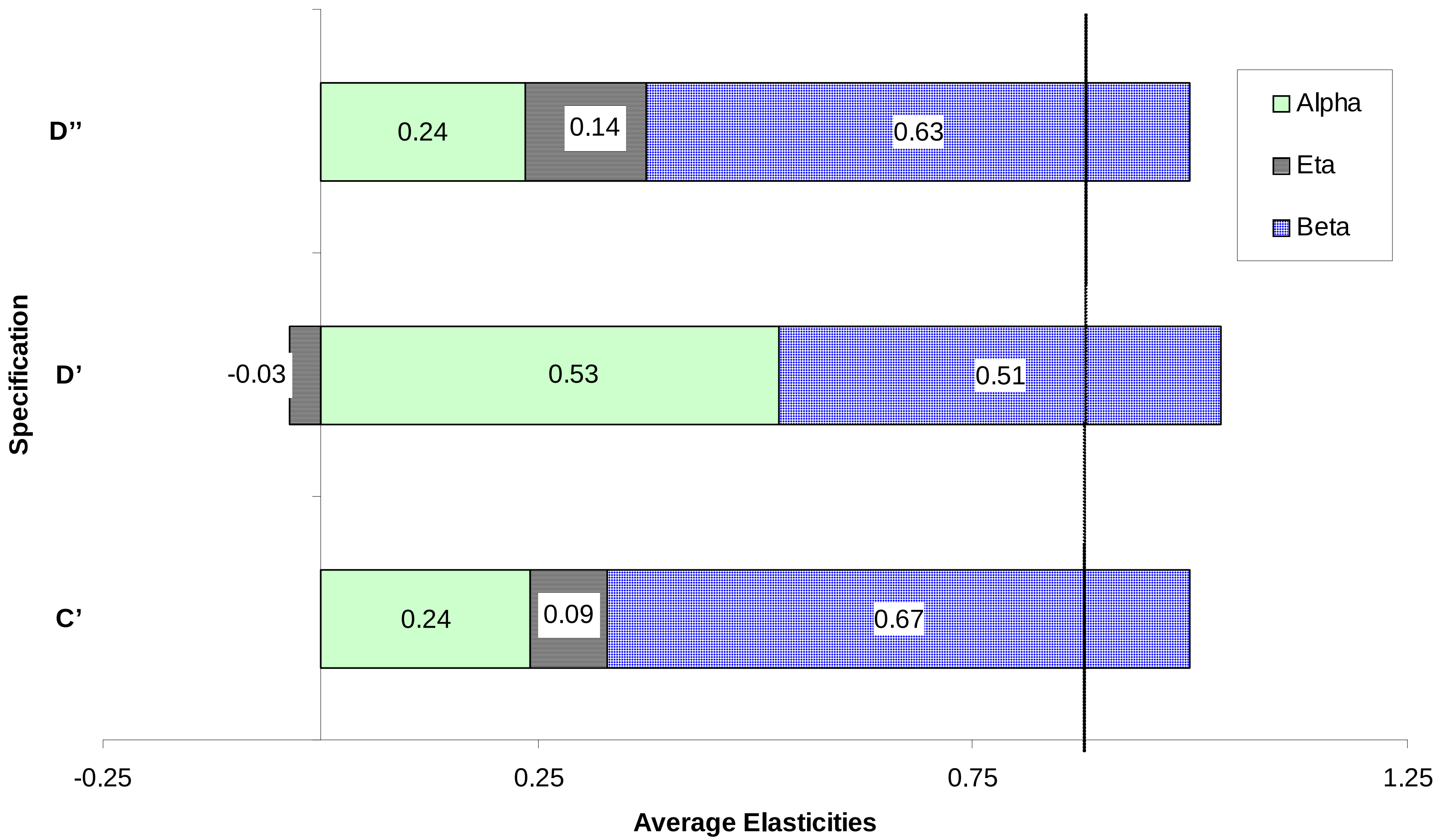
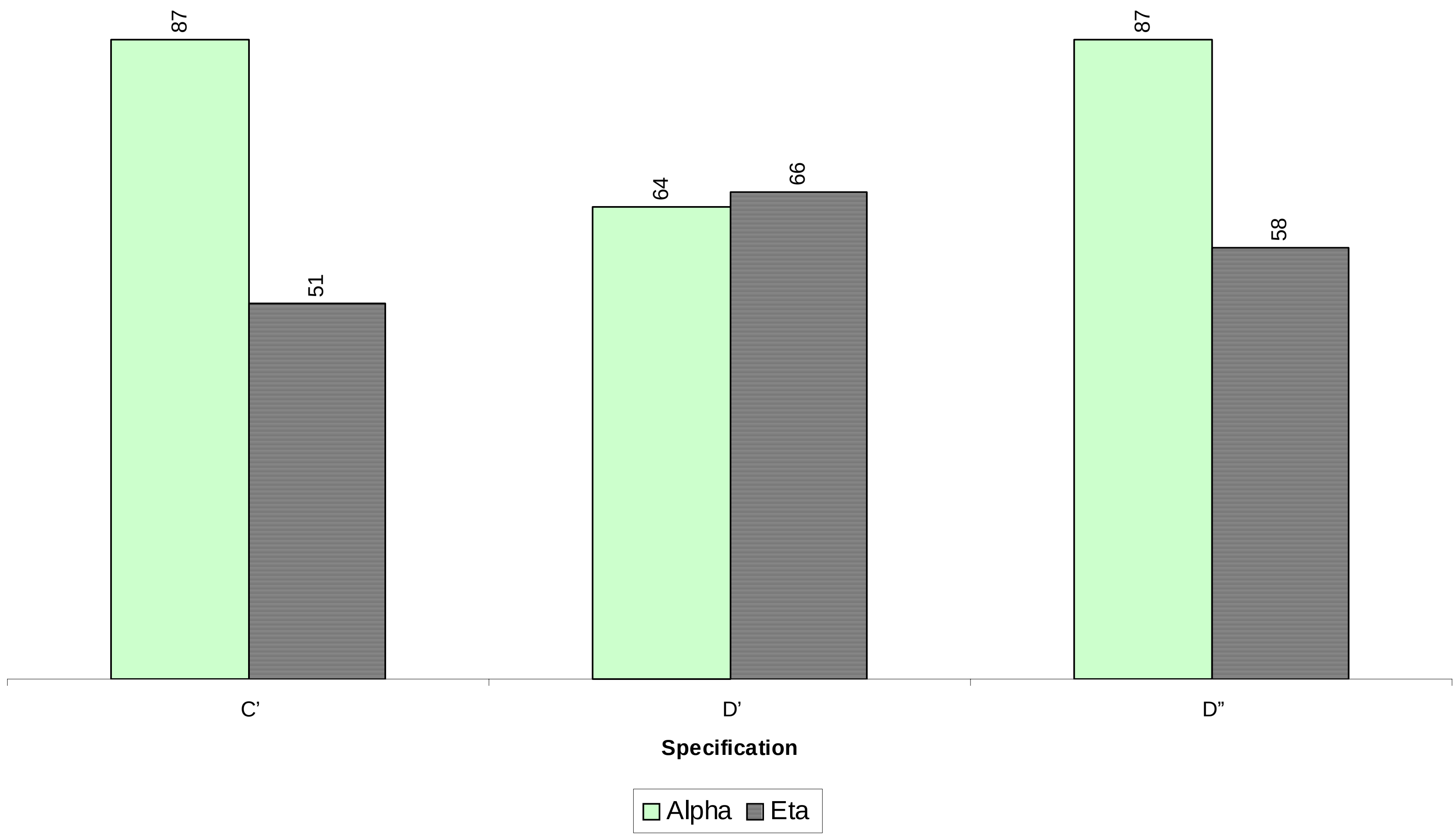


Figure 7, Percent of Elasticities that are Positive -- Specifications Not Including Materials



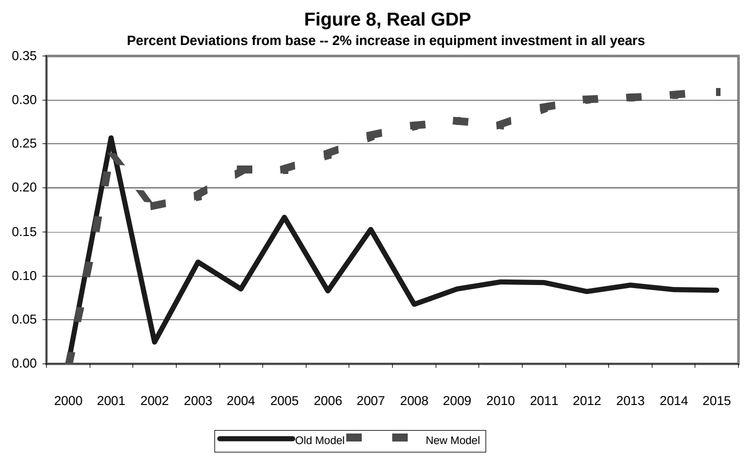


Figure 8, Real GDP
Percent Deviations from base -- 2% increase in equipment investment in all years
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015
Old Model
New Model

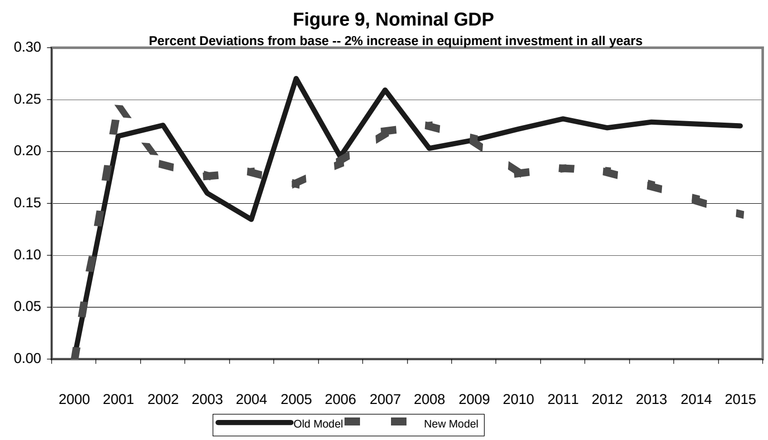


Figure 9, Nominal GDP
Percent Deviations from base -- 2% increase in equipment investment in all years
0.30
0.25
0.20
0.15
0.10
0.05
0.00
2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015
Old Model
New Model

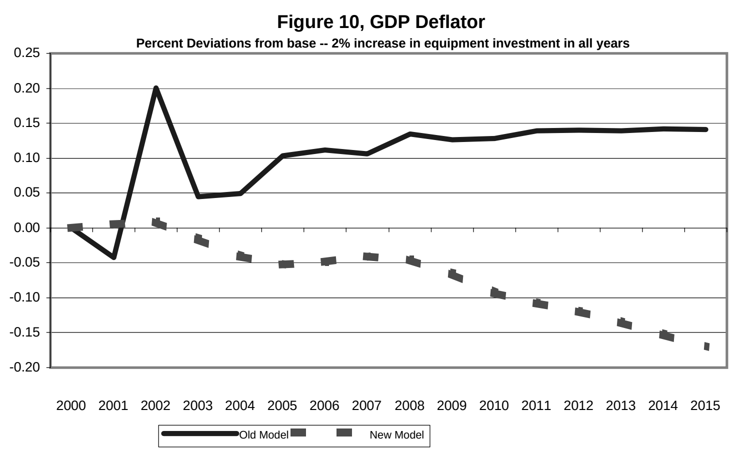


Figure 10, GDP Deflator
Percent Deviations from base -- 2% increase in equipment investment in all years
0.25
0.20
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.20
2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015
Old Model
New Model

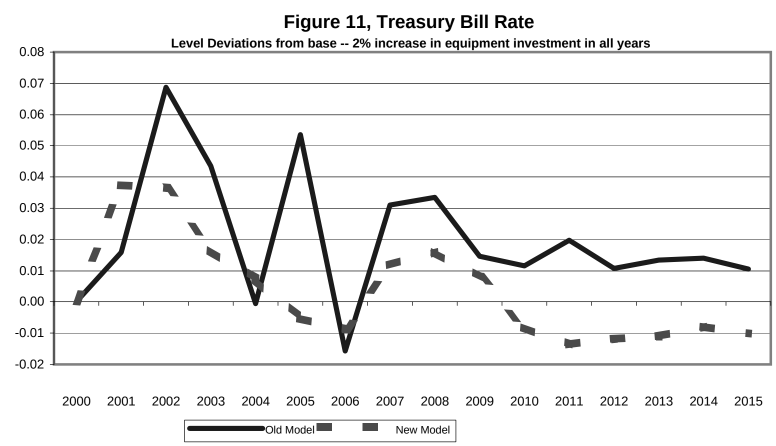


Figure 11, Treasury Bill Rate
Level Deviations from base -- 2% increase in equipment investment in all years
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
-0.01
-0.02
2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015
Old Model
New Model

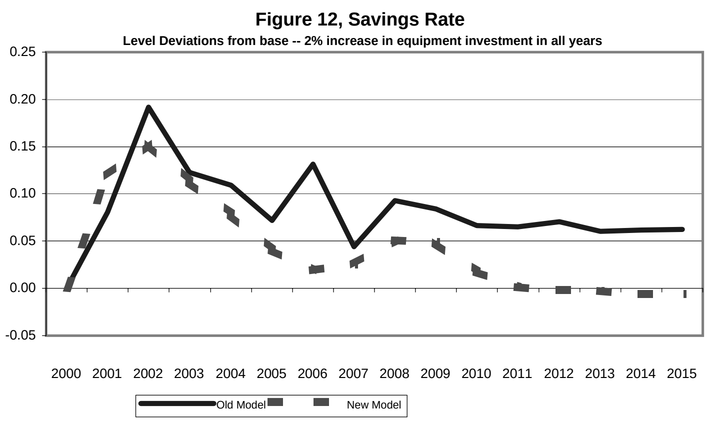


Figure 12, Savings Rate
Level Deviations from base -- 2% increase in equipment investment in all years
0.25
0.20
0.15
0.10
0.05
0.00
-0.05
2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015
Old Model
New Model

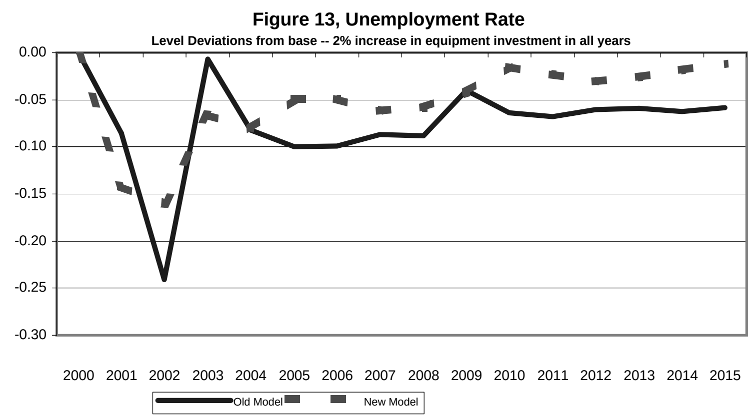


Figure 13, Unemployment Rate
Level Deviations from base -- 2% increase in equipment investment in all years
0.00
-0.05
-0.10
-0.15
-0.20
-0.25
-0.30
2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015
Old Model
New Model

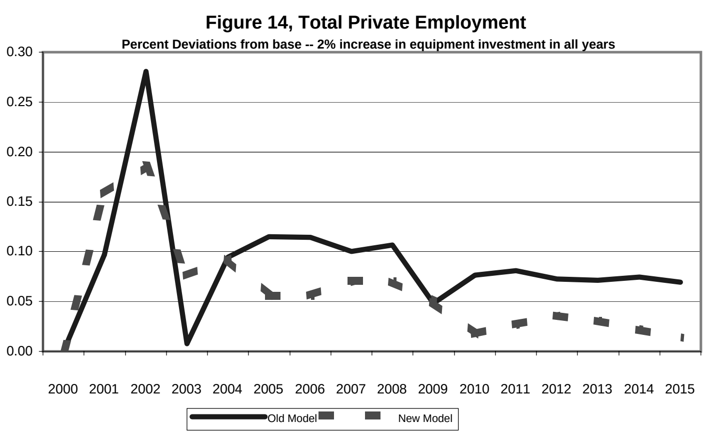


Figure 14, Total Private Employment
Percent Deviations from base -- 2% increase in equipment investment in all years
0.30
0.25
0.20
0.15
0.10
0.05
0.00
2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015
Old Model
New Model

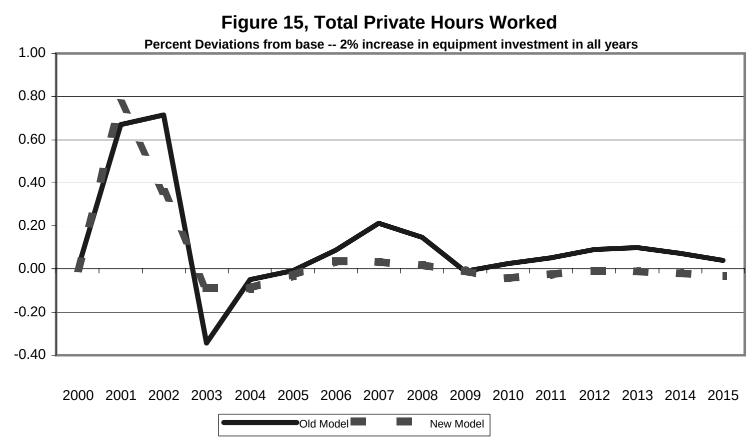


Figure 15, Total Private Hours Worked
Percent Deviations from base -- 2% increase in equipment investment in all years
1.00
0.80
0.60
0.40
0.20
0.00
-0.20
-0.40
2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015
Old Model
New Model

Figure 16, Average Real Wage

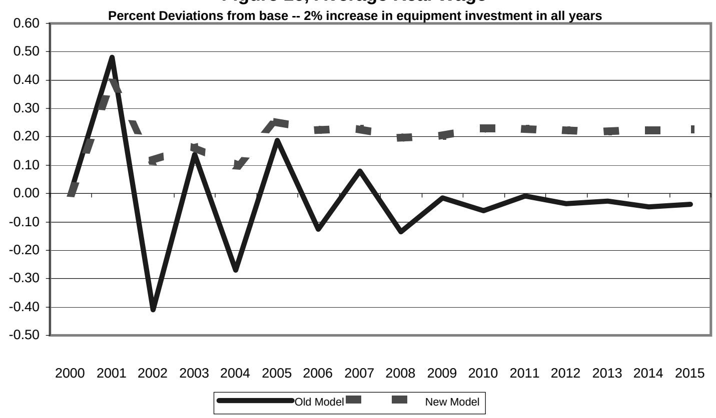


Figure 17, Labor Productivity

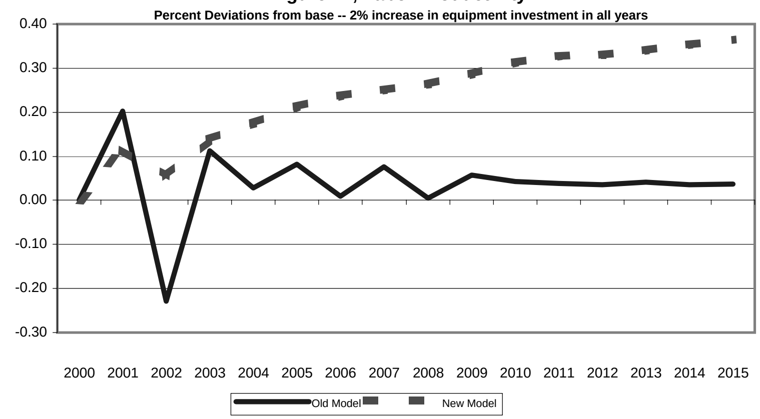


Figure 18, Real GDP

Percent Deviations from base -- 10% increase in equipment investment in 2001

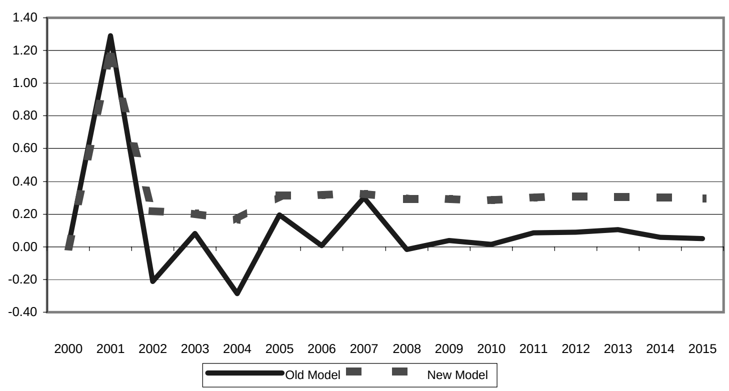


Figure 19, Nominal GDP

Percent Deviations from base -- 10% increase in equipment investment in 2001

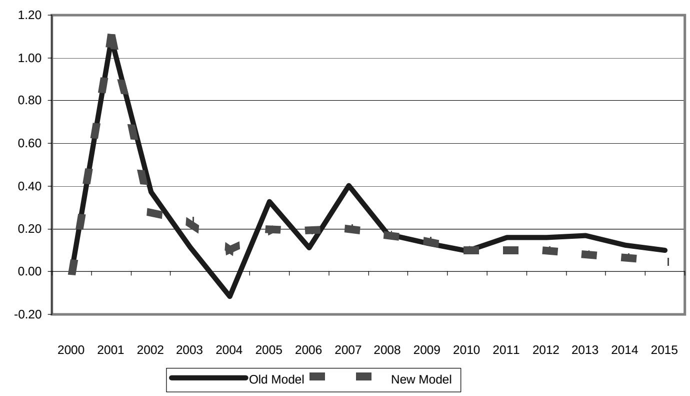


Figure 20, GDP Deflator

Percent Deviations from base -- 10% increase in equipment investment in 2001

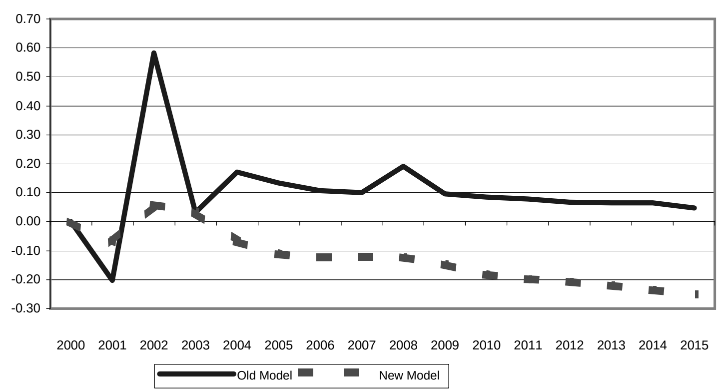
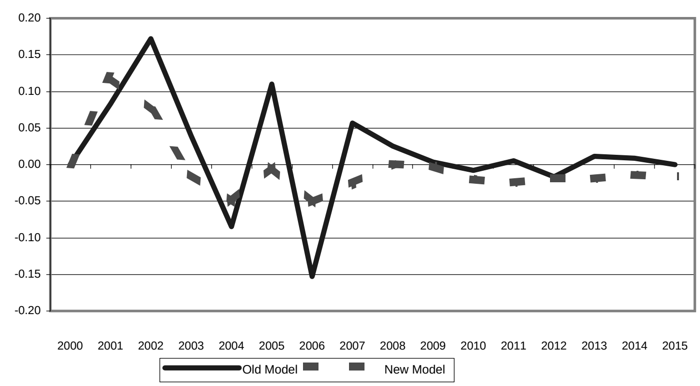


Figure 21, Treasury Bill Rate

Level Deviations from base -- 10% increase in equipment investment in 2001



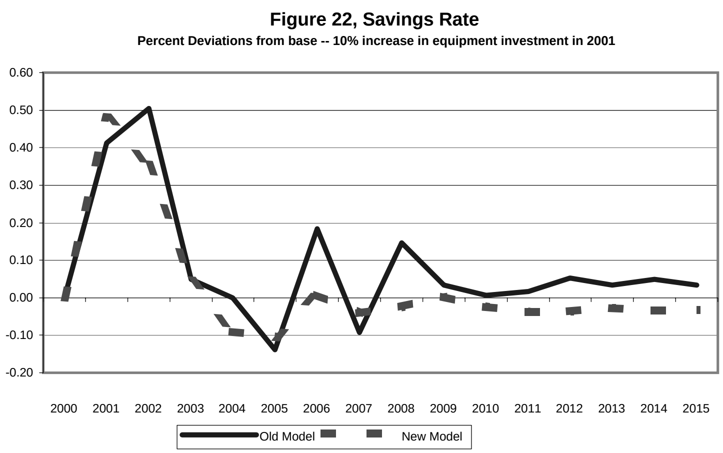


Figure 22, Savings Rate
Percent Deviations from base -- 10% increase in equipment investment in 2001
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015
Old Model
New Model

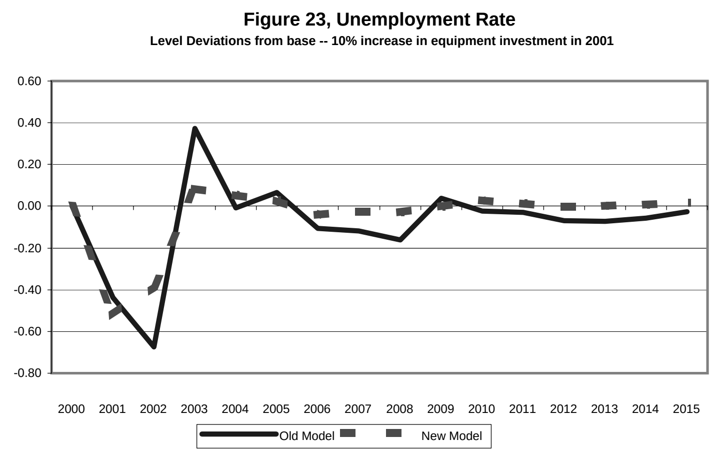


Figure 23, Unemployment Rate
Level Deviations from base -- 10% increase in equipment investment in 2001
0.60
0.40
0.20
0.00
-0.20
-0.40
-0.60
-0.80
2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015
Old Model
New Model

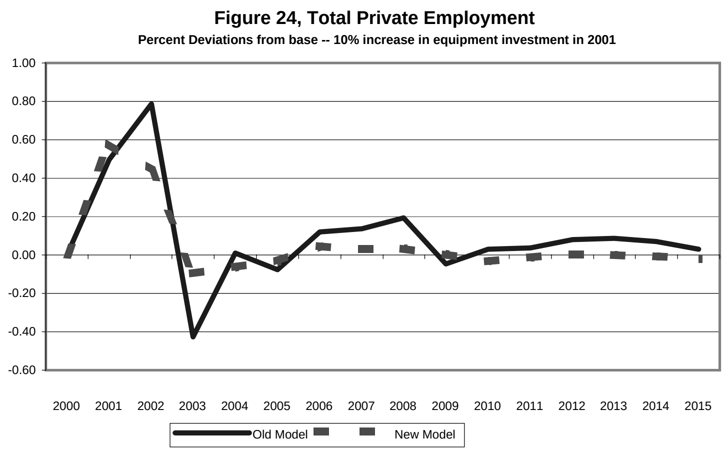


Figure 24, Total Private Employment
Percent Deviations from base -- 10% increase in equipment investment in 2001
1.00
0.80
0.60
0.40
0.20
0.00
-0.20
-0.40
-0.60
2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015
Old Model
New Model

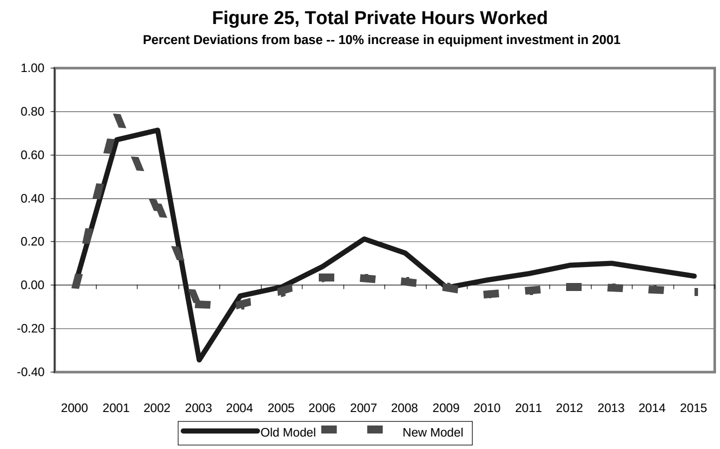


Figure 25, Total Private Hours Worked
Percent Deviations from base -- 10% increase in equipment investment in 2001
1.00
0.80
0.60
0.40
0.20
0.00
-0.20
-0.40
2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015
Old Model
New Model

Figure 26, Average Real Wage

Percent Deviations from base -- 10% increase in equipment investment in 2001

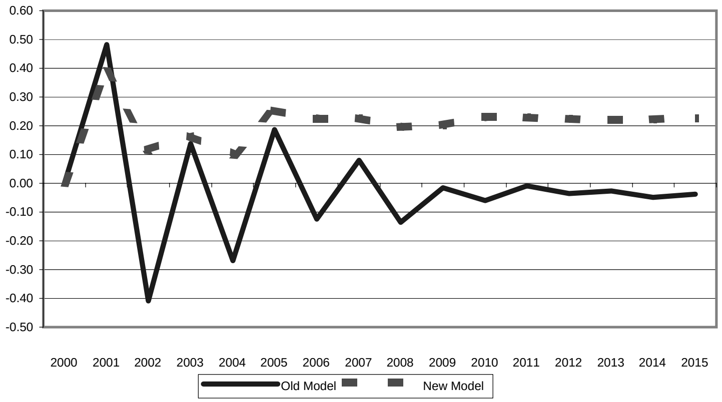


Figure 27, Labor Productivity

Percent Deviations from base -- 10% increase in equipment investment in 2001

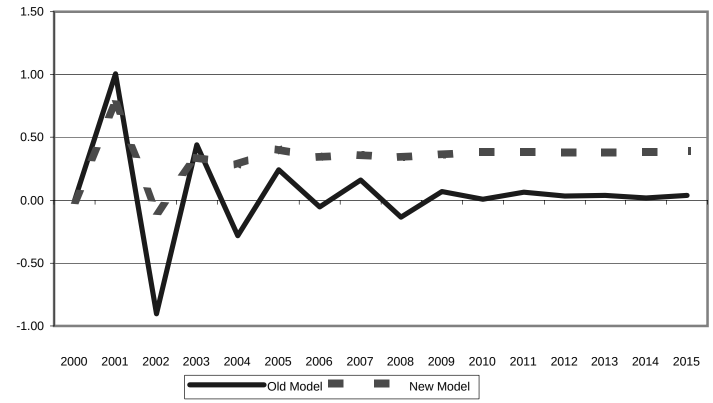


Figure 28, Equipment Investment
Percent Deviations from base -- 10% increase in equipment investment in 2001

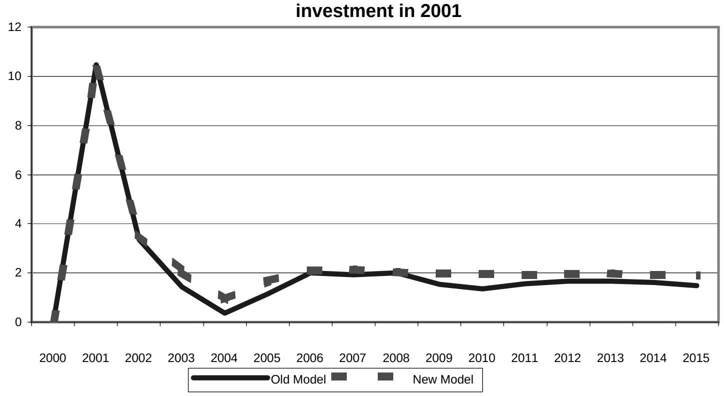


Figure 29, Equipment Capital
Percent Deviations from base -- 10% increase in equipment investment in 2001