A Perhaps Adequate Demand System

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Long-term, multisectoral modeling requires calculation of consumer expenditures in some detail by product. Finding a functional form to represent the market demand functions of consumers for this work has proven a surprisingly thorny problem. Clearly, the form must deal with significant growth in real income, the effects of demographic and other trends, and changes in relative prices. Both complementarity and substitution should be possible among the different goods. Increasing income should certainly not necessarily, by the form of the function, force the demand for some good to go negative. Prices should affect the marginal propensity to consume with respect to income, and the extent of that influence should be an empirical question, not decided by the form of the function.

This paper will present a form which meets these requirements and extends a form suggested earlier by the author [Almon 1979]. Applications of the new form to a 42-sector demand system for Spain and a 93-sector system for the USA are reported.

Before presenting this form, however, it may be well to see just how tricky it can be to find a form with these simple requirements by looking at another form, the "Almost Ideal Demand System" (AIDS) suggested by Deaton and Muellbauer [1980]. Its name, the eminence of its authors and its place of publication have led to wide usage. It has, however, a most peculiar property which is likely to vitiate any growth model in which it is used. Like many others, it is derived from utility maximization; its problems will therefore emphasize the important fact that such derivation does not automatically imply reasonable properties. One of the properties it does imply, however, is Slutsky symmetry in the market demand functions. This property was not mentioned above. Should it have been? What role should this symmetry play in market demand functions? This question also needs to be examined before presenting the new form, for it plays a key role in its formulation.

1. Problems and lessons of the AIDS form.

The AIDS form can be written as an equation for the budget share of good i:

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$$s_i = a_i + \sum_{j=1}^{n} d_{ij} \cdot \log(p_j) + b_i \cdot \log(y/P) \qquad (1)$$

where s_i is the budget share of product i, p_j is the price of product j, y is nominal income and P is an overall price index, the matrix of d's is symmetric and has zero row and column sums, the sum of all the a_i is 1, and the b_i sum to zero. Consequently, if any b_i is positive, then one or more must be negative. Thus *increasing real income must ultimately drive the consumption of one or more goods negative*, unless, of course, it has no effect at all on budget shares. This property seems rather less than "ideal". Moreover, the partial derivative of the share with respect to real income is independent of the relative prices, whereas common sense suggests that it should depend on them. Because of these properties, the AIDS form, while possibly "almost ideal" from some point of view, is surely absolutely inadequate for use in any growth model. Since it is derived from utility maximization, it also serves as a clear warning that the mere fact of such ancestry is no assurance whatsoever of the adequacy of a form. Perhaps there is also in the AIDS story a lesson for modesty in naming a form, a lesson which has been heeded in "PADS" form proposed here.

A number of other forms derived from utility maximization were reviewed in the article cited and found wanting relative to the simple properties set out above. The only study which to my knowledge has estimated these forms, AIDS, and my earlier suggestion all on the same data and compared the results is Gauyacq [1985]. Using French data for 1959-1979, he estimated "the linear expenditure system of Stone; the model with real prices and income of Fourgeaud and Nataf; the additive quadratic model of Houthakker and Taylor; the logarithmically additive model of Houthakker, ... the Rotterdam model of Theil and Barten, the Translog model based on a logarithmic transformation of the utility function; the AIDS model of Deaton and Muellbauer; .... [and] the model proposed by Clopper Almon." The conclusion was not surprising to anyone who had compared the properties of the forms to the simple requirements stated above: "De l'étude que nous avons effectué, il apparaît en définitive que seul le modèle de C. Almon constitue un système que satisfasse approximativement aux attendus théoriques et présente un réel intérêt pour l'étude économétrique de fonctions de demande détaillées." (p. 119). (From the study which we have done, it appears that definitely only the model of C. Almon offers a system which satisfies approximately theoretical expectations and is of real interest for the econometric study of detailed demand functions.) Elegant theoretical derivations, apparently, are of little help in finding adequate forms. Despite this relative success, there is a problem with my earlier suggestion, as we will see in section 3, where we will also see a way to fix it.

2. Slutsky Symmetry and Market Demand Functions

Just about the only non-obvious implication of the theory of the single consumer who maximizes utility subject to a budget constraint is the Slutsky symmetry shown in equation (2).
Here x_i^k is the consumption of product i by individual k, y^k is the nominal income of individual k, and p_j is the price of product j. A comparable relation, however, need not hold for the market

$$\frac{\partial x_i^k}{\partial p_j} + \frac{\partial x_i^k}{\partial y^k} \cdot x_j^k = \frac{\partial x_j^k}{\partial p_i} + \frac{\partial x_j^k}{\partial y^k} \cdot x_i^k \tag{2}$$

demand functions, the sum over all k of individuals' demand functions. Summing the above equation over the individuals gives equation (3),

$$\frac{\partial \sum_k x_i^k}{\partial p_j} + \sum_k \frac{\partial x_i^k}{\partial y^k} \cdot x_j^k = \frac{\partial \sum_k x_j^k}{\partial p_i} + \sum_k \frac{\partial x_j^k}{\partial y^k} \cdot x_i^k \tag{3}$$

which is in general not the same as -- and does not imply -- equation (4),

$$\frac{\partial \sum_k x_i^k}{\partial p_j} + \frac{\partial \sum_k x_i^k}{\partial \sum_k y^k} \cdot \sum_k x_j^k = \frac{\partial \sum_k x_j^k}{\partial p_i} + \frac{\partial \sum_k \partial x_j^k}{\partial \sum_k y^k} \cdot \sum_k x_i^k \tag{4}$$

which is what Slutsky symmetry of the market demand functions would imply. Thus, strict micro theory does not imply Slutsky symmetry of market demand functions. Consequently, there is in general no "representative consumer." To suppose that market demand functions derived by maximizing the utility of this non-existent entity have "micro foundations" not enjoyed by functions not so derived is hardly respectful of micro theory. Rather, any market demand functions so derived are on exactly the same theoretical footing as market demand functions made up without any reference to utility maximization. Both kinds of functions must meet the same "adequacy" criteria.

With that point clearly established, we may, however, ask Are there restrictive conditions under which equation (3) would imply equation (4)? One condition is, of course, that all individuals should have not only the same utility function but also the same income, and that the increase in aggregate income is accomplished by giving each the same increase. That condition is hardly interesting for empirical studies. A less restrictive condition is that the marginal propensity to consume a given product with respect to income should be the same for all individuals, or in effect, that the Engel curves for all products should be straight lines. If, for example,

$$\frac{\partial x_i^k}{\partial y^k} = a_i \tag{5}$$

then the second term on each side of equation (2) can be factored to yield

$$\frac{\partial \sum_k x_i^k}{\partial p_j} + a_i \cdot \sum_k x_j^k = \frac{\partial \sum_k x_j^k}{\partial p_i} + a_j \cdot \sum_k x_i^k \tag{6}$$

This is exactly what equation (4) states, for in this case it makes no difference to whom the "infinitesimal" increase in income is given and

$$\frac{\partial \sum_k x_i^k}{\partial \sum_k y^k} = a_i \ . \tag{7}$$

Now the assumption that all Engel curves are straight lines is generally contradicted by cross-section budget studies, even when one uses total expenditure in place of income in the Engel curves. (See, for example, Chao [1991] where Figure 2.2 shows Engel curves for 62 products). On the other hand, many products have virtually straight Engel curves over a considerable middle range of total expenditure where most households find themselves. Thus, one gets the impression that while Slutsky symmetry is certainly not a necessary property of market demand curves, it probably does no great violence to reality to impose it to reduce the number of parameters to be estimated.

3. A Perhaps Adequate Form

My earlier article introduced a form with a multiplicative relation between the income terms and the price terms. Its general form is:

$$x_i(t) = (a_i(t) + b_i(y/P)) \prod_{k=1}^{n} p_k^{c_{ik}} \tag{8}$$

where the left side is the consumption *per capita* of product i in period t and $a_i(t)$ is a function of time. The b_i is a positive constant. The y is nominal income *per capita*; p_k is the price index of product k; P is an overall price index defined by

$$P = \prod_{k=1}^{n} p_k^{s_k} \tag{9}$$

where s_k is the budget share of product k in the period in which the price indexes are all 1, and the c_{ik} are constants satisfying the constraint

$$\sum_{k=1}^{n} c_{ik} = 0. \tag{10}$$

Any function of this form is homogeneous of degree 0 in all prices and income and satisfies all of the properties set out in the first paragraph. It has three problems:

1 It is not certain that expenditures will add up to income.

2 There is no way to choose the parameters to guarantee Slutsky symmetry *at all prices* if we want to. We can, however, arrange to have symmetry in some particular base period. As long as the shares of various products in total expenditure do not change very much from those of that base period, we will continue to have approximate symmetry.

3 There are a lot of c's to be estimated.

Problem 1 can be easily fixed by adding on a "spreader," that is, by summing all expenditures, comparing them with y, and allocating the difference in proportion to the marginal propensities to consume with respect to y at the current prices. The amount to be spread is usually small and the form with spreader has essentially the same properties as the form without, plus the adding up property. We need not complicate the mathematics here by adding the spreader, but in practice it should be added when the equations are used in forecasting.

Problem 2, in view of section 2, is more a cautionary note than a real problem. Symmetry in a base year is probably quite adequate.

Problem 3 -- which occurs in all forms which provide for varying degrees of substitution and complementarity -- can be quite severe. If we have 80 categories of expenditures, we have 6,400 c's less the 80 determined by equation (10). If we have 20 years of annual data, we have 1,600 data points from which to determine these 5,600 parameters, or 3.5 parameters per data point! Clearly, we have to have employ some restrictions. Even if we had only one parameter per data point, we would probably want restrictions to insure reasonableness of the parameters. Indeed, the principal theoretical problem in consumption analysis is find ways to specify what is "reasonable."

Part of the solution of problem 3 can be found, if we wish, in the point noted in problem 2, namely that we can impose Slutsky symmetry at some prices. The Slutsky condition may be derived either from equation (2) or, more simply, by assuming that the compensating change in income is that which keeps y/P constant. Either approach gives as the symmetry condition equation (11).

$$c_{ij} \cdot x_i / p_j \;=\; c_{ji} \cdot x_j / p_i \tag{11}$$

Multiplying both sides by $p_i p_j / y$ gives equation (12).

$$c_{ij} / s_j \;=\; c_{ji} / s_i \tag{12}$$

If we then define

$$\lambda_{ij} \;=\; c_{ij} / s_j \tag{13}$$

then the form can be written as

$$x_i(t) = (a_i(t) + b_i(y/P)) \prod_{k=1}^{n} p_k^{\lambda_{ik} s_k} \tag{14}$$

where

$$\lambda_{ij} = \lambda_{ji}. \tag{15}$$

This restriction cuts the number of parameters by a half. That reduction is a big help but is clearly insufficient.

Further help with this problem can be found through the idea of groups and subgroups of commodities. The accompanying box shows an example with fifteen basic commodity categories. These are subdivided into three groups and several categories which are not in any group. The first group is divided into two subgroups; the second, into one subgroup and a category not in the subgroup; the third group has no subgroup.

Illustration of Groups and Subgroups

	Product	Group	Subgroup
1.	Meat	I	A
2.	Fish	I	A
3.	Dairy products	I	A
4.	Cereal products	I	B
5.	Fruits and vegetables	I	B
6.	Other food products	I	B
7.	Automobiles	II	C
8.	Gasoline and oil	II	C
9.	Tires, batteries, repair	II	C
10.	Public transportation	II	
11.	Clothing	III	
12.	Shoes	III	
13.	Other durables		
14.	Other non-durables		
15.	Other services		

The idea of the earlier article was to assume that $\lambda_{ij} = \lambda_o$ if i and j are not members of the same group or subgroup, while if they are in the same group, G, $\lambda_{ij} = \lambda_o + \mu'_G$, and if they are in the same subgroup, g, of the group G, $\lambda_{ij} = \lambda_o + \mu'_G + \nu'_g$. Thus, there were as many parameters to estimate as there were groups + subgroups + 1. Estimation was fairly simple because, given a value of λ_o, estimation of the other parameters had to involve only products within the same group or subgroup. Several values of λ_o were chosen, all equations estimated, and the value of λ_o chosen which gave the best over-all fit.

The problem with this form was that products which had no natural partners with which to form a group all ended up either in very strange groups or, if they were given no group at all, all with nearly the same own price elasticity, namely $-\lambda_o$. It is often difficult to find groups for such goods as Telephone service, Medical service, Education, or Religious services. A specification which forces them all to have, for that reason, nearly the same own price elasticity is certainly inadequately flexible.

An adequate form, it now seems, should allow every product to have its own own-price elasticity. We will then have as many price-exponent parameters as there are products plus groups plus subgroups. A simple way to achieve this generalization is to introduce n parameters, $\lambda_1, \ldots, \lambda_n$, and use them to define the λ_{ij} as follows. If i and j are not members of the same group or subgroup, then

$$\lambda_{ij} = \lambda_i + \lambda_j \tag{16}$$

while if they are in the same group, G, $\lambda_{ij} = \lambda_i + \lambda_j + \mu'_G$, and if they are in the same subgroup, g, of the group G, $\lambda_{ij} = \lambda_i + \lambda_j + \mu'_G + \nu'_g$. The definitions apply only for i not equal to j. The λ_{ii} are each determined by equation (10), the homogeneity requirement.

Using these definitions, for product i, a member of group G and subgroup g, the equation becomes

$$x_i(t) = (a_i(t) + b_i(y/P)) \prod_{k \neq i}^{n} p_k^{(\lambda_i + \lambda_k)s_k} \prod_{k \in G, k \neq i} p_k^{s_k \mu'} \prod_{k \in g, k \neq i} p_k^{s_k \nu'} \cdot p_i^{c_{ii}} \tag{17}$$

Equation (10) requires

$$\sum_{k \neq i} \lambda_k s_k + \lambda_i \sum_{k \neq i} s_k + \mu'_G \sum_{k \in G, k \neq i} s_k + \nu'_g \sum_{k \in g, k \neq i} s_k + c_{ii} = 0. \tag{18}$$

If we solve this equation for c_{ii} and substitute in equation (17), we obtain, after a bit of simplification,

$$x_i(t) \;=\; (a_i(t)+b_i(y/P))(p_i/P)^{-\lambda_i}\prod_{k=1}^{n}(p_k/p_i)^{\lambda_k s_k}\left(\prod_{k\in G}(p_k/p_i)^{s_k}\right)^{\mu'_G}\left(\prod_{k\in g}(p_k/p_i)^{s_k}\right)^{\nu'_g} \tag{19}$$

where we have inserted the terms involving p_i/p_i into all of the products, because this term is always 1.0 no matter to what power it is raised. We can make the form even simpler by introducing price indexes for the group G and subgroup g defined by

$$P_G \;=\; \left(\prod_{k\in G} p_k^{s_k}\right)^{1/\sum_{k\in G} s_k} \quad and \quad P_g \;=\; \left(\prod_{k\in g} p_k^{s_k}\right)^{1/\sum_{k\in g} s_k} \tag{20}$$

We then obtain simply equation (21)

$$x_i(t) \;=\; \left(a_i(t)+b_i(y/P)\right)\cdot\left(\frac{p_i}{P}\right)^{-\lambda_i}\prod_{k=1}^{n}\left(\frac{p_i}{p_k}\right)^{-\lambda_k s_k}\cdot\left(\frac{p_i}{P_G}\right)^{-\mu_G}\left(\frac{p_i}{P_g}\right)^{-\nu_g} \tag{21}$$

where

$$\mu \;=\; \mu'\sum_{k\in G} s_k \quad and \quad \nu \;=\; \nu'\sum_{k\in g} s_k \tag{22}$$

This is the form for estimation. Note that it has one parameter, a λ, for each good, plus one parameter, a μ, for each group, plus one parameter, a ν, for each subgroup. Thus, it appears to have an adequate number of parameters. The Slutsky symmetry of (21) at the initial prices and income may be verified directly by taking partial derivatives of (21).

A special case of some interest arises when all the λ_i are the same and equal to $\lambda_o/2$, for in that case equation (21) simplifies to

$$x_i(t) \;=\; \left(a_i(t)+b_i(y/P)\right)\cdot\left(\frac{p_i}{P}\right)^{-\lambda_o}\left(\frac{p_i}{P_G}\right)^{-\mu_G}\left(\frac{p_i}{P_g}\right)^{-\nu_g} \tag{23}$$

which is exactly the form suggested in the earlier article. Thus, the present suggestion is a simple generalization of the earlier one.

In practice, there are apt to be a few commodities, such as Tobacco, Sugar, or Medical care which show so little price sensitivity that they cannot be fit well by this system. For them, we will assume that all the λ_{ij} in their rows and columns are 0. Note that this assumption is perfectly consistent with the symmetry of the lambda's. When there are such "insensitive" commodities in the system, equation (21) is modified in two ways. For these items, there are no price terms at all, while for other items the product term which in (21) is shown with k running from 1 to n is modified so that k runs only over the "sensitive" and not the "insensitive" commodities.

It is useful in judging the reasonableness of regression results to be able to calculate the compensated own and the cross price elasticities. ("Compensated" here means that y has been increased so as to keep y/P constant.) Their derivation is straight-forward but complicated enough to make the results worth recording. In addition to the notation already introduced, we need

u_{ij} = the share in the base year of product j in the group which contains product i, or 0 if i is not in a group with j.

w_{ij} = the share in the base year of product j in the subgroup which contains product i or 0 if i is not in a subgroup with j.

μ_i = the μ for the group which contains product i, or 0 if i is not in a group. (Note that μ_i is the same for all i in the same group.)

ν_i = the ν for the subgroup which contains product i, or 0 if i is not in a subgroup. (Similarly, note that η_i is the same for all i in the same subgroup.)

L = The share-weighted average of the λ_i:

$$L = \sum_{k=1}^{n} \lambda_k s_k \tag{24}$$

The compensated own price elasticity of product i is then

$$\eta_{ii} = -\lambda_i(1-s_i) - L + \lambda_i s_i - \mu_i(1-u_{ii}) - \nu_i(1-w_{ii}) \tag{25}$$

while the cross price elasticity, the elasticity of the demand for good i with respect to the price of good j, is

$$\eta_{ij} = \lambda_i s_j + \lambda_j s_j + u_{ij}\mu_i + w_{ij}\nu_i \tag{26}$$

Two tables are produced by the estimation program. One shows, for each product, its share in total expenditure in the base year, the group and subgroup of which it is a member and its share in them, its λ and the μ and ν of its subgroups, its own price elasticity, and various information on the income parameters. Thus, it contains all the data necessary for calculating any of the cross elasticities. It is small enough to be reasonably reproduced. The other table shows the complete matrix of own and cross elasticities. It is generally too large to be printed except in extract.

It should be noted that the complexity in estimating equation (21) comes from term indicated by the product sign. Without this term, the equation could be estimated separately for each product or group of products. On the other hand, it is this term which gives Slutsky symmetry at the base point. If one did not care about this symmetry, then this term could omitted from the equation, with a great reduction in complexity in estimation. Once the programming has been done to estimate with this term, however, it is little trouble to use the program.

So far, we have said little about the "income" term, the term within the first parenthesis of equation (21). In the equations reported below we have used just a constant, real income per capita, the first difference of real income per capita, and a linear time trend. Furthermore, we have used the same population measure, total population, for computing consumption per capita for all items. The estimation program, however, allows much greater diversity. By use of adult-equivalency weights, different weighted populations can be used for computing the per capita consumption of different items. Further, if the size distribution of income is known, it can be used to compute income-based indicators of consumption more appropriate to each item than just average income. Thus, the program allows a different income variable to be used for each consumer category. Finally, instead of just a linear time trend, one can use a "trend" variable appropriate to a particular category. For example, the percentage of the population which smokes could be used in explaining spending on tobacco. The estimation program allows for all these possibilities. On the other hand, in view of this diversity, it seemed pointless to try to place constraints on the parameters of the income terms to make the income terms add up to total income. Instead, in applying the estimated functions, one should calculate the difference between the assumed total expenditure and that implied by the equations and allocate it to the various items.

4. The Mathematics of Estimation

The function in equation (21) is nonlinear in all its parameters. In a system with 80 consumption categories there will be over 400 parameters involved in the simultaneous non-linear estimation. This size makes it worthwhile to note in this section some simplifying structure in the problem. All non-linear estimation procedures take some guess of the parameters, evaluate the functions with these values to obtain vectors of predicted values, $\hat{x}_i$, and subtract these from the vectors of observed values, x_i, to obtain vectors of residuals, r_i, thus:

$$r_i = x_i - \hat{x}_i$$

They then, in some way, pick changes in the parameters, and re-evaluated the function with the new values. The only difference in the various methods is how the changes in the parameters are picked. The Marquardt algorithm, which we use, is very nearly the same as regressing the residuals on the partial derivatives of the predicted values with respect to the parameters. It requires, in particular, these derivatives. For equation (21), they are reasonably easy to calculate if one remembers the formula from the table of derivatives:

$$\frac{d\,a^x}{dx} = a^x \ln a \tag{28}$$

where ln denotes the natural logarithm. Then for the derivative of the demand for the i^{th} good with respect to its own lambda is

$$\frac{\partial \hat{x}_i}{\partial \lambda_i} = \hat{x}_i \left(\ln \left(\frac{\prod p_i^{s_k}}{p_i} \right) \right) = \hat{x}_i \; (\sum s_k \ln p_k - \ln p_i) \tag{29}$$

and for j not equal to i

$$\frac{\partial \hat{x}_i}{\partial \lambda_j} = \hat{x}_i \ln \left(\frac{p_j}{p_i} \right) s_j = \hat{x}_i \; (\ln p_j - \ln p_i) s_j \tag{30}$$

and if i is a member of the group G

$$\frac{\partial \hat{x}_i}{\partial \mu_G} = \hat{x}_i \; \ln \left(\frac{P_G}{p_i} \right) = \hat{x}_i \; (\ln P_G - \ln p_i) \tag{31}$$

and if further i is a member of the subgroup g

$$\frac{\partial \hat{x}_i}{\partial \nu_g} = \hat{x}_i \; \ln \left(\frac{P_g}{p_i} \right) = \hat{x}_i \; (\ln P_g - \ln p_i) \tag{32}$$

To explain the estimation process, we shall denote the vector of parameters of the "income-and-time term," the term preceding the first dot in equation (21), for product i by $\mathbf{a_i}$ and the vector of parameters of the "price term", the rest of the formula, by **h**. Thus, **h** consists of all values of λ, μ, and ν. Note that **h** is the same for all products, though a particular μ or ν may not enter the equation a given commodity. If we let $\mathbf{A_i}$ be the matrix of partial derivatives of the predicted values for product i with respect to the $\mathbf{a_i}$ and similarly let $\mathbf{B_i}$ be the matrix of partial derivatives of the predicted values of product i with respect to **h**, and finally let $\mathbf{r_i}$ be the residuals, all evaluated at the current value of the parameters, then the regression data matrix, (X,y) in the usual notation, for three commodities is:

$$(X,y) = \begin{pmatrix} A_1 & 0 & 0 & B_1 & r_1 \\ 0 & A_2 & 0 & B_2 & r_2 \\ 0 & 0 & A_3 & B_3 & r_3 \end{pmatrix} \tag{33}$$

If we now form the normal equations, X’Xb = X’y in the usual notation, we find

$$\begin{pmatrix} A_1^{/} A_1 & 0 & 0 & A_1^{/} B_1 \\ 0 & A_2^{/} A_2 & 0 & A_2^{/} B_2 \\ 0 & 0 & A_3^{/} A_3 & A_3^{/} B_3 \\ B_1^{/} A_1 & B_2^{/} A_2 & B_3^{/} A_3 & \sum_{i=1}^{3} B_i^{/} B_i \end{pmatrix} \begin{pmatrix} da_1 \\ da_2 \\ da_3 \\ dh \end{pmatrix} = \begin{pmatrix} A_1^{/} r_1 \\ A_2^{/} r_2 \\ A_3^{/} r_3 \\ \sum_{i=1}^{3} B_i^{/} r_i \end{pmatrix} \tag{34}$$

After initial values of the parameters have been chosen and the functions evaluated with these values and the sum of squared residuals (SSR) calculated, the Marquardt procedure consists of picking a scalar, which we may call M, and following these steps:

1. Compute the matrices of equation 34, multiply the diagonal elements in the matrix on the left by 1 + M and solve for the changes in the $\mathbf{a_i}$ and **h** vectors. Make these changes and evaluate the functions at the new values.

2. If the SSR has decreased, divide M by 10 and repeat step 1.

3. If the SSR has increased, multiply M by 10, go back to the values of the parameters before the last change, evaluate the functions again at these values, and repeat step 1.

The process is stopped when very little reduction in the SSR is being achieved and the changes in the parameters are small. (As M rises, the method turns into the steepest descent method, which can usually find a small improvement if one exists, while as M diminishes, the method turns into Newton’s method, which gives rapid convergence when close enough to a solution that the quadratic approximation is good.)

To economize on space in the computer and to speed the calculations, we can take advantage of the structure of the matrix on the left side of equation (34). To do so, let $\mathbf{Z_i}$ be the inverse of $\mathbf{A_i'A_i}$. Then by Gaussian reduction (34) can be transformed into

$$\begin{pmatrix} I & 0 & 0 & Z_1 A_1' B_1 \\ 0 & I & 0 & Z_2 A_2' B_2 \\ 0 & 0 & I & Z_3 A_3' B_3 \\ 0 & 0 & 0 & \sum_{i=1}^{3} B_i' B_i - B_i' A_i Z_i A_i' B_i \end{pmatrix} \begin{pmatrix} da_1 \\ da_2 \\ da_3 \\ dh \end{pmatrix} = \begin{pmatrix} Z_1 A_1' r_1 \\ Z_2 A_2' r_2 \\ Z_3 A_3' r_3 \\ \sum_{i=1}^{3} B_i' r_i - B_i' A_i Z_i A_i' r_i \end{pmatrix} \tag{35}$$

The columns of the matrix on the left which are just columns of the identiy matrix do not need to be stored in the computer. Instead, the program computes the terms in the last column of this matrix and in the vector on the right, stores only them, and at the same time builds up the sums in the lower right corner of the matrix and in the bottom row of the vector on the right. Once the matrix and vector of equation (35) are ready, the program solves the equations in the last row for **dh** and then substitutes back into the other equations to solve them for the $\mathbf{da_i}$.

The estimation program initializes the income parameters by regressing the dependent variables on the just the constant, income, and trend terms. Then all lambda's are started at .25 and all mu and nu at 0. The program was written in Borland C++ 4.5 with DOS extender and with a double-precision version of the BUMP library of matrix and vector objects and operators. The required to do the estimation seems to be roughly proportional to the fourth power of the number of sectors. Evaluating the B matrices and taking B'B grows roughly with the cube of the number of sectors, so the time required for a single iteration grows with the cube. The number of iterations, however, seems to grow at least linearly with the number of sectors, so the total time required should grow with the fourth power of the number of sectors. Thus, a 90-sector study can be expected to take about 16 times as long to estimate as a 45-sector study. This is roughly what we have experienced, with the 93-sector USA system requiring about 100 minutes and the 42-sector Spanish study five or six minutes on a 133 MHz pentium. The USA study required about 120 iterations. The big drops in the objective function started to appear after about 80 iterations.

5. Results for Spain

The system has been applied thus far to Italian, French, Spanish, and USA data. We will look at the Spanish and USA results here. The Spanish data had 42 sectors, four of which covered medical items which are generally paid for by third parties, generally the state medical system. Thus, these items do not really enter into the consumer's budget constraint, so it is hardly surprising that including them in the system causes trouble. Their relative prices have been rising but the amount spent on them has also been going up so that it was hardly surprising that they came out with strong positive price elasticities. In the nature of our system, these price parameters then affect demands for all other goods and, needless to say, make trouble. These products were therefore treated as "insensitive," and the remaining problems became much more manageable. Sugar had a graph that showed that it had been insensitive to price changes, so it was also put in the "insensitive" group, thus cutting its error in half.

Table 1 shows the results of applying to system to the remaining categories. The data extended from 1971 to 1994, providing 23 annual observations. The "income" variables were a constant, income, change in income, and time. The first column of Table 1, labeled G, is the number of the group to which a product was assigned. Group 1 is food, 2 is beverages, 3 is clothing and shoes, 4 is household durables, 5 is transportation, and 6 is recreation. Fourteen products were not assigned to a group, and have a 0 in this first column. The second column, labeled S, is the subgroup. Only two subgroups were used, 1 for protein-source foods -- meat, fish, and milk products -- and 2 for private transportation. The values for the mu and nu for these groups and subgroups appear at the top of the table. The following four columns of integers indicate:

P the population used for the category

C the income (Cstar) variable used.

T the time trend variable used.

I whether or not the category is "included" in the price sensitive group (1 = yes).

The column labeled "lamb" shows the values of the lambda for each sector, while "share" gives the share in the base year. The "IncEl" column reports the income elasticity in 1988. The "DInc" column shows the coefficient on the change in income divided by income coefficient. Thus, a value of .5 here means that the coefficient on the change in income was half the value of the income coefficient. The "time%" column presents the annual change due to the time trend expressed as a percentage of the 1988 value. The "PrEl" column gives the own price elasticity. The "Err%" column reports the standard error of estimate as a percentage of the 1988 value, and the "rho" column shows the autocorrelation coefficient of the residuals.

Table 1 shows the results of just estimating the system without any sort of constraint on the values of the coefficients. The fits, as indicated by the Err% column are fairly good, and the graphs (not reproduced here) look good. But there are many problems with the reasonableness of the coefficients. What constitutes reasonableness here? In the first place, the own price elasticities, since they are "compensated," should all be negative. About half are positive. The income elasticities for these broad groups should be positive or zero, but four are negative and offset by a positive time coefficient. Finally, if the DInc column shows a value below -1.0 for an item, an increase in income will reduce spending on this category during the first year of the higher income. Since that effect seems implausible, there are a number of problems here also.

It is, in fact, hardly to be expected that all parameters would come out with reasonable values when so many of the variables have similar trends. Thus the use of soft constraints on the coefficients is an integral part of the estimation process. The estimation program allows the user to specify the desired value of any parameter except the constant term and to specify a "trade-off parameter" to express the user’s trade-off between closeness of fit and conformity with desired values of the parameters. For the Spanish study, I first worked on the own price elasticities to get them all negative. Then there were a number of negative income elasticities, so I put in constraints to make them positive. Frequently, they could be made positive by constraining the time trend towards zero. Finally, some of the coefficients on the change in income had to be

Table 1. Results for Spain. No constraints

The value of L is 0.07
The mu: 0.10 -0.90 -0.33 -1.97 0.95 -1.84
The nu: 0.14 -1.87

nsec	title	G	S	P	C	T	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
1	Pan y cereales	1	0	1	1	1	1	-0.34	0.026	0.21	-0.95	-0.18	0.16	2.26	0.55
2	Carne	1	1	1	1	1	1	-0.67	0.066	0.43	-2.22	1.53	0.37	2.04	-0.10
3	Pescado	1	1	1	1	1	1	0.89	0.024	0.40	1.82	0.14	-1.12	4.97	0.72
4	Leche, queso y huevos	1	1	1	1	1	1	-0.52	0.033	-0.10	2.65	1.94	0.22	2.19	0.39
5	Aceites y grasas	1	0	1	1	1	1	0.11	0.011	0.02	-8.80	-0.95	-0.27	2.63	0.57
6	Frutas y verduras	1	0	1	1	1	1	0.30	0.033	0.68	0.45	-1.04	-0.44	3.27	0.49
7	Patatas y tubérculos	1	0	1	1	1	1	-0.36	0.005	0.27	-3.16	-1.54	0.19	4.77	0.39
8	Azúcar	0	0	1	1	1	0	0.00	0.002	-0.21	0.26	0.12	0.00	2.86	0.65
9	Café, té y cacao	2	0	1	1	1	1	0.91	0.006	0.49	0.34	-2.30	-0.29	4.82	0.44
10	Otros alimentos	1	0	1	1	1	1	-0.48	0.007	0.35	-0.45	1.43	0.30	3.08	0.78
11	Bebidas no alcohólicas	2	0	1	1	1	1	-2.27	0.005	1.10	-0.41	-0.03	2.91	8.02	0.65
12	Bebidas alcohólicas	2	0	1	1	1	1	-0.24	0.014	-0.48	0.30	-0.49	0.54	3.71	0.40
13	Tabacos	0	0	1	1	1	1	-0.09	0.016	0.31	-1.44	0.61	0.01	3.03	0.32
14	Vestido	3	0	1	1	1	1	0.64	0.064	1.03	0.35	-0.91	-0.54	1.83	-0.01
15	Calzado	3	0	1	1	1	1	0.42	0.024	1.00	-0.26	-1.61	-0.23	2.36	0.05
16	Alquileres y agua	0	0	1	1	1	1	0.34	0.111	-0.04	20.31	1.54	-0.34	2.12	0.54
17	Calefacción y alumbrado	0	0	1	1	1	1	-0.20	0.025	0.92	-0.22	2.04	0.12	2.36	0.62
18	Muebles	4	0	1	1	1	1	1.36	0.021	1.55	-0.43	-4.06	-0.32	3.17	0.47
19	Artículos textiles	4	0	1	1	1	1	1.27	0.009	0.74	-1.40	0.06	0.24	4.62	0.64
20	Electrodomésticos	4	0	1	1	1	1	0.92	0.010	0.99	0.11	2.42	0.55	3.01	0.50
21	Utensilios domésticos	4	0	1	1	1	1	1.32	0.005	1.67	-1.85	-5.05	0.39	8.58	0.61
22	Mantenimiento	0	0	1	1	1	1	0.53	0.015	0.76	0.68	-0.82	-0.58	4.08	0.69
23	Servicio doméstico	0	0	1	1	1	1	0.94	0.007	1.32	-0.74	-1.47	-1.00	6.37	0.65
24	Medicamentos	0	0	1	1	1	0	0.00	0.016	3.19	-0.83	-1.70	0.00	16.39	0.85
25	Aparatos terapéuticos	0	0	1	1	1	0	0.00	0.003	1.22	-1.18	0.57	0.00	10.03	0.84
26	Servicios médicos	0	0	1	1	1	0	0.00	0.010	1.84	-0.74	-0.55	0.00	9.73	0.84
27	Atención hospitalaria	0	0	1	1	1	0	0.00	0.003	1.07	-1.41	-0.76	0.00	6.86	0.75
28	Seguro médico privado	0	0	1	1	1	1	-0.72	0.003	0.81	-1.82	-4.04	0.64	7.06	0.55
29	Compra de vehículos	5	2	1	1	1	1	-0.06	0.036	1.88	0.47	1.83	0.56	6.79	0.17
30	Gasto de uso de vehícul	5	2	1	1	1	1	0.33	0.075	0.89	0.23	0.88	-0.14	1.35	-0.04
31	Servicios de transporte	5	0	1	1	1	1	-1.08	0.018	0.49	1.82	0.95	0.15	3.12	0.30
32	Comunicaciones	0	0	1	1	1	1	-0.04	0.008	1.29	-0.19	2.92	-0.04	4.09	0.46
33	Artículos de esparcimie	6	0	1	1	1	1	1.29	0.024	1.43	-0.49	-0.09	-0.54	2.87	0.20
34	Servicios de esparcimie	6	0	1	1	1	1	1.34	0.017	0.54	-0.24	-2.71	-0.29	3.86	0.72
35	Libros, periódicos y re	0	0	1	1	1	1	0.20	0.008	0.59	-1.65	0.28	-0.27	4.64	0.68
36	Enseñanza	0	0	1	1	1	1	-0.03	0.019	0.72	0.10	-0.95	-0.05	2.73	0.70
37	Cuidados y efectos pers	0	0	1	1	1	1	-0.94	0.014	0.62	-0.89	1.32	0.84	2.97	0.30
38	Otros artículos n.c.o.p	0	0	1	1	1	1	0.15	0.013	0.40	-2.06	2.26	-0.22	3.76	0.32
39	Restaurantes cafés y ho	0	0	1	1	1	1	-0.45	0.153	0.45	0.50	1.48	0.24	1.28	0.22
40	Viajes turísticos todo	0	0	1	1	1	1	-0.76	0.006	0.42	0.41	-1.09	0.68	3.73	0.44

constrained to keep them from being more negative that the income term is positive.

The values of the soft constraint parameters are shown in Table 2. For each product, there can be specified desired values of the income elasticity, the change in income in elasticity units, the time trend as a percent of the base year (1988) value, lambda, and the mu and nu of the group and subgroup. The table shows for each of these a pair of numbers, the desired value and the trade-off parameter. If the trade-off parameter is 0, the desired value has no effect on the estimation. The higher the parameter, the stronger the constraint relative to the data. A value of 1.0 for the trade-off parameter gives about equal weight to the constraint and to the data. Constraints on mu and nu values can be specified on the line for any member of the group or subgroup, but I have always placed them on the line of the first item in the group or subgroup. This table is, in fact, precisely the way the constraints are entered into the program; the table shows the contents of the file softcon.dat, which is read by the estimation program.

Table 2. Soft Constraints

#sec	Title	Income		DIncome		Time		lambda		mu		nu	
1	Pan y cereales	0	0	0	0	0	0	.25	10.	.25	1.		
2	Carne	0	0	0	0	0	0	.25	1.	0	0	.2	1.
3	Pescado												
4	Leche, queso y huevos	.1	1.	0	0	0	0	.25	1.				
5	Aceites y grasas	.1	1.	-.04	1.	0	0	.25	1.				
6	Frutas y verduras												
7	Patatas y otros tubérculos	.0	1.	-.02	1.	0	0	.25	1.				
8	Azúcar	.1	1.	-.06	1.	0	0						
9	Café, té y cacao	.05	1.	-.03	1	0	0	.25	1.	.25	1.		
10	Otros alimentos	0	0	0	0	0	0	.25	1.				
11	Bebidas no alcohólicas	0	0	0	0	0	0	.25	1.				
12	Bebidas alcohólicas	.05	1.	0	0	0	0	.25	1.				
13	Tabacos	0	0	-.2	1.	0	0	.25	1.				
14	Vestido	0	0	0	0	0	0	.25	1.				
15	Calzado												
16	Alquileres y gasto de agua	0	0	-.05	1.	0	1.0						
17	Calefacción y alumbrado	0	0	0	0	0	0	.25	1.				
18	Muebles	0	0	0	0	0	0	.25	0.	.25	1.		
19	Art. textiles para el hogar	0	0	0	0	0	0	.25	1.				
20	Electrodomésticos	0	0	0	0	0	0	.25	1.				
21	Utensilios domésticos	.1	1.	-.06	1.								
22	Mantenimiento	0	0	0	0	0	0	.25	1.				
23	Servicio doméstico	0	0	-1.2	1								
24	Medicamentos												
25	Aparatos terapéuticos												
26	Servicios médicos												
27	Atención hospitalaria												
28	Seguro médico privado	0	0	0	0	0	0	.25	1.				
29	Compra de vehículos	0	0	0	0	0	0	.25	2.	.1	1.	-.3	1.
30	Gasto de uso de vehículos	0	0	0	0	0	0	.25	1.				
31	Servicios de transporte	0	0	0	0	0	0	.25	1.				
32	Comunicaciones	0	0	0	0	0	0	.25	1.				
33	Artículos de esparcimiento	0	0	0	0	0	0	.25	0.	.25	1.		
34	Servicios de esparcimiento	0	0	-.30	1.	0	0						
35	Libros, periódicos y revist	0	0	-.25	1.	0	0	.25	1.				
36	Enseñanza	0	0	0	0	0	0	.25	1.				
37	Cuidados y efectos personal	0	0	0	0	0	0	.25	1.				
38	Otros artículos n.c.o.p.	0	0	-.40	1.	0	2.						
39	Restaurantes cafés y hotele	0	0	0	0	0	0	.25	1.				
40	Viajes turís. todo incluido	1.0	1.	0	0	0	0	.25	1.				

With these constraints, the system appears to have worked quite satisfactorily with results shown in Table 3. Aside from the medical sectors, whose peculiar nature has already been noted, only one sector, Financial Services, shows a standard error above 7 percent. This is a sector which showed huge changes in connection with Spain's entry into the Common Market in 1985. This event had pronounced effects on a number of sectors, such as Clothing (Vestidos) and Furniture (Meubles), whose graphs are shown at the end of this paper. The decided V-shaped course of these items -- which was not due to a V in total spending -- gave a good test of the functional form. All income and price elasticities in Table 3 are of the right sign and generally plausible values. Time trends are under three percent per year for all but the last two items. The results also show that the new features of this form are used. Values of λ_i range from -.49 to +1.15, quite different from the constant λ_o of the earlier form. Table 4 shows cross elasticities in two groups, each with a subgroup. Here again we see that the features of the form are being used. All members of the Food group are rather weakly substitutes with one another, but the three elements of the Protein subgroup (2, 3, and 4) are strongly substitutes. The two elements of the Private transportation subgroup (29 and 30), on the other hand, are strongly complementary, while both are substitutes with Public transportation (30). Among products not in the same group or subgroup, the cross-price elasticities are mainly positive (substitutes) and quite small, less than

Table 3. Results for Spain -- with Soft Constraints

The value of L is 0.20
The mu: 0.14 -0.17 0.10 0.12 -0.04 0.27
The nu: 0.20 -0.42

nsec	title	G	S	P	C	T	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
1	Pan y cereales	1	0	1	1	1	1	0.23	0.026	0.45	-0.37	-0.84	-0.54	2.55	0.42
2	Carne	1	1	1	1	1	1	0.12	0.066	0.56	-0.16	-0.82	-0.49	3.28	0.53
3	Pescado	1	1	1	1	1	1	-0.49	0.024	0.32	-0.37	-0.37	-0.01	4.72	0.78
4	Leche, queso y huevos	1	1	1	1	1	1	0.05	0.033	0.06	1.89	0.45	-0.50	4.01	0.77
5	Aceites y grasas	1	0	1	1	1	1	-0.03	0.011	0.07	-0.82	-1.08	-0.30	3.56	0.74
6	Frutas y verduras	1	0	1	1	1	1	-0.12	0.033	0.50	0.23	-0.99	-0.20	3.17	0.57
7	Patatas y tubérculos	1	0	1	1	1	1	-0.13	0.005	0.03	-0.74	-2.64	-0.21	7.33	0.66
8	Azúcar	0	0	1	1	1	0	0.00	0.002	0.09	-0.65	-0.48	0.00	5.18	0.88
9	Café, té y cacao	2	0	1	1	1	1	0.11	0.006	0.05	-0.73	-0.70	-0.18	3.73	0.69
10	Otros alimentos	1	0	1	1	1	1	0.04	0.007	0.30	0.67	0.79	-0.37	2.14	0.65
11	Bebidas no alcohólicas	2	0	1	1	1	1	0.14	0.005	1.27	-0.36	-1.36	-0.19	3.31	0.60
12	Bebidas alcohólicas	2	0	1	1	1	1	-0.07	0.014	0.09	5.09	-1.37	-0.05	4.87	0.67
13	Tabacos	0	0	1	1	1	1	-0.04	0.016	0.33	-0.63	0.14	-0.16	2.50	0.41
14	Vestido	3	0	1	1	1	1	0.22	0.064	1.05	0.18	-1.27	-0.42	2.25	0.22
15	Calzado	3	0	1	1	1	1	-0.11	0.024	0.99	-0.53	-1.93	-0.17	2.78	0.24
16	Alquileres y agua	0	0	1	1	1	1	0.76	0.111	0.16	-0.99	0.13	-0.79	5.19	0.91
17	Calefacción y alumbrado	0	0	1	1	1	1	0.00	0.025	0.58	0.21	2.74	-0.20	2.50	0.53
18	Muebles	4	0	1	1	1	1	0.16	0.021	1.35	-0.63	-1.67	-0.42	3.13	0.40
19	Artículos textiles	4	0	1	1	1	1	0.15	0.009	1.16	0.47	-1.26	-0.44	5.05	0.77
20	Electrodomésticos	4	0	1	1	1	1	0.03	0.010	1.72	0.69	-2.21	-0.32	5.02	0.66
21	Utensilios domésticos	4	0	1	1	1	1	0.67	0.005	0.14	-0.50	-0.90	-0.97	6.76	0.64
22	Mantenimiento	0	0	1	1	1	1	0.13	0.015	0.84	0.24	-1.00	-0.32	3.23	0.64
23	Servicio doméstico	0	0	1	1	1	1	1.15	0.007	1.36	-0.89	-1.02	-1.33	6.81	0.63
24	Medicamentos	0	0	1	1	1	0	0.00	0.016	3.07	-0.89	-2.08	0.00	14.34	0.83
25	Aparatos terapéuticos	0	0	1	1	1	0	0.00	0.003	0.87	-1.90	1.20	0.00	7.55	0.75
26	Servicios médicos	0	0	1	1	1	0	0.00	0.010	1.78	-0.84	-0.50	0.00	8.38	0.82
27	Atención hospitalaria	0	0	1	1	1	0	0.00	0.003	1.02	-1.63	-0.44	0.00	5.87	0.68
28	Seguro médico privado	0	0	1	1	1	1	0.27	0.003	0.68	-0.82	1.46	-0.47	4.68	0.67
29	Compra de vehículos	5	2	1	1	1	1	0.18	0.036	1.63	1.13	1.61	-0.05	7.92	0.41
30	Gasto de uso de vehícul	5	2	1	1	1	1	0.25	0.075	0.85	-0.20	1.10	-0.25	1.76	0.22
31	Servicios de transporte	5	0	1	1	1	1	0.16	0.018	0.37	0.65	1.31	-0.32	1.70	0.01
32	Comunicaciones	0	0	1	1	1	1	0.08	0.008	1.08	-0.25	2.86	-0.28	3.17	0.35
33	Artículos de esparcimie	6	0	1	1	1	1	-0.08	0.024	1.47	0.06	0.05	-0.23	4.08	0.60
34	Servicios de esparcimie	6	0	1	1	1	1	0.68	0.017	0.56	-0.60	1.98	-1.02	2.97	0.71
35	Libros, periódicos y re	0	0	1	1	1	1	0.26	0.008	0.40	-0.71	1.06	-0.45	6.96	0.82
36	Enseñanza	0	0	1	1	1	1	0.17	0.019	0.78	-0.35	-1.10	-0.36	2.58	0.67
37	Cuidados y efectos pers	0	0	1	1	1	1	0.12	0.014	0.75	-0.82	0.97	-0.32	2.90	0.43
38	Otros artículos n.c.o.p	0	0	1	1	1	1	0.05	0.013	0.39	-1.12	2.25	-0.25	5.19	0.70
39	Restaurantes cafés y ho	0	0	1	1	1	1	0.18	0.153	0.51	0.78	2.65	-0.32	2.14	0.59
40	Viajes turísticos todo	0	0	1	1	1	1	0.25	0.006	0.87	0.44	2.87	-0.45	7.82	0.82

.01. No estimates of standard errors or t-statistics are shown, for though they could be easily calculated, they would be totally misleading in view of the soft constraints. It should be borne in mind here that we have no interest at all in "testing hypotheses" in the usual way. Our only question is whether we have a functional form which can do justice to the diversity of historical data with values of parameters which enable it to make reasonable forecasts. It appears that we do for Spain.

Table 4. Results for Spain -- Cross Price Elasticities

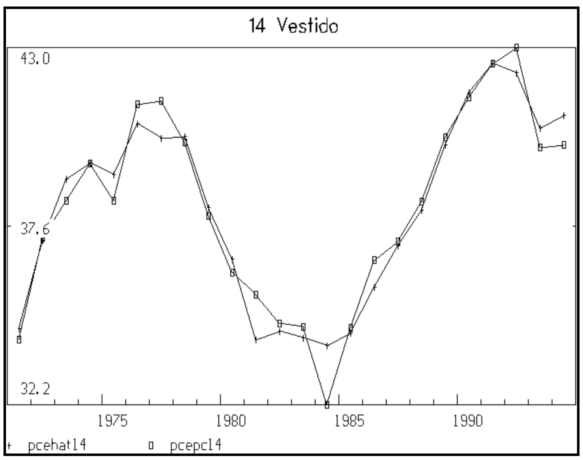
The number in row i and column j is the elasticity of product i with respect to the price of product j.

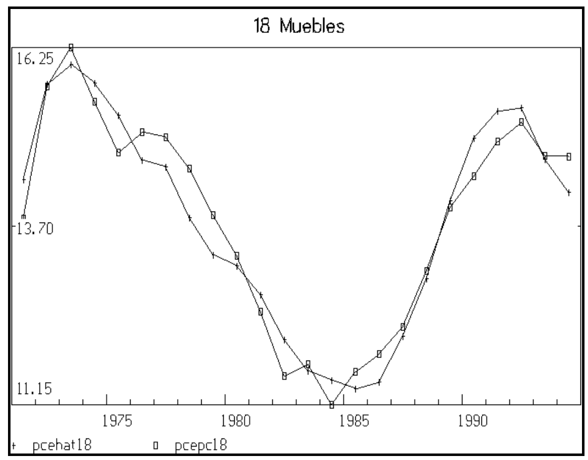
The Food Group

	1	2	3	4	5	6	7	8	9	10
1	-0.54	0.08	0.00	0.03	0.01	0.02	0.00	0.00	0.00	0.01
2	0.03	-0.49	0.02	0.08	0.01	0.02	0.00	0.00	0.00	0.01
3	0.02	0.14	-0.01	0.06	0.00	0.00	0.00	0.00	0.00	0.00
4	0.03	0.18	0.02	-0.50	0.01	0.02	0.00	0.00	0.00	0.01
5	0.03	0.06	-0.01	0.02	-0.30	0.01	0.00	0.00	0.00	0.01
6	0.03	0.05	-0.01	0.02	0.01	-0.20	0.00	0.00	0.00	0.00
7	0.03	0.05	-0.01	0.02	0.01	0.01	-0.21	0.00	0.00	0.00
8	0.01	0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
9	0.01	0.02	-0.01	0.01	0.00	0.00	0.00	0.00	-0.18	0.00
10	0.03	0.06	-0.01	0.03	0.01	0.01	0.00	0.00	0.00	-0.37

The Transportation group

	29	30	31
29	-0.05	-0.24	0.00
30	-0.12	-0.25	0.00
31	0.01	0.03	-0.32





6. Results for the USA

In the USA, a rich source of time-series data is available from the Bureau of Economic Analysis in an "unpublished" table with 325 categories. An idea of the detail is given by such categories as "pork", "eggs", "film developing," or "Watch, clock, and jewlry repair." The detail considerably exceeds what is needed to support the input-output model in which the functions were to be used. Comparison of the input-output sectors with the "unpublished" consumption sectors led to the identification the 93 sectors shown in Table 5. (Actually, there were 95 sectors, but two of these were negative items relating to purchases in the USA by foreign visitors.) A program was then written to take the sector definitions in terms of "unpublished" sector numbers and produce command files for the G regression program to aggregate the unpublished series and write data matrices in the form needed for input into the estimation program. The existence of this program makes it easy to experiment with different aggregations of the "unpublished" data.

Table 5. Results by for the USA

The value of L is 0.26

The mu: 0.29 -0.09 1.71 0.20 0.08 0.67 -0.13 -0.01 -0.20

The nu: -0.09 -0.28 -0.11 -0.41

nsec	title	G	S	P	C	T	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
1	Meat	1	1	1	1	1	1	0.12	0.014	0.09	-0.74	-2.58	-0.59	4.05	0.56
2	Dairy products	1	1	1	1	1	1	0.09	0.009	0.11	-0.64	-2.77	-0.55	5.98	0.74
3	Poultry and eggs	1	1	1	1	1	1	0.14	0.004	0.13	-0.50	0.10	-0.60	4.26	0.63
4	Fresh fruit and vegetab	1	2	1	1	1	1	0.23	0.008	-0.16	-0.10	-0.22	-0.57	6.88	0.83
5	Processed fruit and veg	1	2	1	1	1	1	0.01	0.006	-0.06	-1.42	-0.09	-0.33	6.27	0.86
6	Cereal and bakery produ	1	0	1	1	1	1	-0.03	0.014	0.08	-0.69	0.00	-0.49	4.16	0.78
7	Fats & oils	1	0	1	1	1	1	0.09	0.002	0.00	4.43	-0.84	-0.63	7.89	0.54
8	Sugar & sweets	1	0	1	1	1	1	0.01	0.006	-0.14	0.11	-0.31	-0.55	7.54	0.57
9	Nonalcoholic beverages	1	0	1	1	1	1	0.09	0.010	0.65	1.70	0.00	-0.61	4.82	0.63
10	Other prepared food	1	0	1	1	1	1	1.72	0.011	1.18	-0.32	1.00	-2.21	5.23	0.67
11	Fish & seafood	1	1	1	1	1	1	0.34	0.002	0.55	-0.93	-0.04	-0.80	11.49	0.78
12	Pet food	0	0	1	1	1	1	0.47	0.003	0.70	-0.48	0.51	-0.73	9.61	0.87
13	Wine and distilled spir	2	0	1	1	1	1	-0.14	0.005	0.04	-1.33	-0.27	-0.05	6.76	0.84
14	Beer & ale	2	0	1	1	1	1	1.06	0.009	0.24	1.31	0.02	-1.25	4.40	0.79
15	Food in purchased meals	1	0	1	1	1	1	0.17	0.050	0.93	-0.03	0.00	-0.60	2.30	0.56
16	Alcohol in purchased me	2	0	1	1	1	1	0.76	0.007	-0.09	6.76	-0.17	-0.95	4.41	0.91
17	Tobacco	0	0	1	1	1	1	0.19	0.015	0.00	-1.14	-1.09	-0.44	2.68	0.25
18	Footwear	3	0	1	1	1	1	0.14	0.008	0.71	-0.72	0.00	-1.91	4.70	0.73
19	Clothing, Women's & gir	3	0	1	1	1	1	-0.74	0.029	1.65	-0.37	0.00	-0.53	2.26	0.59
20	Clothing, Men's & boys'	3	0	1	1	1	1	0.07	0.015	0.75	-0.54	0.00	-1.67	2.24	0.43
21	Luggage	0	0	1	1	1	1	0.30	0.001	0.67	-0.62	-0.23	-0.56	7.05	0.57
22	Cleaning, laundering, &	3	0	1	1	1	1	0.12	0.003	1.38	-0.51	-0.01	-2.03	11.67	0.87
23	Jewelry & watches	3	2	1	1	1	1	-0.79	0.009	2.15	-0.61	0.01	-0.79	3.85	0.57
24	Watch repair; misc pers	3	2	1	1	1	1	-0.04	0.003	2.82	-0.59	0.02	-1.61	10.14	0.69
25	Toilet articles & preps	0	0	1	1	1	1	0.17	0.009	1.04	-0.51	0.00	-0.42	2.49	0.73
26	Barbershops, beauty, &	0	0	1	1	1	1	1.29	0.005	0.23	-0.51	-0.03	-1.54	6.07	0.69
27	Owner occupied nonfrm s	4	0	1	1	1	1	0.54	0.103	1.24	-0.86	0.00	-0.75	1.53	0.70
28	Tenant occupied nonfrm	4	0	1	1	1	1	0.14	0.039	0.87	-0.24	0.00	-0.53	2.59	0.71
29	Hotels & motels	0	0	1	1	1	1	1.10	0.005	1.29	-0.49	0.06	-1.35	5.72	0.78
30	Other housing	4	0	1	1	1	1	-0.18	0.003	-0.22	-0.02	-1.48	-0.27	4.83	0.93
31	Furniture	5	0	1	1	1	1	0.08	0.010	0.82	-0.52	-0.01	-0.41	4.34	0.55
32	Kitchen & hh appliances	5	0	1	1	1	1	0.06	0.006	0.15	-0.75	-0.03	-0.40	5.27	0.67
33	China & glaswr, tablwr	5	0	1	1	1	1	-0.05	0.004	1.12	-0.56	0.07	-0.28	4.16	0.70
34	Floor coverings and tex	5	0	1	1	1	1	0.03	0.005	1.40	-0.52	-0.04	-0.37	7.79	0.79
35	Clocks, lamps, & artwor	5	0	1	1	1	1	0.17	0.004	0.82	-0.53	0.06	-0.51	3.95	0.80
36	Hand tools	5	0	1	1	1	1	-0.15	0.001	1.77	-0.47	0.95	-0.19	6.05	0.81
37	Semi-durable house furn	5	0	1	1	1	1	0.42	0.006	1.88	-0.78	-1.93	-0.75	2.54	0.67
38	Cleaning, lighting, pap	5	0	1	1	1	1	0.44	0.010	0.74	-0.57	0.01	-0.76	2.79	0.66
39	Stationery, writing sup	0	0	1	1	1	1	0.10	0.003	1.40	-0.51	0.02	-0.36	3.58	0.74
40	Electricity	6	0	1	1	1	1	0.10	0.018	0.99	-0.50	0.00	-0.59	2.53	0.74
41	Natural gas	6	0	1	1	1	1	-0.31	0.007	-0.04	-0.45	-0.15	-0.46	4.77	0.39
42	Water & sanitary servic	0	0	1	1	1	1	0.06	0.008	1.26	-0.49	0.00	-0.32	3.07	0.79
43	Fuel oil & coal	6	0	1	1	1	1	-0.06	0.003	-0.57	0.03	-2.71	-0.79	27.03	0.91
44	Telephone & telegraph	0	0	1	1	1	1	0.36	0.013	1.66	-0.49	0.00	-0.61	2.83	0.78
45	Domestic services	0	0	1	1	1	1	1.47	0.002	0.69	-0.46	-5.76	-1.73	10.58	0.73
46	HH insurance premiums	0	0	1	1	1	1	-0.05	0.002	1.17	-0.67	-0.11	-0.21	5.09	0.62
47	Other household operati	0	0	1	1	1	1	0.13	0.007	1.01	-0.52	0.01	-0.39	2.42	0.33
48	Drug preparations and s	7	0	1	1	1	1	-0.11	0.017	1.34	-0.53	0.00	-0.03	3.17	0.70
49	Opthalmic & orthopedic	7	0	1	1	1	1	-0.04	0.003	2.12	-0.49	-0.09	-0.08	9.39	0.78
50	Physicians	7	0	1	1	1	1	0.08	0.036	1.62	-0.84	0.00	-0.23	4.27	0.77
51	Dentists	7	0	1	1	1	1	0.00	0.010	0.80	-0.69	0.00	-0.13	3.08	0.75
52	Other professional medi	7	0	1	1	1	0	0.20	0.013	3.23	-0.47	0.01	0.12	13.11	0.91
53	Hospitals	7	0	1	1	1	1	0.16	0.060	1.56	-0.59	0.00	-0.31	4.17	0.85
54	Nursing homes	7	0	1	1	1	1	0.10	0.010	2.21	-0.96	0.02	-0.23	5.41	0.77
55	Health insurance premiu	7	0	1	1	1	1	-0.02	0.010	0.97	-0.54	0.01	-0.11	3.42	0.80
56	Brokerage & invest coun	0	0	1	1	1	1	1.33	0.006	2.88	0.78	0.00	-1.57	9.20	0.46
57	Bank service charges	0	0	1	1	1	1	-0.04	0.007	2.01	0.07	0.01	-0.22	6.10	0.77
58	Imputed service charges	0	0	1	1	1	1	0.15	0.039	1.42	-0.23	0.00	-0.39	3.32	0.48
59	Expense of handlng life	0	0	1	1	1	1	1.02	0.013	1.97	-0.90	0.01	-1.25	5.34	0.39
60	Legal services	0	0	1	1	1	1	0.06	0.010	1.06	-0.49	0.00	-0.31	4.61	0.77
61	Funeral & burial expens	0	0	1	1	1	1	0.00	0.002	0.25	-0.86	-1.59	-0.26	4.77	0.72
62	Other personal business	0	0	1	1	1	1	0.15	0.004	0.76	-0.55	0.01	-0.41	3.18	0.84

Table 5 (continued)

nsec	title	G	S	P	C	T	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
63	New autos	8	3	1	1	1	1	0.08	0.026	0.25	3.72	0.00	-0.25	11.75	0.63
64	Net purchases of used a	8	3	1	1	1	1	-0.33	0.008	1.04	-0.51	-0.01	0.17	7.23	0.78
65	Other motor vehicles	8	3	1	1	1	1	0.11	0.014	2.58	-0.53	-0.01	-0.27	9.08	0.68
66	Tires & tubes, accessor	8	3	1	1	1	1	-0.18	0.007	1.45	-1.00	0.01	0.02	2.89	0.58
67	Auto repair, rental, le	8	3	1	1	1	1	1.03	0.022	1.44	-1.04	0.00	-1.15	3.23	0.57
68	Gasoline & oil	8	3	1	1	1	1	-0.04	0.027	0.30	-0.37	-0.39	-0.14	2.06	0.56
69	Tolls	8	3	1	1	1	1	-0.29	0.001	0.30	-0.70	-1.38	0.14	4.30	0.28
70	Net auto insurance prem	8	3	1	1	1	1	-0.04	0.007	0.35	-0.60	0.05	-0.11	3.86	0.83
71	Mass transit	8	4	1	1	1	1	0.02	0.001	0.27	-0.80	-2.61	0.08	5.80	0.68
72	Taxicab	8	4	1	1	1	1	1.39	0.001	0.32	-0.63	-6.28	-1.26	18.61	0.77
73	Intercity rail	8	4	1	1	1	1	1.09	0.000	0.31	-0.65	-1.03	-0.94	16.64	0.77
74	Intercity bus	8	4	1	1	1	1	-0.62	0.000	-0.02	0.75	-8.40	0.76	19.82	0.66
75	Airline	8	4	1	1	1	1	0.39	0.006	1.15	-0.96	-0.02	-0.48	4.77	0.80
76	Other transport service	8	4	1	1	1	1	1.00	0.001	2.08	-0.81	0.24	-0.87	6.39	0.72
77	Books & maps, Magazines	9	0	1	1	1	1	0.17	0.010	0.29	-0.77	0.00	-0.26	3.67	0.59
78	Toys, dolls, & games	9	0	1	1	1	1	0.09	0.004	1.70	-0.48	0.05	-0.17	3.33	0.68
79	Bicycles, motorcycles	9	0	1	1	1	1	0.00	0.001	0.23	-0.57	-1.53	-0.07	14.07	0.59
80	Cameras, film, and deve	9	0	1	1	1	1	0.08	0.003	0.88	-0.27	-0.03	-0.15	6.93	0.87
81	Guns, ammunition, sport	9	0	1	1	1	1	0.13	0.007	1.70	-0.49	0.00	-0.22	6.09	0.63
82	Electronic entertainmen	9	0	1	1	1	1	0.96	0.010	0.84	-0.49	0.00	-1.03	6.04	0.76
83	Home computers	9	0	1	1	1	0								
84	Flowers seeds & potted	9	0	1	1	1	1	0.12	0.003	1.55	-0.47	0.04	-0.19	4.66	0.65
85	Live entertainment, spo	9	0	1	1	1	1	0.06	0.004	0.53	-0.95	0.05	-0.13	3.98	0.51
86	Clubs & fraternal organ	9	0	1	1	1	1	0.20	0.002	1.85	-0.41	0.06	-0.27	6.20	0.82
87	Participant amusements	9	0	1	1	1	1	0.10	0.007	1.38	-0.52	0.04	-0.18	4.93	0.79
88	Other recreation	9	0	1	1	1	1	0.14	0.014	2.68	-0.50	0.00	-0.24	3.55	0.69
89	Higher education	0	0	1	1	1	1	0.02	0.012	0.89	-0.58	0.00	-0.28	3.14	0.87
90	Private lower education	0	0	1	1	1	1	0.10	0.005	0.30	-0.69	0.01	-0.36	1.78	0.30
91	Other education & resea	0	0	1	1	1	1	1.58	0.004	1.77	-0.47	0.04	-1.82	5.88	0.77
92	Religious & welfare	0	0	1	1	1	1	1.52	0.023	1.81	-0.50	0.00	-1.71	2.77	0.56
93	Foreign travel, purchas	0	0	1	1	1	1	1.61	0.012	0.73	-0.55	0.00	-1.84	3.93	0.25

One of the sectors was Home computers. This item gave problems for a number of reasons and was excluded from the system. Initially, I had intended to begin the fitting of the equations in 1970 and extend it through 1995. The equations fit relatively badly in the first five years, quite possibly because of the experiments in price control during those years. Consequently, the fit period was reduced to 1975 - 1995.

The initial, unconstrained estimates gave a many wrong signs and implausible values. That is not necessarily bad behavior for a system. It simply shows that the form gives the fitting program lots of room for finding values which fit. But we have to look for nonsense coefficients and put in soft constraints to guide the computer to sensible equations. We might find, for example, that an item had a large negative income elasticity but a large positive time trend, an unlikely combination. The first step in the development of the soft constraints, therefore, was to indicate that 0 was about the right value of each time trend coefficients. In other words, I had a considerable preference for explaining most developments with income and prices instead of with time trends. The estimation with just those constraints still produced some negative income elasticities. While these may be appropriate for a few commodities such as "fuel oil and coal," it seemed inappropriate for most products. Soft constraints were added to bring income elasticities just into the positive range for those products which had negative elasticities. In all, 26 of categories had such constraints. Of these, 19 were food or transportation categories. At this point, many own price elasticities were positive, and constraints had to be put on about two-thirds of the λ's. Finally, where necessary, constraints were added to avoid values on the change in income which would make consumption temporarily drop if income were increased. (Such drops were allowed on Automobile repair and a few similar sectors, because the rise in income could lead to purchases of new cars which reduce the need for repairs to old ones.) The final soft constraints are shown in Table 6 and the final results appear in Table 5.

Of the 93 categories, only 11 had standard error of the misses in excess of ten percent of the 1988 value. The largest error, 27%, was in Fuel oil and coal, while three sectors of public transportation had errors of about 19%. In several of these, it appears that time trends have changed over the period. If so, the problem sectors can cured by using special "trend" variables for each of them, a proceedure already provided for in the program. The various components of the private transportation group came out as fairly strongly complimentary. The components of the public transportation group were also compliments, but less strongly than in the private transportation group. The various foods were rather weakly substitutes.

On balance, the success stories considerably outnumber the remaining problems. Thus, it again appears that fairly satisfactory fits can be obtained with reasonable parameter values. So perhaps we do have a adequate form.

Table 6. Soft Constraint File for USA Equations

#		Income	DelIncome	Time	Lambda	mu	nu
1	Meat	.1 2.	-.05 1.	0 0.	.2 1.		
2	Dairy products	.1 1.	-.05 1.	0 0.	.2 1.		
3	Poultry and eggs	.1 1.	-.05 1.	0 1.	.2 1.		
4	Fresh fruit and vegetabl	.1 1.	-.05 0.	0 .5	.2 1.	.2 4.	
5	Processed fruit and vege	.1 1	.1 1.	0 1.	.2 1.		
6	Cereal and bakery produc	.1 1	-.05 1.	0 1.			
7	Fats & oils	.1 1	.0 1.	0 .5	.2 1.		
8	Sugar & sweets	.1 1	.0 1.	0 .5			
9	Nonalcoholic beverages	0 0	.0 0.	0 2.	.2 1.		
10	Other prepared food	0 0	.0 0.	1. 1.			
11	Fish & seafood	0 0	-.5 1.	0. 4.	.2 1.		
12	Pet food	.2 1.	-.3 1.	0 4.	.2 1.		
13	Wine and distilled spiri	.3 1.	.0 0.	0 1.	.2 1.		
14	Beer & ale	.2 1.	.0 0.	0 1.			
15	Food in purchased meals,	0 0	.0 1.	0 1.	.2 1.		
16	Alcohol in purchased mea	.1 1.	.0 0.	0 .5			
17	Tobacco	0 1.	.0 1.	0 0.			
18	Footwear	0 0	-.5 1.	0 1.	.2 1.		
19	Clothing, Women’s & girl	0 0	-.8 0.	0 1.			
20	Clothing, Men’s & boys’	0 0	-.4 1.	0 1.	.2 1.		
21	Luggage	0 0	-.4 1	0 1.	.2 1.		
22	Cleaning, laundering, &	0 0	-.5 0.	0 4.	.2 1.		
23	Jewelry & watches	0 0	.0 0.	0 1.			
24	Watch repair; misc perso	0 0	.0 0.	0 1.	.2 1.		
25	Toilet articles & preps	0 0	-.5 1.	0 1.	.2 1.		
26	Barbershops, beauty, & h	0 0	-.1 1.	0 1.			
27	Owner occupied nonfrm sp	0 0	.0 0.	0 1.	.2 0.	.2 1.	
28	Tenant occupied nonfrm s	0 0	.0 0	0 1.	.2 1.		
29	Hotels & motels	0 0	-.6 1.	0 1.			
30	Other housing	.1 1.	.0 1.	0 .5	.2 1.		
31	Furniture	0 0	-.4 1.	0 1.	.2 1.	.2 1.	
32	Kitchen & hh appliances	0 0	-.1 2.	0 1.	.2 1.		
33	China & glaswr, tablwr &	0 0	-.6 1.	0 1.			
34	Floor coverings and text	0 0	-.7 1.	0 2.	.2 1.		
35	Clocks, lamps, & artwork	0 0	-.4 1.	0 1.	.2 2.		
36	Hand tools	0 0	-.8 1.	0 1.	.2 1.		
37	Semi-durable house furni	0 0	.0 0.	0 1.	.2 1.		
38	Cleaning, lighting, pape	0 0	-.4 1.	0 1.			
39	Stationery, writing supp	0 0	-.7 1.	0 1.	.2 1.		
40	Electricity	0 0	-.5 1.	0 1.	.2 1.		
41	Natural gas	.1 1.	.0 1.	0 .5			
42	Water & sanitary service	0 0	-.6 1	0 1.	.2 1.		
43	Fuel oil & coal	.1 1.	.0 1.	0 .5			
44	Telephone & telegraph	0 0	-.8 1.	0 1.			
45	Domestic services	.7 1.	-.3 1.	0 0.			
46	HH insurance premiums	0 0	.0 0.	0 1.	.2 1.		
47	Other household operatio	0 0	-.5 1.	0 1.	.2 1.		
48	Drug preparations and su	0 0	-.7 1.	0 1.	.2 0.	.0 1.	
49	Opthalmic & orthopedic e	0 0	-1.0 1.	0 1.	.2 1.		
50	Physicians	0 0	-.8 0.	0 1.	.2 1.		
51	Dentists	0 0	.0 0.	0 1.			
52	Other professional medic	0 0	-1.5 1.	0 1.	.2 2.		
53	Hospitals	0 0	-.6 0.	0 1.	.2 1.		
54	Nursing homes	0 0	.0 0.	0 1.	.2 1.		
55	Health insurance premium	0 0	-.5 1.	0 1.			
56	Brokerage & invest couns	0 0	.0 0.	0 1.			
57	Bank service charges	0 0	.0 0.	0 1.	.2 1.		
58	Imputed service charges	0 0	.0 0.	0 1.			
59	Expense of handlng life	0 0	.0 0.	0 1.			
60	Legal services	0 0	-.5 1.	0 1.	.2 1.		
61	Funeral & burial expense	.1 0	-.2 2.	0 0.	.2 1.		
62	Other personal business	.1 0	-.4 1.	0 .5	.2 1.		
63	New autos	0 0	.0 0.	0 1.	.2 1.	.2 1.	.1 1.
64	Net purchases of used au	0 0	-.5 1.	0 1.			
65	Other motor vehicles	0 0	.0 0.	0 1.	.2 1.		
66	Tires & tubes, accessori	0 0	.0 0.	0 1.			
67	Auto repair, rental, lea	0 0	.0 0.	0 1.			
68	Gasoline & oil	.3 1.	-.1 1.	0 0.			
69	Tolls	.3 1.	-.2 1.	0 .5	.2 2		
70	Net auto insurance premi	.3 1.	-.2 1.	0 .5			
71	Mass transit	.3 1.	-.2 1.	0 0.	.2 0.	.2 0.	.2 1.
72	Taxicab	.3 1.	-.2 1.	0 0.			
73	Intercity rail	.3 1.	-.2 1.	0 0.			
74	Intercity bus	.1 1.	.0 1.	0 1.			

Table 6. Soft Constraint File for USA Equations (cont.)

#		Income	DelIncome	Time	Lambda	mu	nu
75	Airline	0 0	.0 0.	0 1.			
76	other transportation ser	0 0	.0 0.	0 1.			
77	Books & maps, Magazines	0 0	-.2 1.	0 1.	.2 1.		
78	Toys, dolls, & games	0 0	-.8 1.	0 1.	.2 1.		
79	Bicycles, motorcycles	.3 1	-.1 1.	0 .1	.2 1.		
80	Cameras, film, and proce	0 0	-.5 0.	0 1.	.2 1.		
81	Guns, ammunition, sporti	0 0	-.8 1.	0 1.	.2 1.		
82	Electronic entertainment	0 0	-.4 1.	0 1.			
83	Home computers	0 0	.0 0.	0 1.	.2 1.		
84	Flowers seeds & potted p	0 0	-.7 1.	0 1.	.2 1.		
85	Live entertainment, spor	0 0	-.5 1.	0 1.	.2 2.		
86	Clubs & fraternal organi	0 0	.0 0.	0 1.			
87	Participant amusements a	0 0	-.7 1.	0 1.	.2 1.		
88	Other recreation	0 0	.0 0.	0 1.	.2 1.		
89	Higher education	0 0	.0 0.	0 1.	.2 1.		
90	Private lower education	0 0	-.2 1.	0 1.			
91	Other education & resear	0 0	-.8 1.	0 1.			
92	Religious & welfare	0 0	-.9 1.	0 1.			
93	Foreign travel, and purc	0 0	-.4 1.	0 1.			

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Appendix A. Use of the estimation program

The estimation program has two control input matrices, groups.ttl and softcon.dat, and several data matrices, consum.dat, prices.dat, cstar.dat, popul.dat, and time.dat.

The groups.ttl file, as the name suggests, defines the groups. It also specifies which categories are sensitive and which insensitive to price, which weighted population, which income variable, and which trend variable is to be used by each category. This file for the Spanish study is shown in the box below. Its first column consist of simply the integers from 1 to n, the number of

```
The Groups.ttl File

# Groups.ttl.  Columns are
# 1 The consumption category number
#    2 The group number
#       3 The subgroup number
#          4  Which weighted population number to be used with this category
#             5 Which Income (Cstar) variable
#               6 Which Trend variable
#                 7 Use price terms ( 1 = yes, 0 = no)
#                   8 The title of the category
 1  1  0  1  1  1 1 Pan y cereales
 2  1  1  1  1  1 1 Carne
 3  1  1  1  1  1 1 Pescado
 4  1  1  1  1  1 1 Leche, queso y huevos
 5  1  0  1  1  1 1 Aceites y grasas
 6  1  0  1  1  1 1 Frutas y verduras
 7  1  0  1  1  1 1 Patatas y tubérculos
 8  1  0  1  1  1 1 Azúcar
 9  2  0  1  1  1 1 Café, té y cacao
10  1  0  1  1  1 1 Otros alimentos
11  2  0  1  1  1 1 Bebidas no alcohólicas
12  2  0  1  1  1 1 Bebidas alcohólicas
13  0  0  1  1  1 1 Tabacos
14  3  0  1  1  1 1 Vestido
15  3  0  1  1  1 1 Calzado
16  0  0  1  1  1 1 Alquileres y agua
17  0  0  1  1  1 1 Calefacción y alumbrado
18  4  0  1  1  1 1 Muebles
19  4  0  1  1  1 1 Artículos textiles
20  4  0  1  1  1 1 Electrodomésticos
21  4  0  1  1  1 1 Utensilios domésticos
22  0  0  1  1  1 1 Mantenimiento
23  0  0  1  1  1 1 Servicio doméstico
24  0  0  1  1  1 0 Medicamentos
25  0  0  1  1  1 0 Aparatos terapéuticos
26  0  0  1  1  1 0 Servicios médicos
27  0  0  1  1  1 0 Atención hospitalaria
28  0  0  1  1  1 1 Seguro médico privado
29  5  2  1  1  1 1 Compra de vehículos
30  5  2  1  1  1 1 Gasto de uso de vehículos
31  5  0  1  1  1 1 Servicios de transporte
32  0  0  1  1  1 1 Comunicaciones
33  6  0  1  1  1 1 Artículos de esparcimiento
34  6  0  1  1  1 1 Servicios de esparcimiento
35  0  0  1  1  1 1 Libros, periódicos y revistas
36  0  0  1  1  1 1 Enseñanza
37  0  0  1  1  1 1 Cuidados y efectos personales
38  0  0  1  1  1 1 Otros artículos n.c.o.p.
39  0  0  1  1  1 1 Restaurantes cafés y hoteles
```

categories of consumption. The second column carries the number of the group in which the category falls, or a zero if it is not assigned to a group, and the third column carries the number of the subgroup to which the category belongs or a zero if it belongs to none. The fourth is the number of the weighted population to be used for the item, the fifth is the number of the "income" (or Cstar) series to be used, the sixth is the number of the "trend" series to be used, and the seventh is a 1 if the category is a regular, price-sensitive commodity or a 0 if it is not. Although conceptually we have thought of neatly defined groups and subgroups strictly within the groups, the computer program makes no effort to enforce this tidy structure. It is possible to form "subgroups" with categories drawn from more than one group.The second major control file is softcon.dat, which gives soft constraints for the various equations. Since this file for Spain has been shown and fully explained in the text, it need not detain us here.

The consum.dat file begins with some dimensions and dates and then contains the data on consumption in almost exactly the form in which it would be written by the G command matty. The layout is shown in the above box for the Spanish case; the ... show where material has been cut out of the file to make it fit on the page. Notice the four numbers with which it begins. Each should be on its own line. Then come the data, with 20 series at a time across the "page". Comments may be introduced in the data by beginning the line with a #.

The Consum.dat File for Spain

```
42 Sectors
24 years of data
1971 First year
1986 Base year
# Consumption, constant 1986 prices, total (not percapita)
#   Date        kcpi1      kcpi2      kcpi3      kcpi4 ...   kcpi20
#   70.000    499.340   1021.118    549.566    467.982 ...  164.161
    71.000    490.641   1022.712    571.055    466.638 ...  175.657
    72.000    506.050   1017.367    574.642    472.211 ...  201.513
    73.000    548.905   1182.404    582.436    498.514 ...  210.624
    74.000    554.821   1338.080    536.756    562.671 ...  202.528
     ...         ...        ...        ...        ...         ...
    94.000    599.644   1666.945    606.117    735.611 ...  289.384
#  Date        kcpi21     kcpi22     kcpi23     kcpi24 ...   kcpi40
#   70.000    109.692    286.789    184.407    272.749 ...   68.330
    71.000    117.373    299.460    188.615    313.366 ...   74.745
    72.000    134.650    342.775    189.809    337.034 ...   79.912
    73.000    140.738    351.768    187.654    340.828 ...   85.539
    74.000    135.328    350.958    180.607    370.052 ...   86.795
     ...         ...        ...        ...        ...         ...
    94.000    121.689    412.728    183.031    780.092 ...  165.796
# Date        kcpi41     kcpi42
    70.000     34.073    298.342
    71.000     38.251    334.738
    72.000     45.404    395.237
    73.000     48.923    425.776
    74.000     55.430    479.674
     ...         ...        ...
    94.000     48.288    934.576

The ... indicate where data have been removed to fit the into this box.
```

Exactly the same format is followed for the prices.dat file, which give the price indexes, except that the four numbers at the top are omitted. The Cstar.dat, which gives the income series, begins with the number of such series. It then has these series arranged in columns. It has one extra year of data at the beginning so that the first difference of income can be calculated. The Popul.dat file is very similar; it begins with an integer giving the number of populations, followed by data in the same format. It also has the extra year at the beginning. Finally the tempi.dat file gives various series which may be used as the time trend. Like the popul.dat file, it has the number of series at the beginning but does not have the extra year of data at the beginning.

Once the files groups.ttl, consum.dat, prices.dat, cstar.dat, tempi.dat, and softcon.dat are ready, the program is run by the command "symcon [n]" from the DOS prompt. The optional parameter, n, is the number of iterations to be run before turning over control to the user. Thus "symcon" will run only 1 iteration and then give the user the option of quiting (by tapping y) or continuing the Marquardt process another iteration. If the command given is "symcon 40", then 40 iterations are automatically run without pausing for user input. In this case, when the limit is reached, the program sounds three long notes: low, high, low. A symcon calculation started in this way can be put into the background of a multitasking operating system such as OS2. When it has reached the limit, the notes will sound, and the user can turn his attention to it. To check that data has been read correctly, use "symcon d". (The d is for "debug".)